## Exercises Advanced Quantum Field Theory TAE - Madrid 2012 R. Casalbuoni

For most of the exercises see R. Casalbuoni, Lecture notes in Advanced Quantum Field Theory, http://theory.fi.infn.it/casalbuoni/lezioni99.pdf Lecture 1

1) Evaluate the Feynman amplitude in the free case and prove that it is given by:

$$\langle q',t'|q,t\rangle = \left[\frac{m}{2\pi i(t'-t)}\right]^{\frac{1}{2}} e^{iS_{cl}}$$

where

$$S_{\rm cl} = \frac{m}{2} \frac{(q'-q)^2}{(t'-t)}$$

is the classical action for the free particle (see lecture 1).

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2)The wave function is determined in this formalism as the ampitude for going to a point starting from a fixed one  $\psi(q',t') = \langle q',t' | 0,0 \rangle$ 

In the free case evaluate the wave function in momentum space and show that 2

$$\Psi(p,t) = e^{-iEt}, \quad E = \frac{p}{2m}$$

3) Show that the state  $|q,t\rangle$ 

such that (the operator is in the Heisenberg representation)

$$q(t) | q,t \rangle = q | q,t \rangle$$

is given by (q is in the Schrodinger representation)

$$|q,t\rangle = e^{iHt} |q\rangle, \quad q |q\rangle = q |q\rangle$$

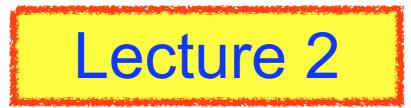
4) Prove the Feynman formula in configuration space given in page 13.

5) Evaluate the functional derivatives of the functionals defined in the lectures

$$F_{1}[\eta] = \int_{-\infty}^{+\infty} dx w(x) \eta(x) \qquad F_{2}[\eta] = e^{-\frac{1}{2} \int dx \eta^{2}(x)}$$

$$F_{3}[\eta] = \eta(x_{0})$$
and of  $F_{4}[\eta] = e^{-\frac{1}{2} \int dx dy K(x,y) \eta(x) \eta(y)}$ 

6) If 
$$S[q] = \int_{t}^{t'} L(q,\dot{q})dt$$
 with  $\delta q(t) = \delta q(t') = 0$   
prove that  $\frac{\delta S}{\delta q(t)} = \frac{\partial L}{\partial q(t)} - \frac{d}{dt} \frac{L}{\partial \dot{q}(t)}$ 



1) Assuming that the set of particles  $\beta = \gamma, p'$ prove the following equation using the procedure outlined in the main text (see lecture 2, pages 13-14)

$$\langle \beta; \operatorname{out} | \alpha, p; \operatorname{in} \rangle = \langle \beta - p; \operatorname{out} | \alpha; \operatorname{in} \rangle +$$
$$+ \frac{i}{\sqrt{Z}} \int d^4 x f_{\vec{p}}(x) (\Box + m^2)_x \langle \gamma; \operatorname{out} | \phi(x) | \alpha - p'; \operatorname{in} \rangle +$$
$$\frac{i}{\sqrt{Z}} \int^2 \int d^4 x \, d^4 y \, f_{\vec{p}}(x) f_{\vec{p}'}^*(y) (\Box + m^2)_x (\Box + m^2)_y \langle \gamma; \operatorname{out} | T(\phi(y)\phi(x)) | \alpha; \operatorname{in} \rangle$$

2) For a free field, evaluate the momentum operator and show that

$$[P_{\mu},\varphi(x)] = -i\partial_{\mu}\varphi(x), \quad [P_{\mu},a(k)] = -k_{\mu}a(k)$$

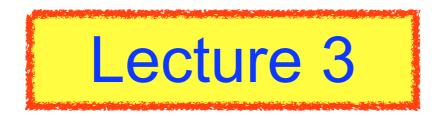
where a(k) is the annihilation operator

3) Evaluate the expression for the action in normal coorfinates given at page 20 in Lecture 2.

4) Evaluate  $G_0^{(4)}$  in the free case, see lecture 2, page 24.

5) Show that W[J] (page 26, lecture 2) generates the connected Green functions (see page 127 of Casalbuoni's lecture notes in the bibliography)

6) Evaluate  $\delta_1$  given in page 30, lecture 2



1) Derive thee expression for the normalization factor  $N(\theta, \theta^*)$  given in Lecture 3, page 7.

2) Prove the translational invariance of the Grassmann integral, ;ecture 3, page 10.

3) Show that a linear invertible change of the generators does not change the algebraic relations defining a Grassmann algebra (see lecture 3, page 11)

4) Show that a real antisymmetric matrix A can be put in the canonical form given il lecture 3, page 14, through an orthogonal transformation.

5) Using the result of the previous exercise prove the following equation

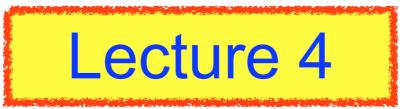
$$I = \int d\theta_n \cdots d\theta_1 e^{\theta^T A \theta/2} = \lambda_1 \cdots \lambda_{\frac{n}{2}} = \sqrt{\det A}$$

6) In a gauge theory consider the gauge invariant operator  $F[A^{\Omega}] = F[A]$ 

and its expectation value in a given gauge f.

$$\langle 0|T^*(F[\mathbf{A}])|0\rangle_f = \int D(A_\mu)\delta[f]\Delta_f e^{iS}F[A]$$

Show that this expression does not depend on the choice of the function f.

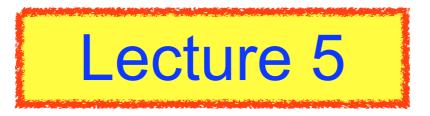


1) Derive the Feynman rules for a non abelian gauge theory, using the functional formalism (see lecture 4, pages 4 and following.

2) In the context of regularization of the scalar theory derive the expressions given in Lecture 4 (pages 23-24) for  $T_2$  and  $T_4$ .

3) Perform explicitly the valculations about the one-loop renormalization of the scalar theory discussed in Lecture 4, pages 30-35)

4) Perform the perturbative expansion at second order discussed Lecture 2 and derive the diagrammatic expansion given in Lecture 4, page 36 and the expressions of page 37.



1) Derive the expression for the 2 and 4 points Γ functions at given in Lecture 5, page 9, and determine the arbitrary functions F and G in the renormalization schemes A and B in Lecture 5, page 10.

2) In the framework of QED derive the expression for the bare electric charge given in lecture 5, page 26 (assume the validity of the Ward identity  $Z_1 = Z_2$ .

3) Repeat the same exercise as in 2) for QCD, that is evaluate  $g_B$ .

4) Check that the Ward identity derived, at first order, in lecture 5, page 35 can be derived from the equal time commutation relation

$$[j_0^A(\vec{x},t),\varphi_i(\vec{y},t)] = -\delta^3(\vec{x}-\vec{y})(T^A)_{ij}\varphi_j(\vec{x},t)$$