Electroweak Symmetry Breaking

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Madrid, July/2012

1. Lecture 1: The Higgs boson in the Standard Model

2. Lecture 2: Standard Model Higgs boson searches

3. Lecture 3: Some avenues beyond Standard Model Higgs

4. Lecture 4: The Higgs sector of the MSSM

The Higgs boson in the Standard Model

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1. The gauge symmetry of the electroweak interactions. Why Higgs?.

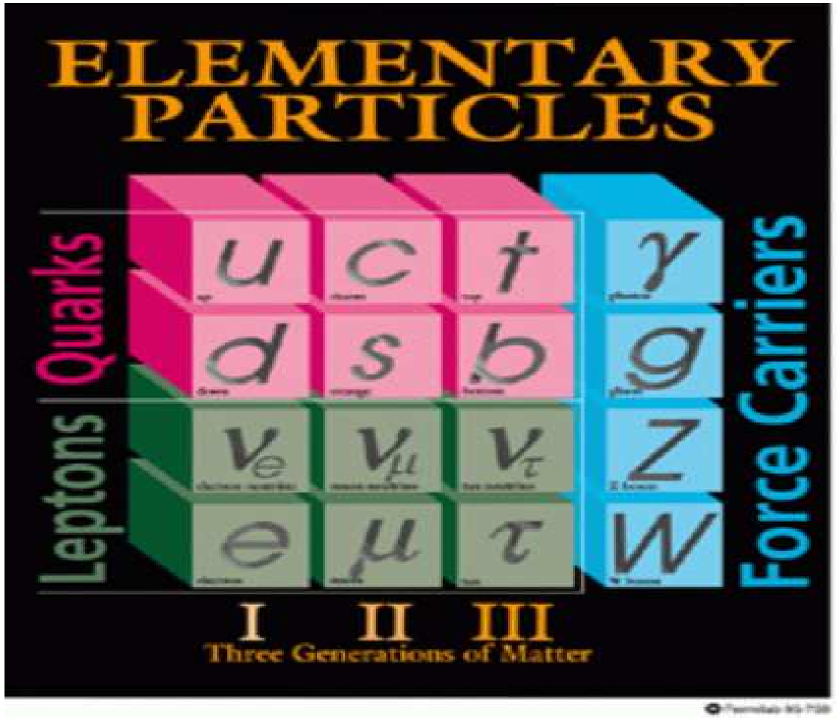
2. Spontaneous symmetry breaking, the Goldstone Theorem

3. Electroweak symmetry breaking, the Higgs Mechanism

4. Properties of the Higgs particle in the Standard Model

Current status of knowledge: the Standard Model (SM)

The SM describes with unprecedent precision (0.1%) the properties of all known elementary particles, Leptons and Quarks, and their fundamental interactions, electromagnetic, strong and weak. Gravity is not included in SM.



⇒ all particles experimentally seen

⇒ the carriers of electromagnetic (photon) and strong interactions (gluons) are massless gauge bosons . But the carriers of weak interactions, $W^\pm$ and Z, are massive: $M_W^{\rm exp} = 80.385 \pm 0.015$ GeV, $M_Z^{\rm exp} = 91.1876 \pm 0.0021$ GeV

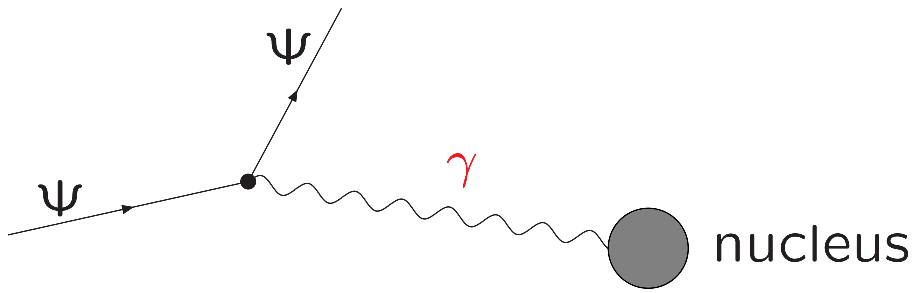
The problem: how to reconcile gauge invariance and massive gauge bosons

SM: Quantum field theory $\Rightarrow$ interaction: exchange of field quanta

Construction principle of the SM: gauge invariance

Example: Quantum electro-dynamics (QED)

Fermion Ψ with electric charge Q (in units of e, the electron charge).
Exchanged field: photon A_μ (γ in the figure)



$\mathcal{L}_{\text{QED}}$ invariant under gauge transformation:
$\Psi \to e^{i\,e\,Q\,\theta(x)}\Psi$ (spacetime dep. rot. in internal space), $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\theta(x)$

mass term for photon: $m^2 A^\mu A_\mu$ is not gauge invariant (E)
$\Rightarrow$ photon is massless: In agreement with data! .
What about W and Z electroweak gauge bosons?
Why are they massive? How do they get their masses?

The building of a gauge theory:

The gauge principle:

In order to get a Lagrangian that is invariant under local (gauge) transformations, massless gauge fields A_μ must be introduced with specific interactions with matter. The concrete prescription is provided by the covariant derivative. Number of gauge bosons = Number of symmetries= Number of generators of the symmetry group.

Steps: 1) Start with the Lagragian for propagating fermion fields without interactions,i.e., for free fields. 2) Replace the usual derivative by the covariant derivative. 3) Add the proper invariant kinetic terms for the gauge fields, such that they can propagate.

QED as an example:

$\mathcal{L}_{\text{free}} = \bar{\Psi}(i\partial\!\!\!/ - m)\Psi$, $\partial\!\!\!/ \equiv \partial_\mu \gamma^\mu$, γ^μ= Dirac matrices
The corresponding eq. of motion for Ψ is Dirac equation: $(i\partial\!\!\!/ - m)\Psi = 0$
$\partial_\mu \Psi \to D_\mu \Psi \equiv (\partial_\mu - ieQA_\mu)\Psi$; $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$\Rightarrow \mathcal{L}_{\text{QED}} = \bar{\Psi}(iD\!\!\!\!/ - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ invariant $\Psi \to e^{ieQ\theta}\Psi$, $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\theta$
Q = generator of $U(1)_{\text{em}}$ group; A_μ gauge field $\equiv$ photon

The gauge symmetry group of electroweak interactions

Gauge group: $SU(2)_L \times U(1)_Y$: 4 generators

$SU(2)_L$ weak isospin group. Non abelian. 3 generators $T_{1,2,3} = \frac{\sigma_{1,2,3}}{2}$

$U(1)_Y$ weak hypercharge group. Abelian. 1 generator $\frac{Y}{2}$

$U(1)_{\text{em}} \subset SU(2)_L \times U(1)_Y$; $Q = T_3 + \frac{Y}{2}$

Quarks and Leptons transform as:

1) Under $SU(2)_L$: $\Psi_L \to e^{i\frac{\vec{\sigma}}{2}\vec{\theta}(x)}\Psi_L$, doublets ; $\Psi_R \to \Psi_R$, singlets

2) Under $U(1)_Y$: $\Psi \to e^{i\frac{Y}{2}\beta(x)}\Psi$

Lepton	T	T_3	Q	Y
ν_L	$\frac{1}{2}$	$\frac{1}{2}$	0	-1
e_L	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-1
e_R	0	0	-1	-2

Quark	T	T_3	Q	Y
u_L	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$
d_L	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$
u_R	0	0	$\frac{2}{3}$	$\frac{4}{3}$
d_R	0	0	$-\frac{1}{3}$	$-\frac{2}{3}$

The Electroweak Theory II:

The particle content and gauge interactions

Matter

1^{st} family: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$, e_R^-, $\begin{pmatrix} u \\ d \end{pmatrix}_L$, u_R, d_R

2^{nd} family: $\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$, μ_R^-, $\begin{pmatrix} c \\ s \end{pmatrix}_L$, c_R, s_R

3^{rd} family: $\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$, τ_R^-, $\begin{pmatrix} t \\ b \end{pmatrix}_L$, t_R, b_R

Gauge

$SU(2)_L$: 3 generators T_i, 3 gauge bosons W_i^μ

$U(1)_Y$: 1 generator $\frac{Y}{2}$, 1 gauge boson B^μ

$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon^{ijk} W^j_\mu W^k_\nu$

$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

Physical EW bosons

$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu)$

$Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu$

$A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu$

Introduce interactions by: $\partial_\mu \Psi \to D_\mu \Psi = (\partial_\mu - ig\vec{T}.\vec{W}_\mu - ig'\frac{Y}{2}B_\mu)\Psi$

$g = SU(2)_L$ gauge coupling; $g' = U(1)_Y$ gauge coupling

$U(1)_{\text{em}} \subset SU(2)_L \times U(1)_Y \Rightarrow g = \frac{e}{\sin\theta_W}, g' = \frac{e}{\cos\theta_W}$

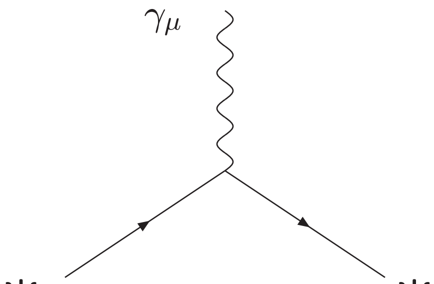
$\mathcal{L}_{\text{EW}} = \sum_\Psi i\overline{\Psi}\gamma^\mu D_\mu \Psi - \frac{1}{4} W^i_{\mu\nu} W_i^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$; $\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{EW}} + \mathcal{L}_{\text{EWSB}}$

Interactions OK in $\mathcal{L}_{\text{EW}}$, but fermions and gauge bosons massless yet

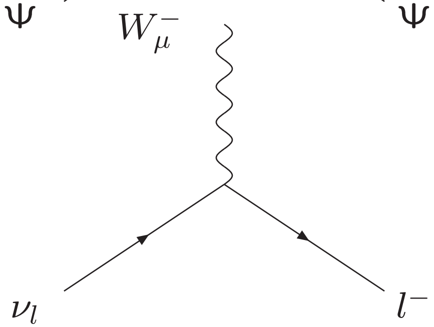
$m\overline{\Psi}\Psi = m(\overline{\Psi}_L \Psi_R + \overline{\Psi}_R \Psi_L)$; $M_W^2 W_\mu W^\mu$, Not gauge invariant (E) $\Rightarrow$ need to find $\mathcal{L}_{\text{EWSB}}$

The Electroweak Theory: Feynman Rules I

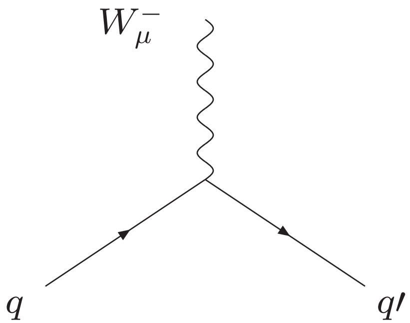
Interactions in $\mathcal{L}_{\rm EW}$



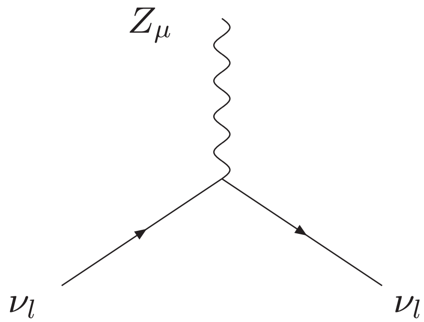
$$ieQ_\Psi \gamma_\mu$$



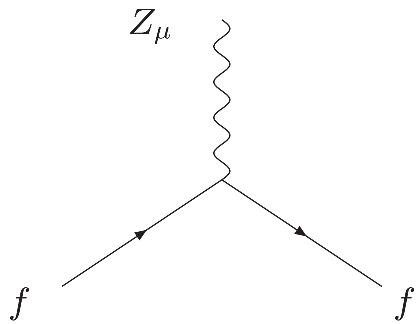
$$i\frac{g}{2\sqrt{2}}\gamma_\mu\,(1-\gamma_5)$$



$$i\frac{g}{2\sqrt{2}}\gamma_\mu\,(1-\gamma_5)\,U_{qq\prime}$$



$$i\frac{g}{2c_W}\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)$$

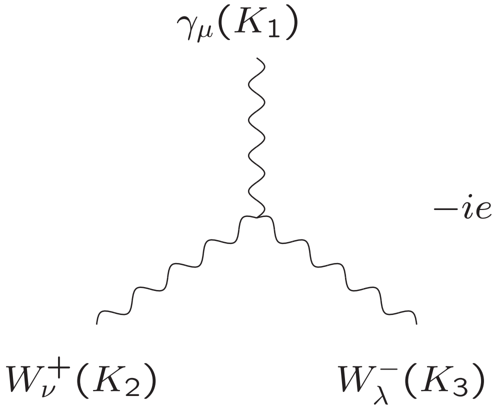


$$i\frac{g}{c_W}\gamma_\mu\left[g_L^f\left(\frac{1-\gamma_5}{2}\right)+g_R^f\left(\frac{1+\gamma_5}{2}\right)\right]$$

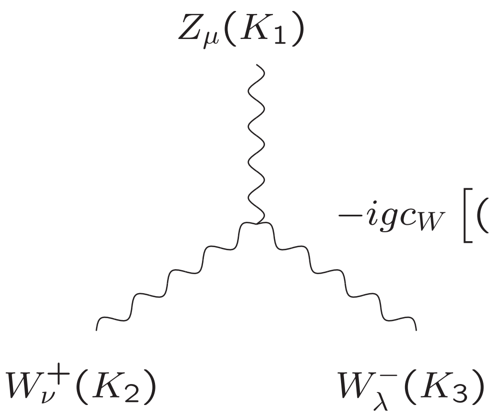
where, $g_L^f = T_3^{f_L} - Q^f s_W^2$, $g_R^f = T_3^{f_R} - Q^f s_W^2$, $s_W = \sin\theta_W$, $c_W = \cos\theta_W$

$U_{qq'}$ is the qq' element of CKM matrix. For $qq' = ud, cs, tb$, $U_{qq'} \sim 1$. Others: $qq' = us, ub, cd, cb, td, ts$, $U_{qq'} << 1$

Interactions in $\mathcal{L}_{\mathrm{EW}}$

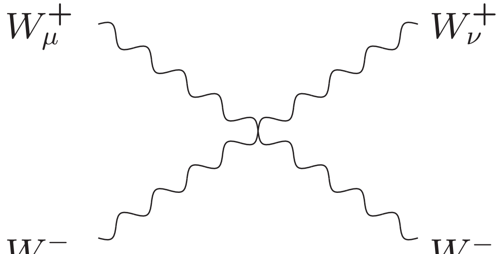


$$-ie\left[(K_1-K_2)_\lambda\, g_{\mu\nu} + (K_2-K_3)_\mu\, g_{\nu\lambda} + (K_3-K_1)_\nu\, g_{\lambda\mu}\right]$$

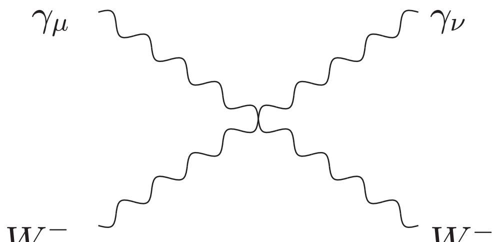


$$-igc_W\left[(K_1-K_2)_\lambda\, g_{\mu\nu} + (K_2-K_3)_\mu\, g_{\nu\lambda} + (K_3-K_1)_\nu\, g_{\lambda\mu}\right]$$

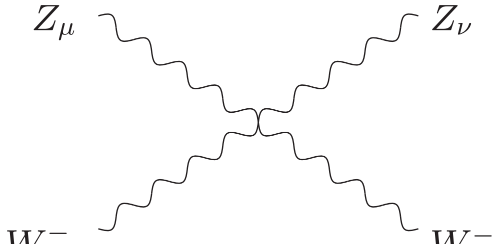
Interactions in $\mathcal{L}_{\text{EW}}$



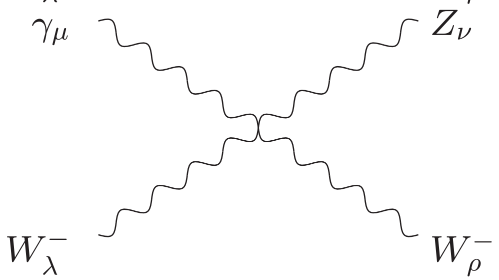
$ig^2 S_{\mu\nu,\lambda\rho}$



$-ie^2 S_{\mu\nu,\lambda\rho}$



$-ig^2 c_W^2 S_{\mu\nu,\lambda\rho}$



$-ig^2 s_W c_W S_{\mu\nu,\lambda\rho}$

where, $S_{\mu\nu,\lambda\rho} = 2g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}$

Electroweak Symmetry Breaking: the building of $\mathcal{L}_{\text{EWSB}}$

1. The Phenomenom of Spontaneous Symmetry Breaking

2. Spontaneous Symmetry Breaking: the Goldstone Theorem

3. Electroweak Symmetry Breaking: the Higgs Mechanism

A simple definition:

A physical system has a symmetry that is spontaneously broken if the interactions governing the dynamics of the system possess such a symmetry but the ground state of this system does not.

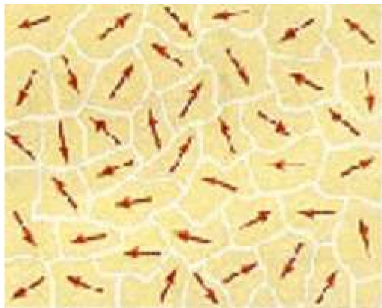
A simple example:

Infinitely extended ferromagnet at T close to Curie temperature T_C

$\Rightarrow$ System described by infinite elementary spins. Interactions rotational invariant.

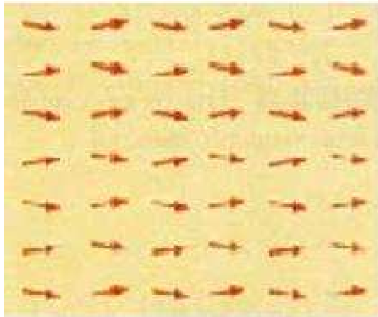
$\Rightarrow$ Ground state presents two different situations depending on T:

Situation I: $T > T_C$



spins randomly oriented
ground state rotationaly invariant
Average Magnetization (order parameter)
$\vec{M}_{\text{average}} = 0$

Situation II: $T < T_C$



spins oriented to some particular (arbitrary) direction
ground state is not rotationaly invariant
$\vec{M}_{\text{average}} \neq 0$ (*Spontaneous Magnetization*)
$\exists$ infinite possible ground states, but system chooses one.

The Theory of Ginzburg-Landau (1950)

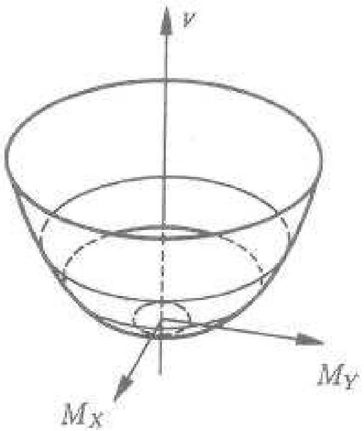
For T near T_C, the magnetization $\vec{M}$ is small and the free energy density $u(\vec{M})$ is:

$$\begin{aligned} u(\vec{M}) &= (\partial_i \vec{M})(\partial_i \vec{M}) + V(\vec{M}) \; ; \; i = 1,2,3 \; ; \text{ here } \vec{M} = (M_X, M_Y) \text{ to simplify} \\ V(\vec{M}) &= \alpha_1 (T - T_C)(\vec{M}.\vec{M}) + \alpha_2 (\vec{M}.\vec{M})^2 \; ; \; \alpha_1, \alpha_2 > 0 \end{aligned}$$

The magnetization of the ground state is obtained from the condition of extremum:

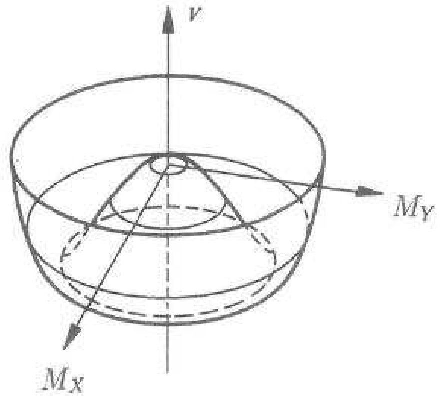
$$\frac{\delta V(\vec{M})}{\delta M_i} = 0 \Rightarrow \vec{M}. \left[\alpha_1 (T - T_C) + 2\alpha_2 (\vec{M}.\vec{M}) \right] = 0 \;\; \Rightarrow \;\; \text{two solutions}$$

Situation I: $T > T_C$, Symmetric phase



Unique minimum at $\vec{M} = 0$ and $V(0) = 0$

Situation II: $T < T_C$, Non symmetric phase



$\vec{M} = 0$ is a local maximum
infinite degenerate minima all having same $|\vec{M}|$
$\alpha_1 (T - T_C) + 2\alpha_2 (\vec{M}.\vec{M}) = 0 \Rightarrow |\vec{M}| = \sqrt{\frac{\alpha_1 (T_C - T)}{2\alpha_2}}$
The choice of a particular minimum (direction) is what generates the spontaneous breaking.

Goldstone Theorem (Nambu,Goldstone,1960-1962)

Goldstone Theorem applies to Quantum Field Theories (QFT) with Spontaneous Symmetry Breaking (SSB).

SSB stated in simple words:

In QFT, a system is said to possess a symmetry that is spontaneously broken if the Lagrangian describing the dynamics of the system is invariant under this symmetry transformation, but the vacuum of the theory is not. The vacuum $|0>$ *is the state where the Hamiltonian expectation value* $<0|H|0>$ *is minimum.*

Goldtone Theorem stated in simple words:

If a QFT has a global symmetry of the Lagrangian which is not a symmetry of the vacuum $\Rightarrow$ *there must exist one massless boson, scalar or pseudoscalar, associated to each generator which does not annihilate the vacuum and having its same quantum numbers. These modes are referred to as Nambu-Goldstone bosons or simply as Goldstone bosons.*

Notice that:

$U|0>=|0>$ with $U=\exp(i\epsilon^a Q^a) \Rightarrow Q^a|0>=0 \;\forall\; a$

and:

$U|0>\neq|0>$ with $U=\exp(i\epsilon^a Q^a) \Rightarrow \exists\; Q^a \;/\; Q^a|0>\neq 0$

QCD as an example (I)

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2} Tr G^{\mu\nu} G_{\mu\nu} + \sum_{u,d} (i\bar{q}\gamma^\mu D_\mu q - m_q \bar{q} q)$$

where,

$$\begin{aligned} G_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - ig_s \left[A_\mu, A_\nu\right] \\ D_\mu q &= (\partial_\mu - ig_s A_\mu) q \\ A_\mu &= \sum_{a=1}^{8} \frac{1}{2} A_\mu^a \lambda_a \end{aligned}$$

Besides the $SU(3)_C$ gauge symmetry, and for $m_{u,d} = 0$, $\mathcal{L}_{\text{QCD}}$ has a global symmetry:

$$SU(2)_L \times SU(2)_R \equiv \text{Chiral Symmetry}$$

defined by:

$$\Psi_L \to \Psi'_L = U_L \Psi_L = \exp(i\alpha_L^a Q_L^a)\Psi_L \; ; \; Q_L^{1,2,3} \text{ generators of } SU(2)_L$$

$$\Psi_R \to \Psi'_R = U_R \Psi_R = \exp(i\alpha_R^a Q_R^a)\Psi_R \; ; \; Q_R^{1,2,3} \text{ generators of } SU(2)_R$$

where,

$$\Psi = \begin{pmatrix} u \\ d \end{pmatrix} \; ; \; \Psi_L = \frac{1}{2}(1 - \gamma_5)\Psi \; ; \; \Psi_R = \frac{1}{2}(1 + \gamma_5)\Psi$$

$m_{u,d} \neq 0$ breaks explicitly the chiral symmetry, but not much since the masses are small. Chiral symmetry is not exact but it is a very good approximate symmetry of QCD.

QCD as an example (II)

In QCD, chiral symmetry is spontaneously broken down to isospin symmetry:

$SU(2)_L \times SU(2)_R = SU(2)_V \times SU(2)_A \to SU(2)_V$; $SU(2)_V = SU(2)_{R+L}$; $SU(2)_A = SU(2)_{R-L}$

$\mathcal{L}_{\rm QCD}$ invariant under $SU(2)_L \times SU(2)_R$ but QCD vacuum is NOT.

The QCD vacuum is only invariant under the subgroup $SU(2)_V \subset SU(2)_L \times SU(2)_R$

But, how do we know from experiment that the QCD vacuum is not $SU(2)_L \times SU(2)_R$ symmetric?

If $|0>$ is chiral invariant $\Rightarrow$

$$U_L|0>=|0> \; ; \; U_R|0>=|0> \Rightarrow Q^a_L|0>=0 \; ; \; Q^a_R|0>=0$$

If $|\Psi>$ is an eigenstate of the Hamiltonian and parity operator such that:

$$H|\Psi>=E|\Psi> \; ; \; P|\Psi>=|\Psi>$$

then,

$$\exists|\Psi'>=\frac{1}{\sqrt{2}}(Q^a_R - Q^a_L)|\Psi> \; / \; H|\Psi'>=E|\Psi'> \; ; \; P|\Psi'>=-|\Psi'>$$

But, no such parity doublets in the hadronic spectrum $\Rightarrow$ $SU(2)_A$ is NOT a symmetry of the vacuum, or equivalently, $Q^a_A|0>\neq 0 (a=1,2,3)$. $\Rightarrow$ chiral symmetry must be spontaneously broken to the reduced symmetry of the vacuum, $SU(2)_V$.

QCD as an example (III)

According to Goldstone Theorem:

If a Theory has a global symmetry of the Lagrangian which is not a symmetry of the vacuum then there must exist one massless boson, scalar or pseudoscalar, associated to each generator which does not annihilate the vacuum and having its same quantum numbers. These modes are referred to as Nambu-Goldstone bosons or simply as Goldstone bosons.

The spontaneous breaking of the chiral symmetry in QCD:

$$SU(2)_L \times SU(2)_R \to SU(2)_V \;\; ; \;\; \text{with, } Q_A^a|0> \neq 0 (a = 1, 2, 3)$$

$\Rightarrow \;\; \exists$ 3 massless GBs, pseudoscalars, $\pi^a(x) \;\; a = 1, 2, 3.$

They are identified with the physical pions . More specificaly, their combinations: π^+, π^- and π^0.

Since, in Nature, $m_\pi \neq 0 \Rightarrow$ chiral symmetry is explicitely broken, and the pions are pseudo-GB. But the hierarchy $m_\pi << m_{\text{hadrons}}$ is explained .

The dynamics of pion interactions is well described by the so-called Chiral Lagrangians .

More in next lectures

The Higgs Mechanism: I

The Goldstone Theorem is for theories with spontaneously broken global symmetries but does not hold for gauge theories. When a spontaneous symmetry breaking takes place in a gauge theory the so-called Higgs Mechanism operates:

Many authors involved: Higgs 1964; Englert, Brout 1964; Guralnik, Hagen, Kibble 1964. Inspired in previous works within Solid State Physics: Anderson 1963. See also Schwinger 1962 where the generation of mass for gauge fields was already mentioned. See also BCS Theory of Superconductivity, Cooper pairs and absence of massless GBs in presence of electromagnetic interactions by Nambu 1960.

How to generate mass for gauge bosons in gauge theories:

The would-be Goldstone bosons associated to the global symmetry breaking do not manifest explicitely in the physical spectrum but instead they 'combine' with the massless gauge bosons and as result, once the spectrum of the theory is built up on the non-symmetric vacuum, there appear massive vector particles. The number of vector bosons that acquire a mass is precisely equal to the number of these would-be-Goldstone bosons.

The Higgs Mechanism: II

An illustrative example: $U(1)$ gauge symmetry breaking: I

Consider $U(1)$ gauge theory, with one complex scalar $\Phi = \frac{1}{\sqrt{2}}(\Phi_1 + i\Phi_2)$, one gauge boson A_μ, and a potential of Ginzburg-Landau type:

$$\mathcal{L} = (D_\mu\Phi)^\dagger(D^\mu\Phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\Phi)$$

$$D_\mu\Phi = (\partial_\mu - igA_\mu)\Phi \ ; \ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$V(\Phi) = \mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 \ ; \ \lambda > 0$$

$\mathcal{L}$ is invariant under $U(1)$ gauge transformations:

$$\Phi \to e^{-i\alpha(x)}\Phi \ ; \ D_\mu\Phi \to e^{-i\alpha(x)}D_\mu\Phi \ ; \ e^{-i\alpha(x)} \subset U(1)$$

$$A_\mu \to A_\mu - \frac{1}{g}\partial_\mu\alpha(x)$$

Compare $V(\Phi)$ with the previous ferromagnet case:

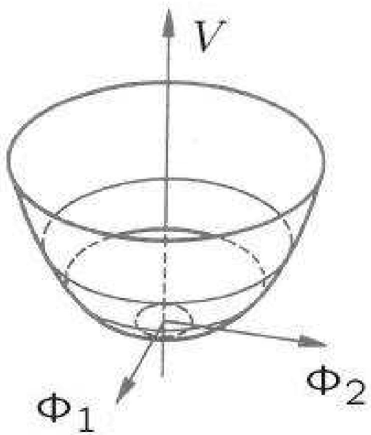
$$V(\vec{M}) = \alpha_1(T - T_C)(\vec{M}.\vec{M}) + \alpha_2(\vec{M}.\vec{M})^2 \ ; \ \alpha_1, \alpha_2 > 0$$

All said applies with the replacements: $(M_X, M_Y) \to \frac{1}{\sqrt{2}}(\Phi_1 + i\Phi_2)$

$$\alpha_1(T - T_C) \to \mu^2; \ \alpha_2 \to \lambda; \ \vec{M}_{\text{ground state}} \to < 0|\Phi|0 > \equiv < \Phi >$$

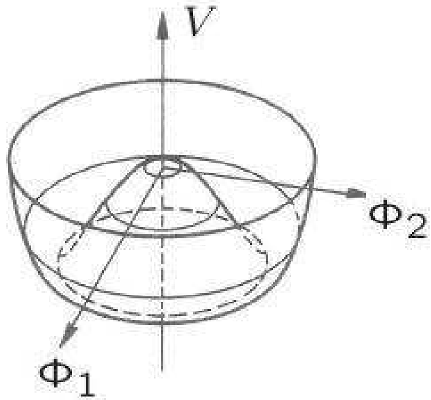
An illustrative example: $U(1)$ gauge symmetry breaking: II

Situation I: $\mu^2 > 0$, Symmetric phase



Unique vacuum (minimum) at $< \Phi >= 0$
and $V(\Phi) = 0$ at $< \Phi >= 0$
The vacuum IS invariant under $U(1)$

Situation II: $\mu^2 < 0$, Non symmetric phase



$< \Phi >= 0$ is a local maximum
infinite degenerate vacua (minima) all having same $| < \Phi > |$ but different complex phases:
$| < \Phi > | = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}} \neq 0$; arg $< \Phi >$ arbitrary
A particular vacuum IS NOT invariant under U(1)

The choice of a particular vacuum (complex phase) is what generates the spontaneous breaking of $U(1)$

Building the spectra on top of this non-invariant vacuum (minimum) is what generates the gauge boson mass (see next)

An illustrative example: $U(1)$ gauge symmetry breaking: III

Building the spectra on top of a particular non-invariant vacuum (minimum) is what generates the gauge boson mass

For instance, let us choose

$|<\Phi>| = \sqrt{\frac{-\mu^2}{2\lambda}} \neq 0$; arg $<\Phi>= 0 \Rightarrow <\Phi_1>= \sqrt{\frac{-\mu^2}{\lambda}} = v$, $<\Phi_2>= 0$

Then, we change coordinates to new fields ($\equiv$ shifting the origen):

$\Phi_1' \equiv \Phi_1 - v$; $\Phi_2' \equiv \Phi_2$ such that $<\Phi_1'>= 0$; $<\Phi_2'>= 0$

Next, write everything in terms of these new $\Phi_{1,2}'$ fields:

$$(D_\mu\Phi)^\dagger(D_\mu\Phi) = \left((\partial_\mu + igA_\mu)\frac{1}{\sqrt{2}}(\Phi_1 - i\Phi_2)\right)\left((\partial_\mu - igA_\mu)\frac{1}{\sqrt{2}}(\Phi_1 + i\Phi_2)\right) = ...$$

$$\frac{1}{2}(\partial_\mu\Phi_1' + gA_\mu\Phi_2')^2 + \frac{1}{2}(\partial_\mu\Phi_2' - gA_\mu\Phi_1')^2 - gvA^\mu(\partial_\mu\Phi_2' + gA_\mu\Phi_1') + \frac{1}{2}g^2v^2A_\mu A^\mu$$

A mass term for A_μ has appeared, but it is not the physical basis yet...
there is a (unphysical) mixing term $\sim gvA^\mu\partial_\mu\Phi_2'$!!!!!

An illustrative example: $U(1)$ gauge symmetry breaking: IV

First, choose the good coordinates

We want to remove the (unphysical) mixing term $\sim gvA^\mu \partial_\mu \Phi_2'$

Choose 'polar' coodinates to describe 'small oscillations' around vacuum configuration:
$$\Phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x)) e^{i\frac{\xi(x)}{v}}$$

Second, choose the proper gauge , i.e., make a gauge transf. to the unitary gauge (by fixing the gauge parameter to $\alpha(x) = \frac{\xi(x)}{v}$) where the unwanted mixing terms do not appear:

$$\Phi(x) \to e^{-i\frac{\xi(x)}{v}}\Phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x))$$
$$A_\mu(x) \to A_\mu(x) - \frac{1}{gv}\partial_\mu \xi(x) \equiv B_\mu(x)$$

In terms of the new fields: B_μ massive gauge , η massive scalar ($\equiv$ Higgs)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)^2 + \mu^2\eta^2 - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + \frac{1}{2}(gv)^2 B_\mu B^\mu + \qquad \text{(E)}$$
$$\frac{1}{2}g^2 B_\mu B^\mu \eta(2v + \eta) - \lambda v \eta^3 - \frac{1}{4}\lambda\eta^4 \quad ; M_{B_\mu} = gv \quad ; m_\eta = \sqrt{2}|\mu|$$

The 'nice' properties of the Higgs Mechanism:

⋆ The gauge symmetry of the interactions (i.e, of $\mathcal{L}$) is preserved

⋆ The renormalizability of the massless gauge theories is preserved

⋆ The total number of polarization degrees is preserved
For instance, in the previous $U(1)$ case:
Before SSB: total polarization degrees = 4 = (2 of A_μ)+(2 of Φ)
After SSB: total polarization degrees = 4 = (3 of B_μ)+(1 of η)

⋆ The unphysical fields (i.e. the would-be-GBs) have dissapeared from the spectrum. In the previous $U(1)$ case: $\xi(x)$

⋆ The n$^{\mathrm{er}}$ of gauge bosons getting a mass = n$^{\mathrm{er}}$ of would-be-GBs= n$^{\mathrm{er}}$ of symmetries of $\mathcal{L}$ that are not of vacuum. In the previous $U(1)$ case: 1.

⋆ "The would-be-GBs combine with the massless gauge bosons to give them a mass" means the mixing term $\sim gvA^\mu\partial_\mu\Phi_2'$

⋆ Notice that: The Higgs mechanism does not necessarily imply the existence of a Higgs particle. It appears JUST when required by the polarization degrees preservation property.

The Higgs Mechanism applied to the Standard Model:

We want to generate masses for 3 gauge fields: Z, W^+, W^-
But we want to keep the photon γ massless.

Strategy: Introduce (*ad hoc*) a new scalar field, Φ, and a potential of Ginzburg-Landau type, $V(\Phi)$ that make the job:

$\Rightarrow$ It must provide the 3 needed polarization degrees to play the role of the would-be-GBs

$\Rightarrow$ It must have non-zero $SU(2)_L \times U(1)_Y$ quantum numbers, such that the vacuum is not invariant under the complete symmetry, but just invariant under the subgroup $U(1)_{\rm em}$.

$\Rightarrow$ The field component in Φ acquiring a vev must be elect. neutral to preserve $U(1)_{\rm em}$

Scalar SU(2) doublet: $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

$$T(\Phi) = \frac{1}{2}\,,\; Y(\Phi) = 1$$

$\mu^2 > 0 : SU(2)_L \times U(1)_Y$

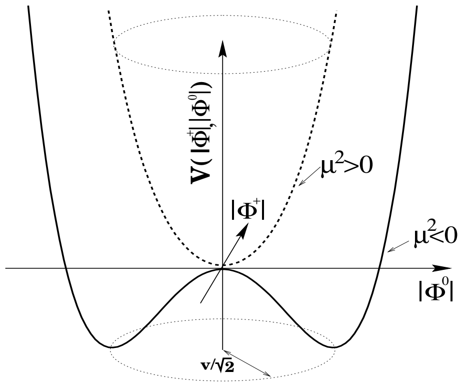
$\mu^2 < 0 : SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \quad \lambda > 0$$

$\mu^2 > 0$ unique minimum at $< 0|\Phi|0 >= 0$

$\mu^2 < 0$ infinite degenerate minima at:

$$| < 0|\Phi|0 > | = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \;;\; \text{arbitrary arg } \Phi \;;\; v \equiv \sqrt{\frac{\mu^2}{\lambda}}$$



The choice of a particular arg Φ produces the breaking

Getting the proper gauge boson and fermion masses: I

The building of $\mathcal{L}_{\text{EWSB}}$: including $V(\Phi)$, covariant derivatives of Φ and Yukawa interactions of Φ with fermions:

$$
\begin{aligned}
\mathcal{L}_{\text{EWSB}} &= \mathcal{L}_{\text{SBS}} + \mathcal{L}_{\text{YW}} \\
\mathcal{L}_{\text{SBS}} &= (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \\
V(\Phi) &= \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \\
\mathcal{L}_{\text{YW}} &= \lambda_e \bar{l}_L \Phi e_R + \lambda_u \bar{q} \widetilde{\Phi} u_R + \lambda_d \bar{q}_L \Phi d_R + h.c. + 2^{\text{nd}} \text{ and } 3^{\text{rd}} \text{ families}
\end{aligned}
$$

$$
\begin{aligned}
l_L &= \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \; ; \; q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \\
\Phi &= \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} \; ; \; \widetilde{\Phi} = i\tau_2 \Phi^* = \begin{pmatrix} \phi_0^* \\ -\phi^- \end{pmatrix} \\
D_\mu \Phi &= (\partial_\mu - \frac{1}{2} i g \vec{\tau} \cdot \vec{W}_\mu - \frac{1}{2} i g' B_\mu) \Phi
\end{aligned}
$$

$\mathcal{L}_{\text{EWSB}}$ is gauge $SU(2)_L \times U(1)_Y$ invariant (E)

Getting the proper gauge boson and fermion masses: II

Follow the steps:

1) Fix a particular non-symmetric vacuum. For instance:

$$< 0|\Phi|0 >= \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad ; \quad \arg\Phi = 0$$

2) Perform 'small oscillations' around this vacuum:

$$\Phi(x) = \exp\left(i\frac{\vec{\xi}(x)\vec{\tau}}{v}\right)\begin{pmatrix} 0 \\ \frac{v+H(x)}{\sqrt{2}} \end{pmatrix}$$

where $\vec{\xi}(x) = (\xi_1(x), \xi_2(x), \xi_3(x))$ and $H(x)$ are 'small' fields.

3) To eliminate the unphysical (would-be-GBs) fields $\vec{\xi}$ make the gauge transformation (unitary gauge):

$$\begin{aligned}
\Phi' &= U(\xi)\Phi = \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} \; ; \; U(\xi) = \exp\left(-i\frac{\vec{\xi}\vec{\tau}}{v}\right) \\
l'_L &= U(\xi)l_L \; ; \; e'_R = e_R \; ; \; q'_L = U(\xi)q_L \; ; \; u'_R = u_R \; ; \; d'_R = d_R \\
\left(\frac{\vec{\tau}\cdot\vec{W}'_\mu}{2}\right) &= U(\xi)\left(\frac{\vec{\tau}\cdot\vec{W}_\mu}{2}\right)U^{-1}(\xi) - \frac{i}{g}(\partial_\mu U(\xi))U^{-1}(\xi) \; ; \; B'_\mu = B_\mu
\end{aligned}$$

Getting the proper gauge boson and fermion masses: III

4) Rotate the weak eigenstates to the mass eigenstates:

$$
\begin{aligned}
W_\mu^\pm &= \frac{W_\mu'^1 \mp i W_\mu'^2}{\sqrt{2}} \; ; \; g = \frac{e}{\sin\theta_W} \; ; \; g' = \frac{e}{\cos\theta_W} \\
Z_\mu &= \cos\theta_W \, W_\mu'^3 - \sin\theta_W \, B'_\mu \\
A_\mu &= \sin\theta_W \, W_\mu'^3 + \cos\theta_W \, B'_\mu
\end{aligned}
$$

5) Read the (tree level) particle masses from proper terms in $\mathcal{L}_{\rm EWSB}$:

$$
\begin{aligned}
(D_\mu \Phi')^\dagger (D^\mu \Phi') &= \left(\frac{g^2 v^2}{4}\right) W_\mu^+ W^{\mu-} + \frac{1}{2}\left(\frac{(g^2+g'^2)v^2}{4}\right) Z_\mu Z^\mu + \ldots \quad \text{(E)} \\
V(\Phi') &= \mu^2 H^2 + \ldots \\
\mathcal{L}_{\rm YW} &= -\left(\lambda_e \frac{v}{\sqrt{2}}\right) \bar{e}'_L e'_R - \left(\lambda_u \frac{v}{\sqrt{2}}\right) \bar{u}'_L u'_R - \left(\lambda_d \frac{v}{\sqrt{2}}\right) \bar{d}'_L d'_R + h.c. + \ldots \\
M_W &= \frac{gv}{2} \; ; \; M_Z = \frac{\sqrt{g^2+g'^2}v}{2} \; ; \; M_H = \sqrt{2}|\mu| \; ; \; v = \sqrt{\frac{-\mu^2}{\lambda}} \\
m_e &= \lambda_e \frac{v}{\sqrt{2}} \; ; \; m_u = \lambda_u \frac{v}{\sqrt{2}} \; ; \; m_d = \lambda_d \frac{v}{\sqrt{2}} \; ; \ldots
\end{aligned}
$$

Counting bosonic degrees: before SSB 12 (4x2gauge+4scalar); after SSB 12 (3x3gauge+1x2gauge+1scalar)

The values of the tree level masses

The vacuum expectation value of the Φ field was known long time ago, indeed, before the discovery of $W^\pm$ and Z.

It was obtained from physical observables, well known from experiment:

For instance, from muon decay, $\mu^- \to \nu_\mu \bar{\nu}_e e^-$

The prediction in V-A Theory (Feynman, Gell-Mann 1958):

$$\frac{1}{\tau_\mu} = \Gamma(\mu^- \to \nu_\mu \bar{\nu}_e e^-) \simeq \frac{G_F^2 m_\mu^5}{192\pi^3}$$

Provides the correct muon life time:

$$\tau_\mu = 2.2 \times 10^{-6} s \quad , \quad \text{for } G_F = 1.167 \times 10^{-5} \text{ GeV}^{-2}$$

Within the SM the muon decay proceeds via an intermediate virtual W exchange: By matching the above Γ to the prediction in the SM:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2} \;\Rightarrow\; \boxed{v = 246 \text{ GeV}}$$

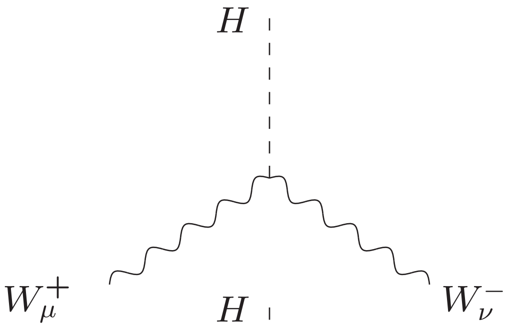
By using $\sin^2\theta_W \simeq 0.23$ from *e.g.* DIS data and $g = e/\sin\theta_W$

$\Rightarrow M_W^{\text{tree}} \simeq 78$ GeV, $M_Z^{\text{tree}} \simeq 89$ GeV... discovered at CERN in 1983 !!

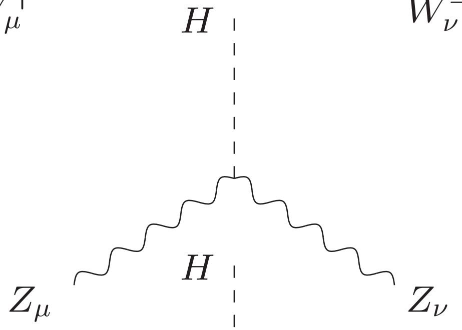
In contrast: exp.fermion masses $\Rightarrow$ Yukawa couplings

M_H and λ unpredicted in SM!!

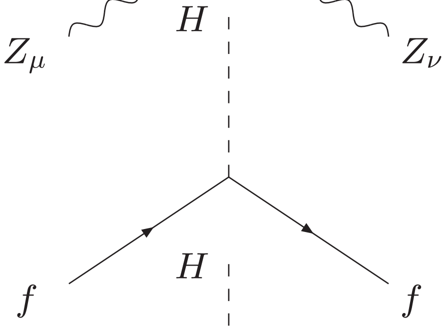
SM Higgs boson interactions (Unitary gauge)



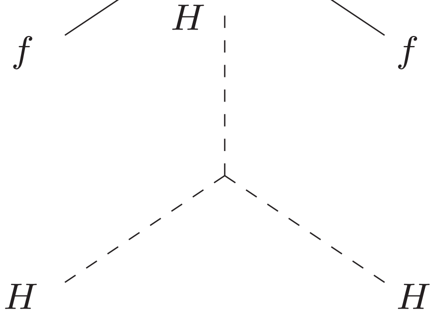
$igM_W g_{\mu\nu}$



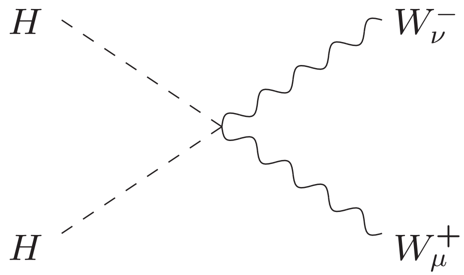
$i\frac{g}{c_W}M_Z g_{\mu\nu}$



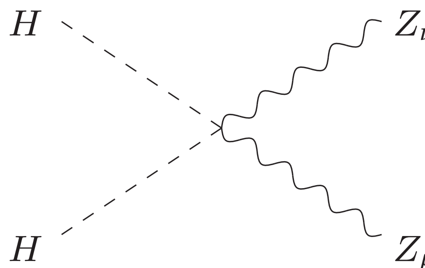
$-i\frac{g}{2}\frac{m_f}{M_W} = -i\frac{\lambda_f}{\sqrt{2}}$



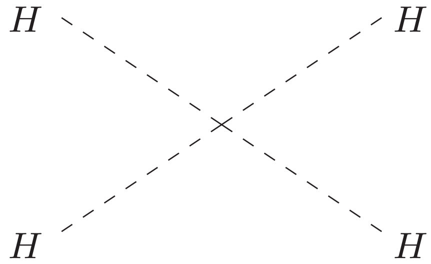
$-ig\frac{3}{2}\frac{M_H^2}{M_W} = -i6\lambda v$



$i\frac{g^2}{2}g_{\mu\nu}$



$i\frac{g^2}{2c_W^2}g_{\mu\nu}$



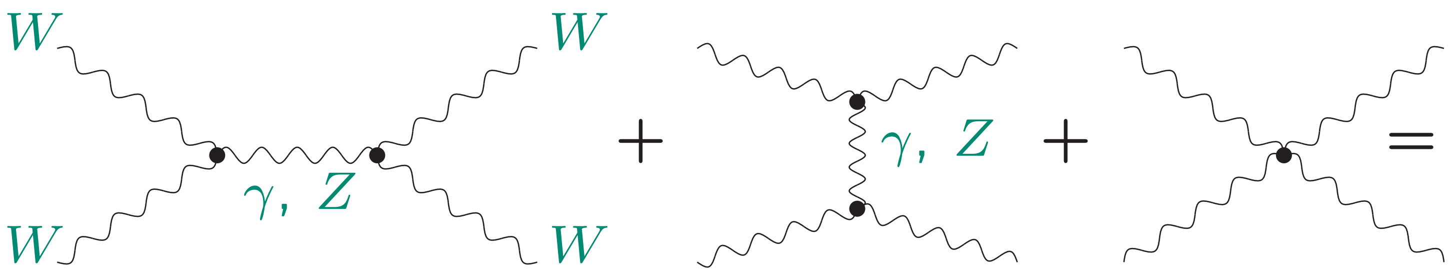
$-ig^2\frac{3}{4}\frac{M_H^2}{M_W^2} = -i6\lambda$

Higgs boson couplings to particle P larger for larger m_P !!!

Higgs boson role in scattering of longitudinal W bosons:

$W_L W_L \to W_L W_L$

$T_V =$

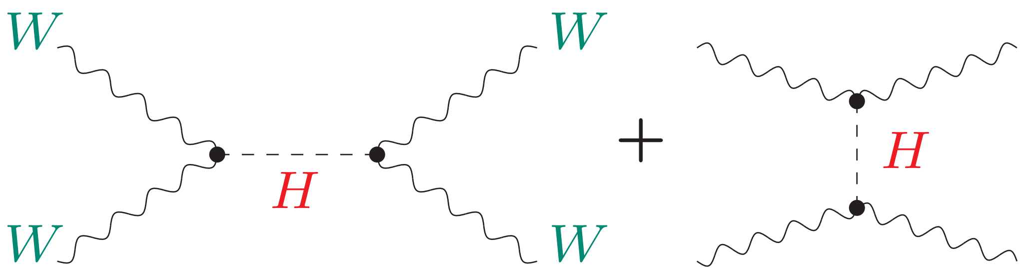


$= -g^2 \frac{E^2}{M_W^2} + \mathcal{O}(1)$ for $E \to \infty$

$\Rightarrow$ violation of unitarity

Contribution of a scalar particle with couplings prop. to the mass:

$T_S =$



$= g^2_{WWH} \frac{E^2}{M_W^4} + \mathcal{O}(1)$ for $E \to \infty$

$$T_{\text{tot}} = T_V + T_S = \frac{E^2}{M_W^4}\left(g^2_{WWH} - g^2\, M_W^2\right) + \ldots$$

$\Rightarrow$ compensation of terms with bad high-energy behavior for

$$g_{WWH} = g\, M_W$$

Compare WW scattering with would-be-GB scattering

An interesting comparison is provided by the so-called

Equivalence Theorem (Cornwall *et al* 1974, Lee *et al* 1977):

The scattering amplitudes of longitudinal gauge bosons V_L $(V = W^{\pm}, Z)$, *at high energies,* $\sqrt{s} >> M_V$, *are equivalent to the scattering amplitudes of their corresponding would-be Goldstnone bosons* w

$$|T(V_L^1 V_L^2 ... V_L^N \to V_L^1 V_L^2 ... V_L^{N'})| \approx |T(w_1 w_2 ... w_N \to w_1 w_2 ... w_{N'})|$$

(E)

Use the more general Feynman rules of R_ξ gauges to demostrate: 1)

$$T(W_L^+ W_L^- \to W_L^+ W_L^-) = T(w^+ w^- \to w^+ w^-) + O(\frac{M^2}{s}), \text{ for } \sqrt{s} >> M_W, M_Z$$

and, 2) for $M_H >> M_{W,Z}$:

$$\begin{aligned} \Gamma(H \to W_L^+ W_L^-) &= \Gamma(H \to w^+ w^-) + O(\frac{M_W}{M_H}) \\ \Gamma(H \to Z_L Z_L) &= \Gamma(H \to zz) + O(\frac{M_Z}{M_H}) \end{aligned}$$

Upper Higgs mass bound from unitarity

Study the behaviour of the complete scattering amplitude with M_H:

$$T(W_L^+W_L^- \to W_L^+W_L^-) = -\frac{1}{v^2}\{-s-t+\frac{s^2}{s-M_H^2}+\frac{t^2}{t-M_H^2}+2M_Z^2+ \frac{2M_Z^2 s}{t-M_Z^2}+\frac{2t}{s}(M_Z^2-4M_W^2)-\frac{8s_W^2 M_W^2 M_Z^2 s}{t(t-M_Z^2)}\}$$

Decompose T in partial waves a_J:

$$T(s,\cos\theta) = 16\pi \sum_{J=0}^{\infty}(2J+1)a_J(s)P_J(\cos\theta) \ , \ P_J = \text{Legendre polynomials}$$

Compute cross-section in terms of partial waves:

$$\sigma_{\text{tot}} \simeq \sigma_{\text{el}} = \frac{16\pi}{s}\sum_{J=0}^{\infty}(2J+1)|a_J(s)|^2$$

Require Optical Theorem (consequence of unitarity $T^\dagger T = TT^\dagger = 1$):

$$\sigma_{\text{tot}}(1+2 \to \text{anything}) = \frac{1}{s}\text{Im}\, T(s,\cos\theta=1)$$

In terms of partial waves:

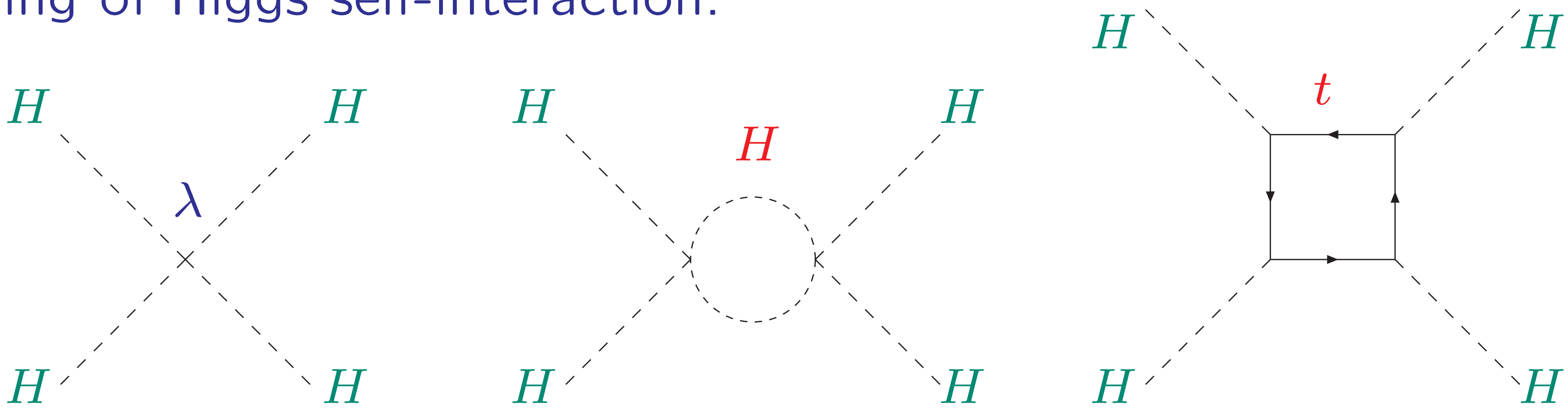
$$|a_J(s)|^2 = \text{Im}\, a_J(s) \ ; \ \forall J \ \Rightarrow \ |a_J|^2 \le 1 \ ; \ 0 \le \text{Im}\, a_J \le 1 \ ; \ |\text{Re}\, a_J| \le \frac{1}{2} \ ; \ \forall J$$

$$a_0(W_L^+W_L^- \to W_L^+W_L^-) = \frac{1}{32\pi}\int_{-1}^{1} T(s,\cos\theta)d(\cos\theta) \ \Rightarrow \ |a_0| \xrightarrow{s>>M_H^2,M_V^2} \frac{M_H^2}{8\pi v^2}$$

$|Re\ a_0| \le \frac{1}{2} \Rightarrow M_H < 860$ GeV (perturb. unitarity bound). Other channels improve this.

What else do we know about the Higgs boson?

Running of Higgs self-interaction:



Renormalization group equation:

$$\frac{d\lambda}{dt} = \frac{3}{16\,\pi^2}\left[4\lambda^2 + 2\lambda g_t^2 - g_t^4 + \frac{1}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right] \,, \qquad t = \log\left(\frac{Q^2}{v^2}\right)$$

Two conditions:

1) avoid landau pole/triviality problem (for large $\lambda \sim M_H^2$)

2) avoid vacuum instability problem (for small/negative λ)

Upper Higgs mass bound from triviality

Avoid Landau pole/triviality problem for large $\lambda \sim M_H^2$

$$\frac{d\,\lambda}{d\,t} = \frac{3}{4\,\pi^2}\left[\lambda^2\right]$$

$$\Rightarrow \qquad \lambda(Q) = \frac{\lambda_0}{1 - \frac{3\lambda_0}{2\,\pi^2}\log\left(\frac{Q}{\Lambda}\right)} \quad ; \quad \lambda_0 \equiv \lambda(\Lambda)$$

Taking the $\Lambda \to \infty$ limit, while fixing λ_0 to a finite value
$\Rightarrow \lambda(Q) \to 0$ (Triviality) $\Rightarrow$ require $\exists \Lambda_{\mathrm{phys}} < \infty$ such that $\lambda(Q) \neq 0 \Rightarrow$

$$M_H^2 = 2\lambda(v)v^2 \;\text{ with }\; \lambda(v) = \frac{\lambda_0}{1 - \frac{3}{2\pi^2}\lambda_0 \log(\frac{v}{\Lambda_{\mathrm{phys}}})}$$

Decreasing (increasing) $\Lambda_{\mathrm{phys}} \Rightarrow$ Increasing (decreasing) M_H and they may cross. This crossing point where $M_H(\Lambda_{\mathrm{phys}}) \simeq \Lambda_{\mathrm{phys}}$ is what gives the upper bound to M_H. It is a cut-off dependent bound.

Lower Higgs mass bound from vacuum stability

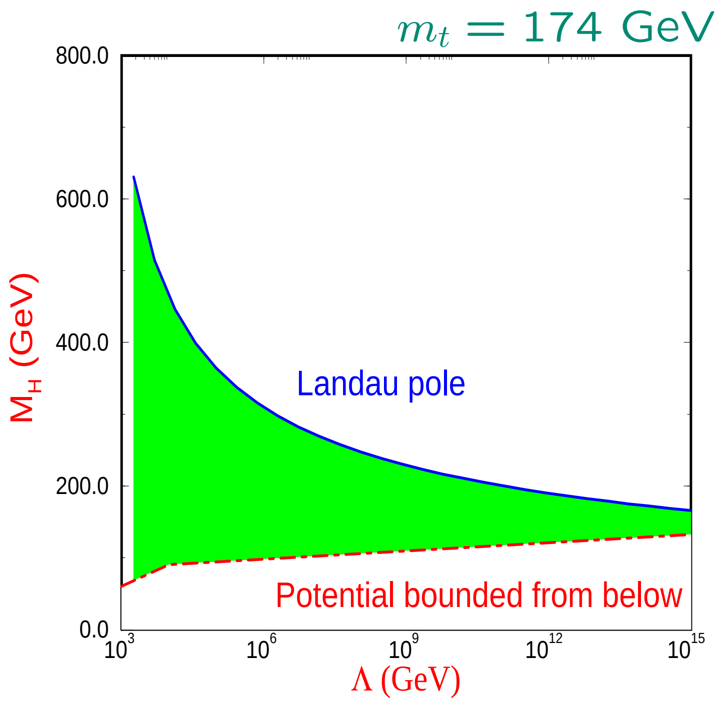
Avoid vacuum instability (for small/negative λ):
The minimum of the effective potential (including loop corrections) changes with $\lambda(Q)$ and, a too small or negative $\lambda(Q)$ may change the true vacuum: $V(v) < V(0)$ may change to $V(v) > V(0) \Rightarrow$ EWSB does NOT take place. It can lead to an effective potential that is not even bounded from below!!
By requiring $V(v) < V(0)$ one gets a lower bound on $\lambda(v)$ and therefore on M_H which is cut-off dependent
For instance, to one loop:

$$\frac{d\lambda}{dt} = \frac{3}{16\,\pi^2}\left[-g_t^4 + \frac{1}{16}\left(2g_2^4 + (g_2^2+g_1^2)^2\right)\right]$$

$$\Rightarrow \qquad \lambda(Q^2) = \lambda(v^2) + \frac{3}{16\,\pi^2}\left[-g_t^4 + \frac{1}{16}\left(2g_2^4 + (g_2^2+g_1^2)^2\right)\right]\log\left(\frac{Q^2}{v^2}\right)$$

$$\lambda(\Lambda) > 0 \Rightarrow M_H^2 > \frac{3v^2}{8\,\pi^2}\left[g_t^4 - \frac{1}{16}\left(2g_2^4 + (g_2^2+g_1^2)^2\right)\right]\log\left(\frac{\Lambda^2}{v^2}\right)$$

Both limits combined:



Λ: scale up to which the SM is valid

For instance, if $\Lambda = M_{\rm GUT} \Rightarrow 130$ GeV $\lesssim M_H \lesssim 180$ GeV

Recent computations of the stability lower bound include a NNLO analysis of the Higgs potential and realistic error estimates.
The condition for absolute stability up to the Planck scale is (Degrassi *et al* 2012):

$$M_H(\text{ GeV}) > 129.4 + 1.4\left(\frac{m_t(\text{ GeV} - 173.1}{0.7}\right) - 0.5\left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007}\right) \pm 1.0_{\rm th} \Rightarrow M_H > 129.4 \pm 1.8 \text{ GeV}$$

$\Rightarrow$ vacuum stability of the SM up to the Planck scale is excluded at $2\sigma(98\% CL)$ for $M_H < 126$ GeV !!!

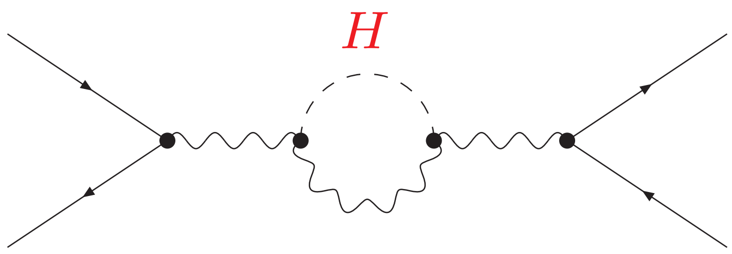
Upper Higgs mass bound from radiative corrections

Comparison of electro-weak precision observables (EWPO) with theory:

EW Precision data:		Theory:
$M_W, \sin^2\theta_{\text{eff}}, a_\mu$	$\leftrightarrow$	SM , . . .

$$\Downarrow$$

Test of theory at quantum level: Sensitivity to loop corrections, e.g. H



$$\Downarrow$$

SM: limits on M_H

Very high accuracy of measurements and theoretical predictions needed

Precision observables in the SM

M_W, $\sin^2\theta_{\text{eff}}$, M_h, $(g-2)_\mu$, b physics, ...

Theoretical prediction for M_W in terms

of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right) = \frac{\pi\,\alpha}{\sqrt{2}\,G_\mu}\left(\frac{1}{1-\Delta r}\right)$$

$\Updownarrow$

loop corrections

Evaluate Δr from μ decay $\Rightarrow M_W$

One-loop result for M_W in the SM:

[*A. Sirlin '80*] , [*W. Marciano, A. Sirlin '80*]

$$\begin{array}{ccccccc} \Delta r_{1-\text{loop}} = & \Delta\alpha & - & \frac{c_W^2}{s_W^2}\Delta\rho & + & \Delta r_{\text{rem}}(M_H) \\ & \sim \log\frac{M_Z}{m_f} & & \sim m_t^2 & & \log(M_H/M_W) \\ & \sim 6\% & & \sim 3.3\% & & \sim 1\% \end{array}$$

Comparison of SM prediction of M_W with data (without LHC)

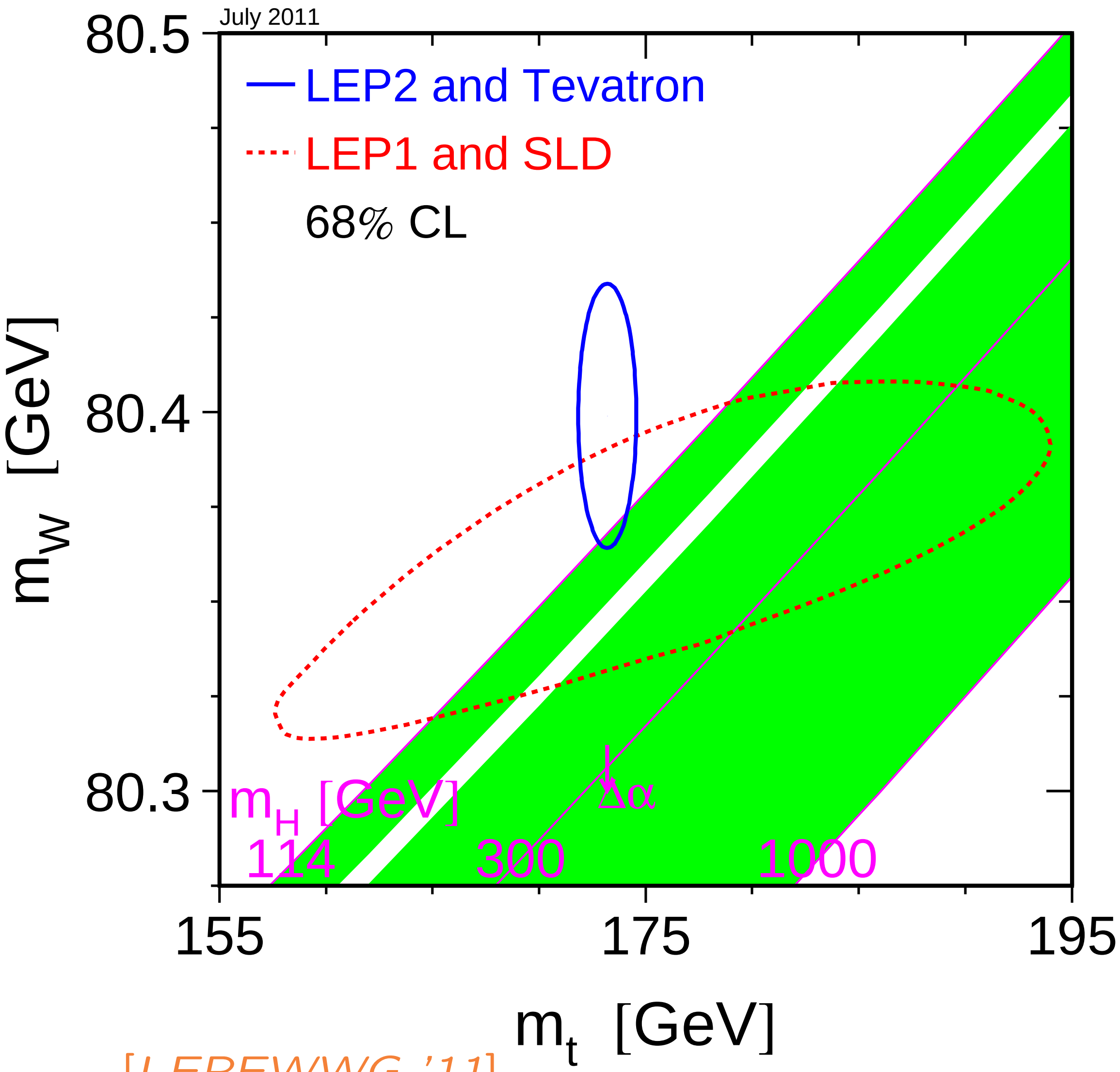
$$\Delta r = -\frac{11 g_2^2}{96\,\pi^2}\frac{s_W^2}{c_W^2} \log\left(\frac{M_H}{M_W}\right)$$

general for EWPO:

$$\Delta \sim g_2^2 \left[\log\left(\frac{M_H}{M_W}\right) + g_2^2 \frac{M_H^2}{M_W^2}\right]$$

leading term: $\log(M_H)$

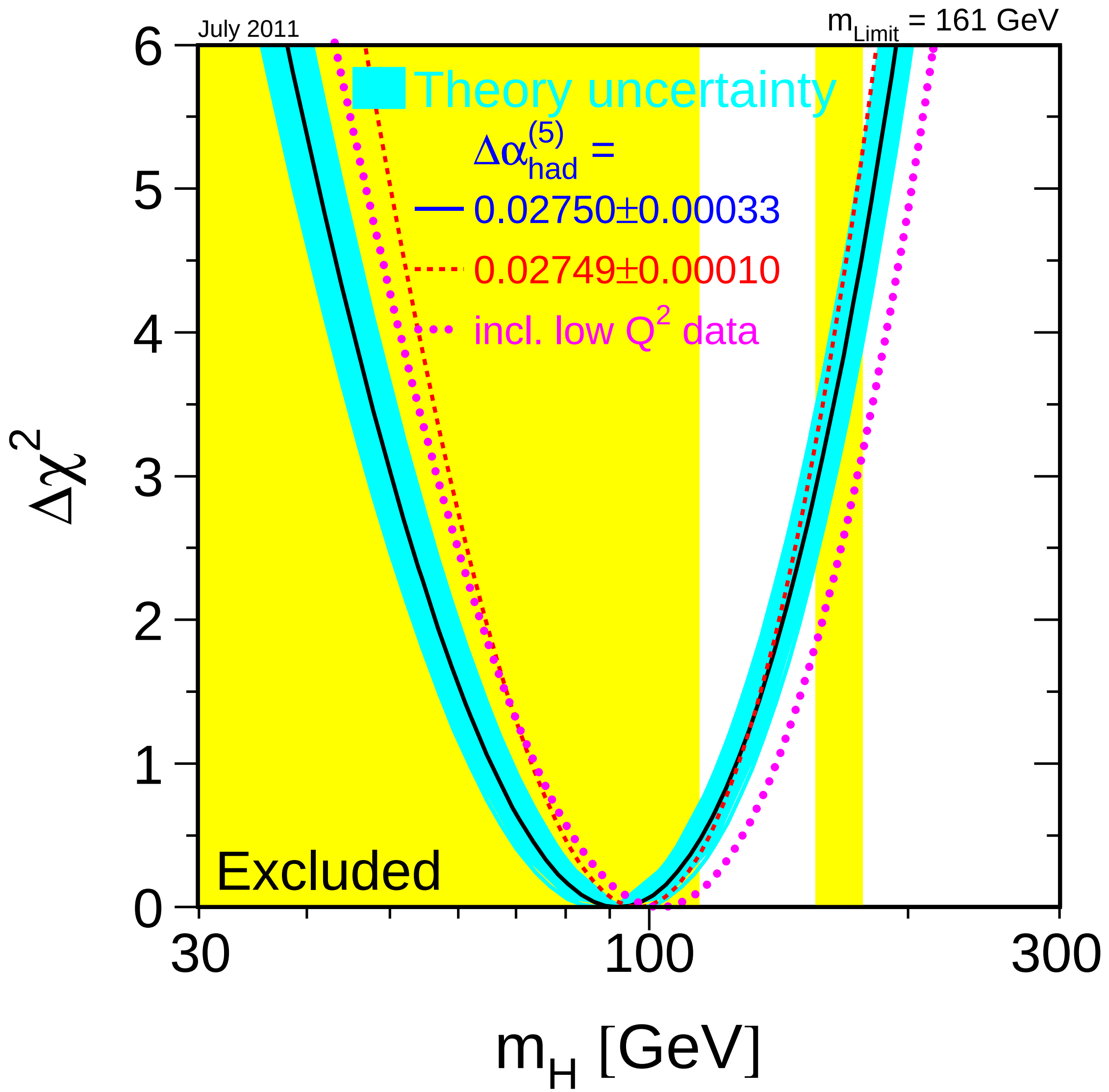
first term $\sim M_H^2$ with g_2^4



[LEPEWWG '11]

⇒ light Higgs boson preferred

Global fit to all SM data:



[*LEPEWWG '11*]

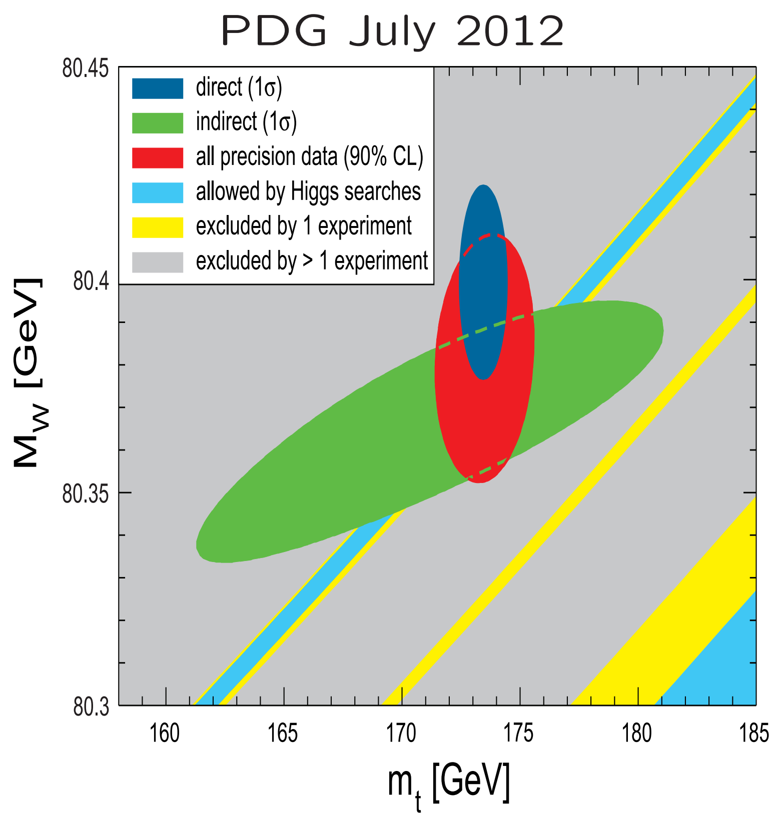
$\Rightarrow\ M_H = 92^{+34}_{-26}$ GeV

$M_H < 161$ GeV, 95% C.L.

Assumption for the fit:
SM incl. Higgs boson

$\Rightarrow$ Higgs boson seems to be light, $M_H \lesssim 160$ GeV

Comparison of SM prediction of M_W with data (including LHC)



Red area: allowed by all precision data at 90%CL

Light blue bands: SM prediction for M_W as a function of m_t, with M_H allowed by Higgs searches at LHC: a) Central band: 115.5 GeV $< M_H <$ 127 GeV, b) band at lower-right corner: $M_H >$ 600 GeV.

Properties of the SM Higgs boson

1.) Decay to fermions:

coupling:

$$g_{f\bar{f}H} = \left[\sqrt{2}\,G_\mu\right]^{1/2} m_f$$

decay width:

$$\Gamma(H \to f\bar{f}) = N_c \frac{G_\mu\, M_H}{4\sqrt{2}\,\pi} m_f^2(M_H^2) \left(1 - 4\frac{m_f^2}{M_H^2}\right)^{3/2} \qquad \text{(E)}$$

with N_c = number of colors

Bulk of QCD corrections for decays to quarks are mapped into

$$m_q^2(\text{pole}) \to m_q^2(M_H^2)$$

Dominant decay process: $H \to b\bar{b}$

2.) Decay to heavy gauge bosons ($V = W, Z$):

coupling:

$$g_{VVH} = 2\left[\sqrt{2}\,G_\mu\right]^{1/2} M_V^2$$

on-shell decay width ($M_H > 2M_V$):

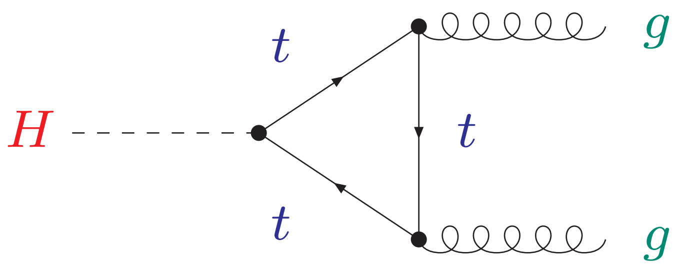
$$\Gamma(H \to VV) = \delta_V \frac{G_\mu M_H^3}{16\sqrt{2}\,\pi}\left(1 - 4\frac{M_V^2}{M_H^2} + 12\frac{M_V^4}{M_H^4}\right)\left(1 - 4\frac{M_V^2}{M_H^2}\right)^{1/2} \qquad \text{(E)}$$

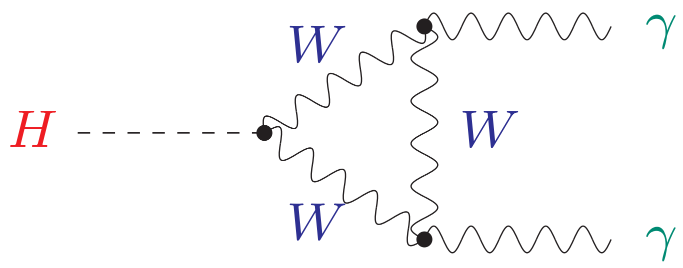
with $\delta_{W,Z} = 2, 1$

off-shell decay width ($M_H < 2M_V$):

$$\Gamma(H \to VV^*) = \delta'_V \frac{3G_\mu^2 M_H}{16\,\pi^3} M_V^4 \times \text{Integral}$$

3.) Decay to massless gauge bosons (gg, $\gamma\gamma$):





$$\Gamma(H \to gg) = \frac{G_\mu \, \alpha_s^2(M_H^2) \, M_H^3}{36\sqrt{2}\,\pi^3}\left[1 + C\frac{\alpha_s(\mu)}{\pi}\right]$$

via the top quark loop with

$$C = \frac{215}{12} - \frac{23}{6}\log\left(\frac{\mu^2}{M_H^2}\right) + \mathcal{O}(\alpha_s)$$

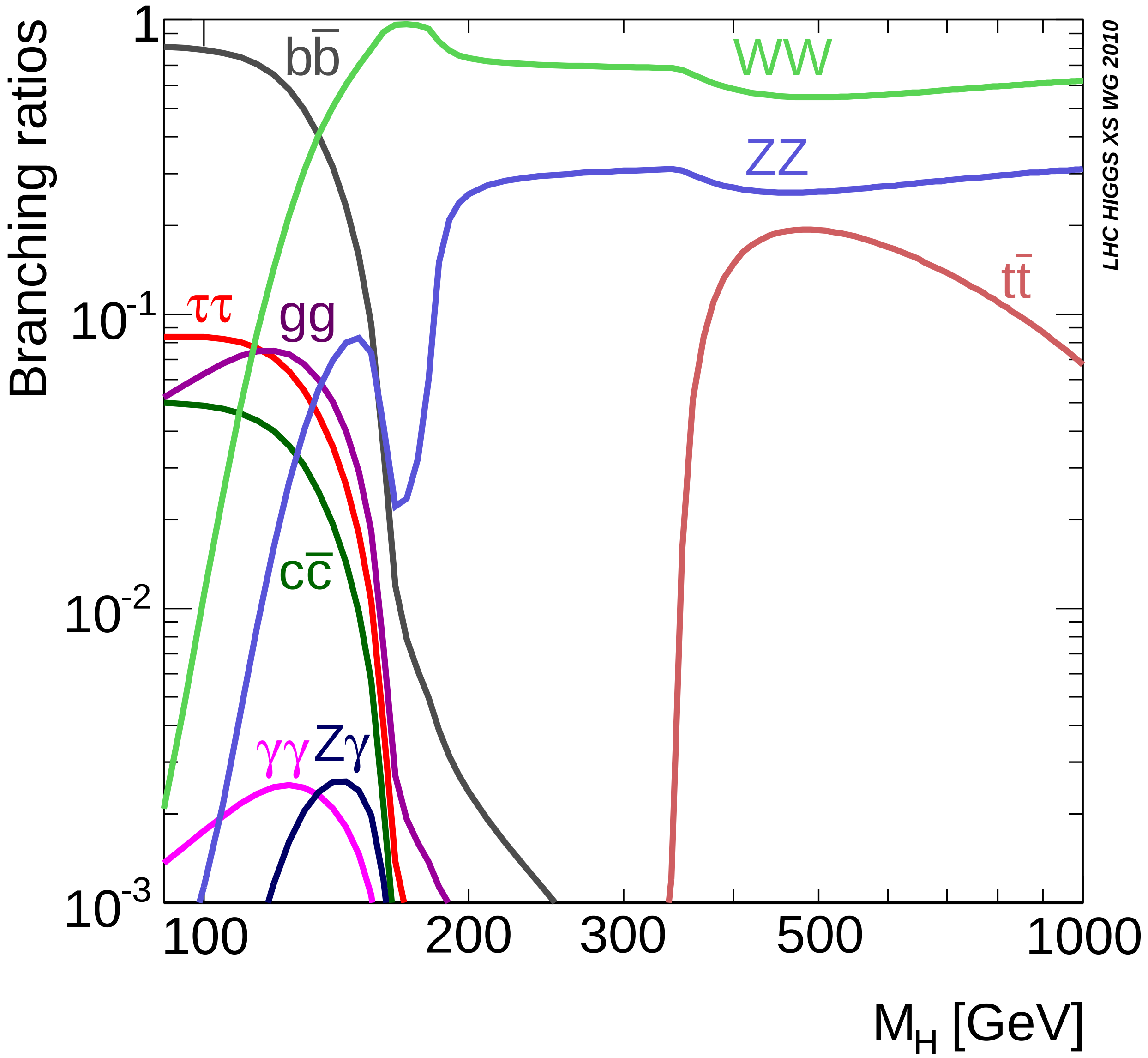
$\Rightarrow$ huge QCD corrections

$$\Gamma(H \to \gamma\gamma) = \frac{G_\mu \, \alpha^2 \, M_H^3}{128\sqrt{2}\,\pi^3}\left|\frac{4}{3}e_t^2 - 7\right|^2$$

via the top quark and W boson loop

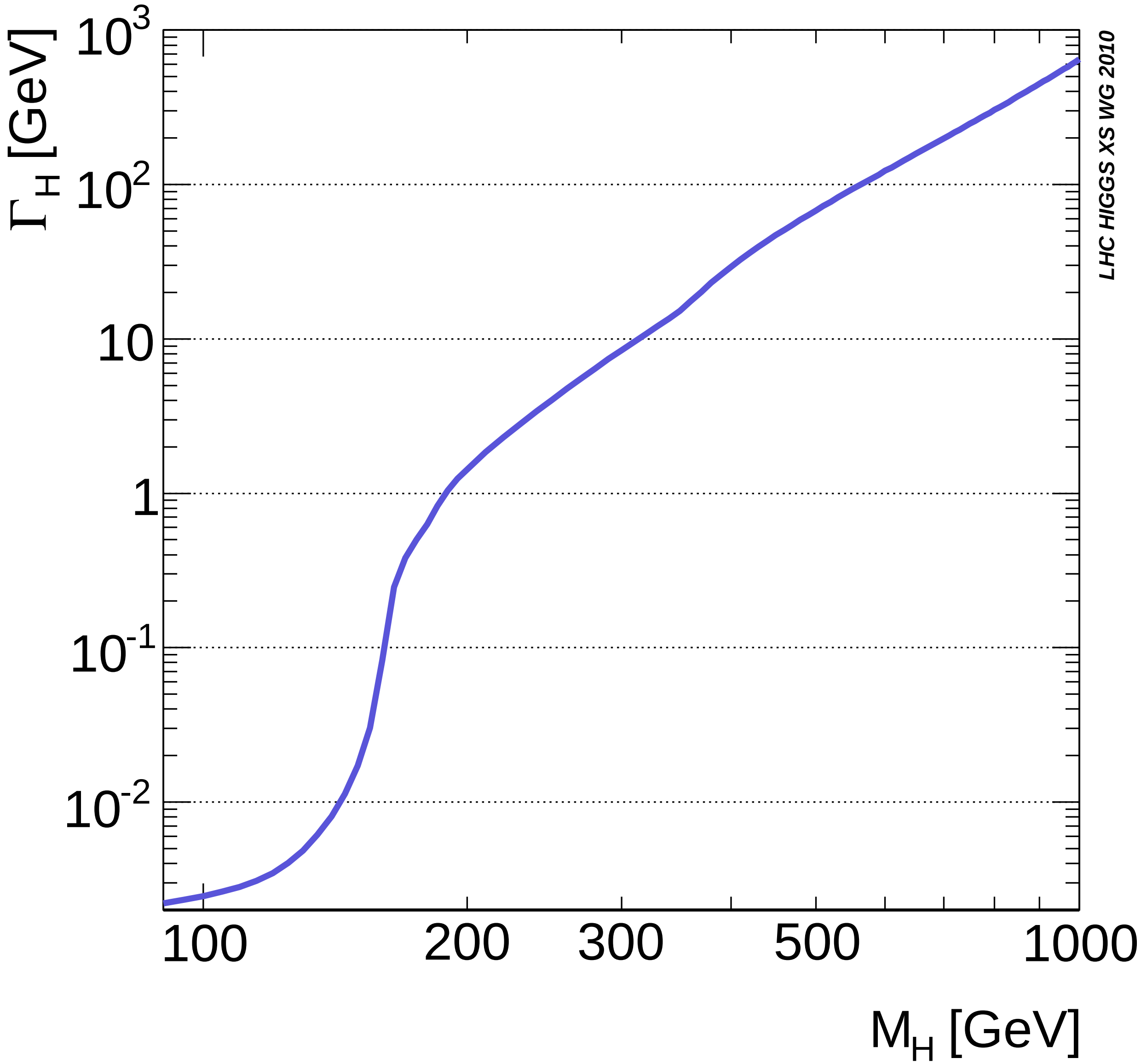
Latest theory predictions for the SM Higgs: branching ratios

[*LHC Higgs XS WG '10*]



Latest theory predictions for the SM Higgs: total width

[*LHC Higgs XS WG '10*]



The ρ parameter and the custodial symmetry I

The ρ parameter was defined as the ratio of neutral to charged current amplitudes at low energies:

$$\rho \equiv \frac{T_{NC}(q^2 << M_Z^2)}{T_{CC}(q^2 << M_W^2)}$$

From ν-scattering experiments and others: $\rho_{\text{exp}} \approx 1$. Last: $\rho_{\text{exp}} = 1.0008^{+0.0020}_{-0.0011}$(PDG 2012)
The SM prediction at tree level is:

$$\rho^{\text{SM}}_{\text{tree}} = \frac{M_W^{2\ \text{tree}}}{M_Z^{2\ \text{tree}} \cos^2 \theta_W^{\text{tree}}} = 1$$

At one loop and keeping just the so-called 'oblique' corrections,

$$\rho = \frac{\rho_{\text{tree}}}{1 - \Delta\rho}\ ;\ \Delta\rho = \frac{\Sigma_Z^{\text{R}}(0)}{M_Z^2} - \frac{\Sigma_W^{\text{R}}(0)}{M_W^2} \quad \text{related to } T \text{ parameter}$$

For instance, the leading top and Higgs loop contributions:

$$\begin{aligned} (\Delta\rho)_t &= \frac{g^2}{64\pi^2} N_C \frac{m_t^2}{M_W^2} + \ldots \quad \text{(E)} \\ (\Delta\rho)_H &= -\frac{g^2}{64\pi^2} 3 \tan^2\theta_W \log\frac{M_H^2}{M_W^2} + \ldots \quad \text{(E)} \end{aligned}$$

The ρ parameter being close to one is due to the so-called custodial symmetry: a global protecting symmetry of the SM Higgs sector in absence of gauge interactions.

The ρ parameter and the custodial symmetry II

Use the alternative way of writing the (ungauged) Lagrangian of SBS:

$$\begin{aligned}
\mathcal{L}_{\text{SBS}} &= \frac{1}{4}\text{Tr}\left[(\partial_\mu M)^\dagger(\partial^\mu M)\right] - V(M)\ ; \\
V(M) &= \frac{1}{4}\lambda\left[\frac{1}{2}\text{Tr}(M^\dagger M) + \frac{\mu^2}{\lambda}\right]^2
\end{aligned}$$

where M is a 2×2 matrix containing the four real scalar fields of Φ:

$$\begin{aligned}
M &\equiv \sqrt{2}(\widetilde{\Phi}\Phi) = \sqrt{2}\begin{pmatrix} \phi_0^* & \phi^\dagger \\ -\phi^- & \phi_0 \end{pmatrix}\ ; \\
\Phi &= \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}\ ; \\
\widetilde{\Phi} &= i\tau_2\Phi^* = \begin{pmatrix} \phi_0^* \\ -\phi^- \end{pmatrix}
\end{aligned}$$

$\mathcal{L}_{\text{SBS}}$ is invariant under the global transformations:

$$M \to g_L M g_R^+\ ;\ \ g_L \subset SU(2)_L\ ;\ \ g_R \subset SU(2)_R$$

This global symmetry $SU(2)_L \times SU(2)_R$ is called chiral symmetry (for analogy with QCD) and it is spontaneously broken down to the diagonal subgroup $SU(2)_{L+R} \equiv SU(2)_{\text{custodial}}$. The pattern of global symmetry breaking is:

$$SU(2)_L \times SU(2)_R \to SU(2)_{\text{custodial}}$$

Once $SU(2)_L \times U(1)_Y$ is gauged, the chiral symmetry (and the custodial) is explicitely broken.