The Cosmic Microwave Background and More on Dark Energy

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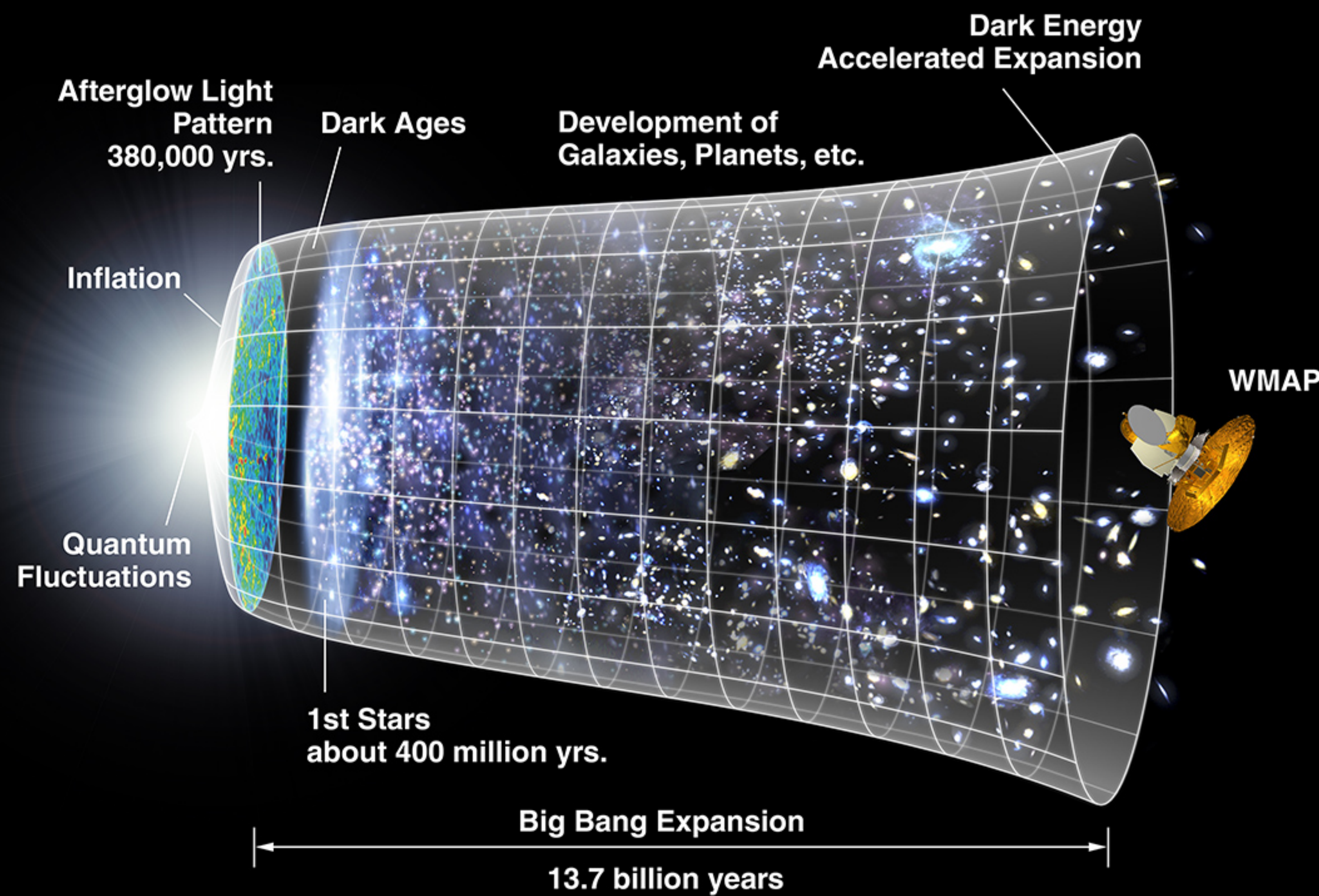
Overview

- **CMB**
  - origin
  - statistical description
  - evolution of the perturbations
  - constraints on parameters
  - in brief: lensing, ISW, polarization
- **Dark Energy**
  - evolving dark energy; constraints on w
  - modified gravity models
- **Summary**

Brief history of the Universe



perturbation evolution

period	scale	CDM	radiation	baryons
$t < t_{eq}$	$k < aH$	grows ~a^2	grows ~a^2	grows ~a^2
$t > t_{eq}$	$k < aH$	grows ~a	grows ~a	grows ~a
$t < t_{eq}$	$k > aH$	~ constant (ln a)	oscillates	oscillates
$t_{eq} < t < t_{dec}$	$k > aH$	grows ~a	oscillates	oscillates
$t_{dec} < t$	$k > aH$	grows ~a	free-streams	grows ~a

CDM: inside horizon grows only after matter-radiation equality -> scale imprinted in power spectrum where power-law will change!

radiation: oscillates, then free-streams after decoupling -> oscillations remain imprinted in power spectrum -> acoustic oscillations in CMB!

baryons: oscillate with photons until decoupling, then fall into CDM potential wells -> small imprint of acoustic oscillations also in matter power spectrum -> BAO

initial conditions and P(k)

HOT TOPIC

(& growth rate, redshift-space distortions, non-linear growth)

Inflationary spectrum: $\delta(k,t_{enter}) \sim k^{n/2-2}$

scales entering before t_{eq}: $\lambda<\lambda_{eq}$

$$\delta_\lambda(t) \simeq \delta_\lambda(t_{enter})(a/a_{eq})$$

scales entering after t_{eq}: $\lambda>\lambda_{eq}$

$$\delta_\lambda(t) = \delta_\lambda(t_{enter})(a/a_{enter})$$

$$= \delta_\lambda(t_{enter})(a/a_{eq})(a_{eq}/a_{enter})$$

horizon: $t_{enter} = \lambda\, a_{enter} \sim \lambda t_{enter}^{2/3}$

$$\rightarrow (a_{eq}/a_{enter}) = (\lambda_{eq}/\lambda)^2$$

in terms of k:

scales entering before teq:

$$|\delta_k(t)|^2 \propto k^{n-4}(a/a_{eq})^2$$

scales entering after teq:

$$|\delta_k(t)|^2 \propto k^{n}(a/a_{eq})^2$$

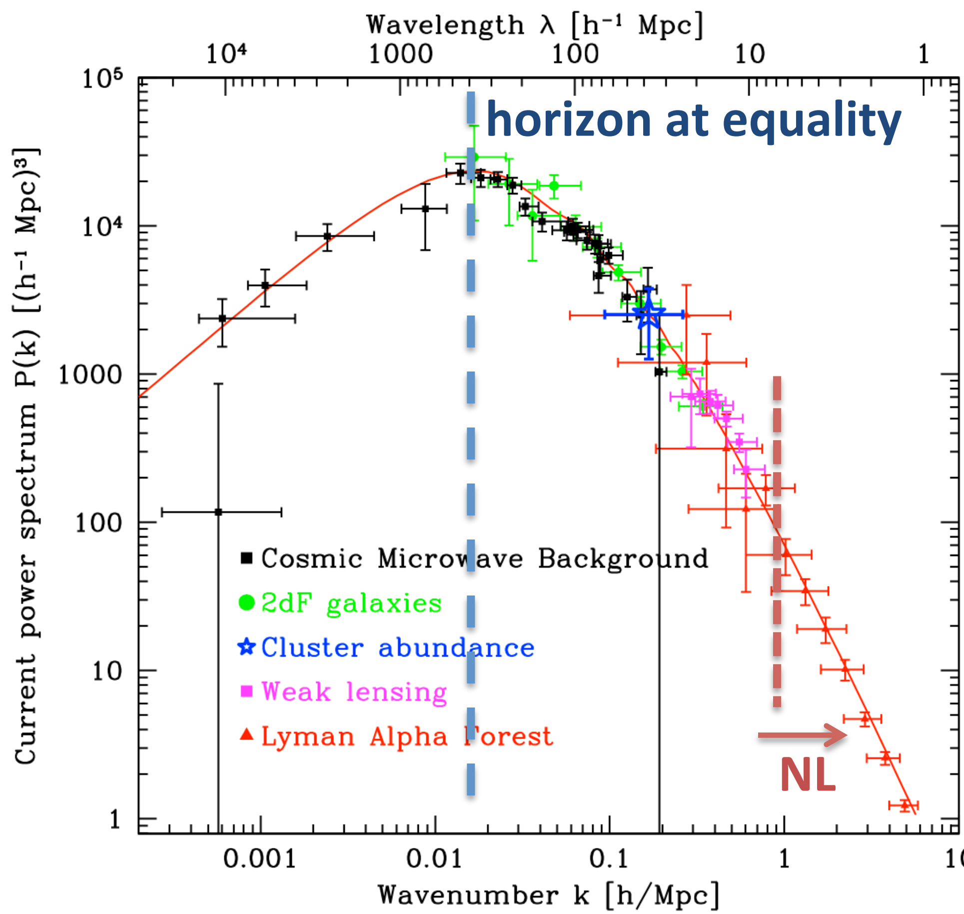
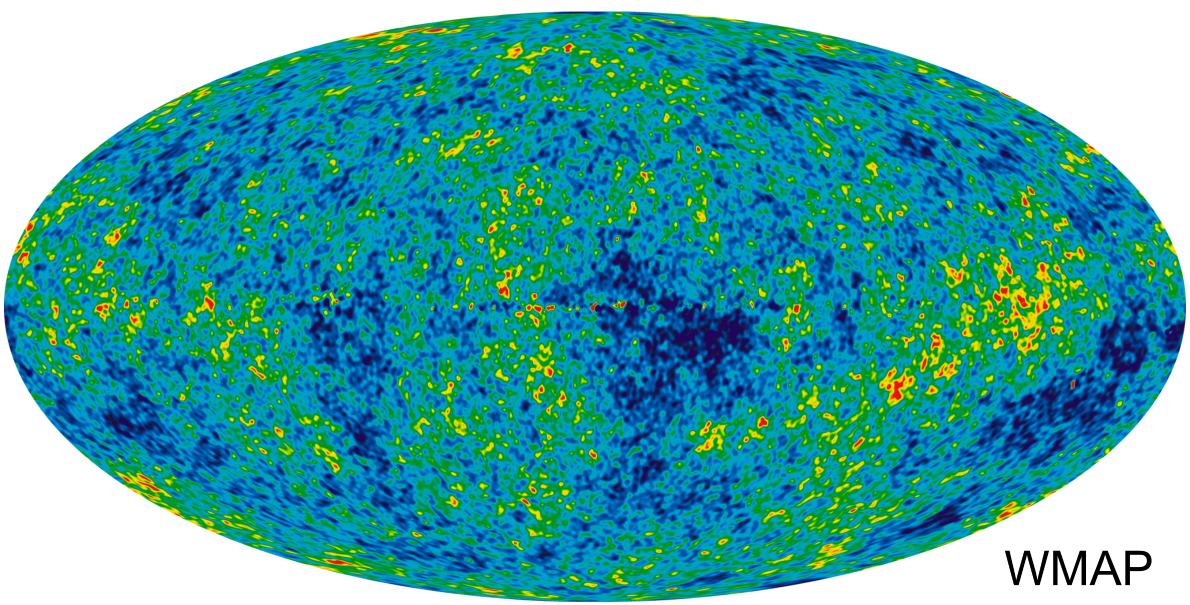


figure from Tegmark & Zaldarriaga

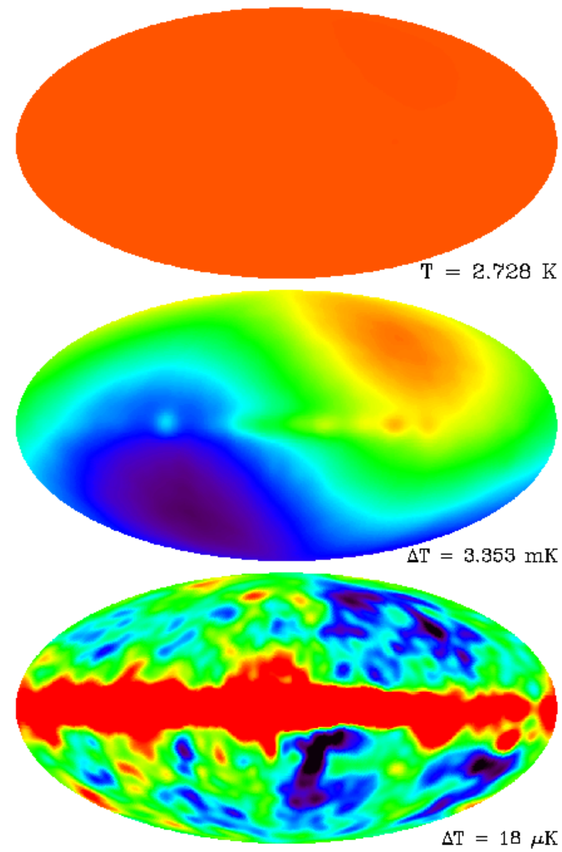
anisotropies in the CMB

You have often seen this picture

- what does it show?
- why?
- what does it tell us about the universe?



WMAP



COBE

origin of the CMB

T > 3000 K :

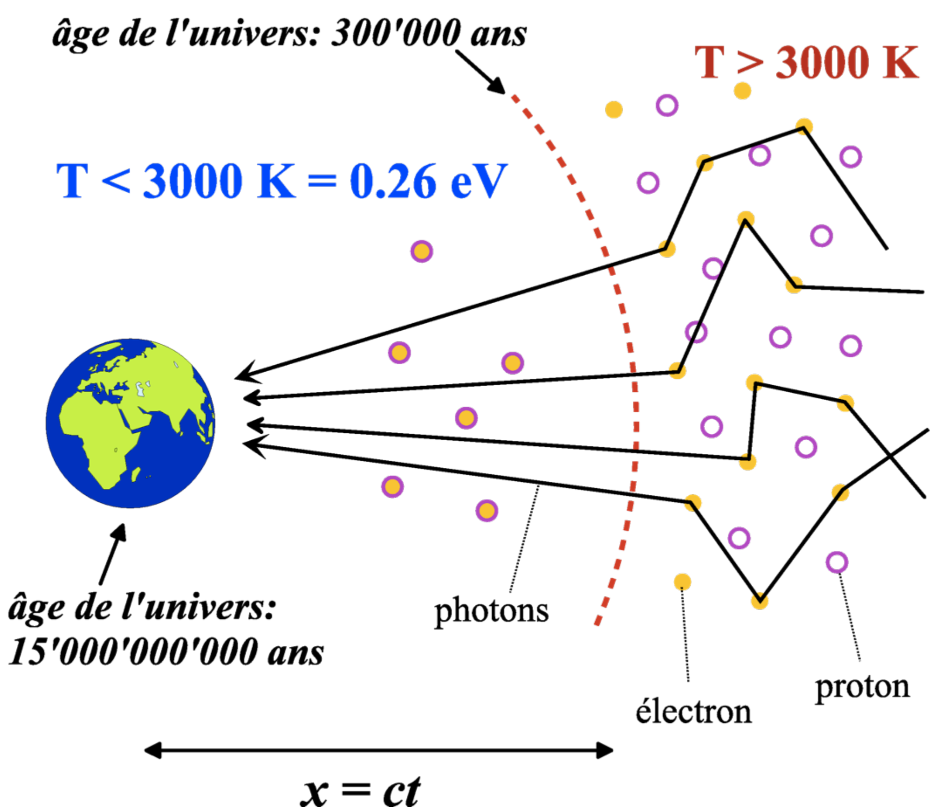
Electrons and protons are free. Light interacts strongly with the electron (baryon-photon plasma), strong scattering as in fog.

T < 3000 K :

Electrons and protons (re-)combine to neutral atoms. The universe becomes transparent for light, which free-streams to us.

We observe:

- 'photo' of last scattering surface
- stuff that happens on the way



statistical description

Temperature T(n) on the sky: Gaussian random field

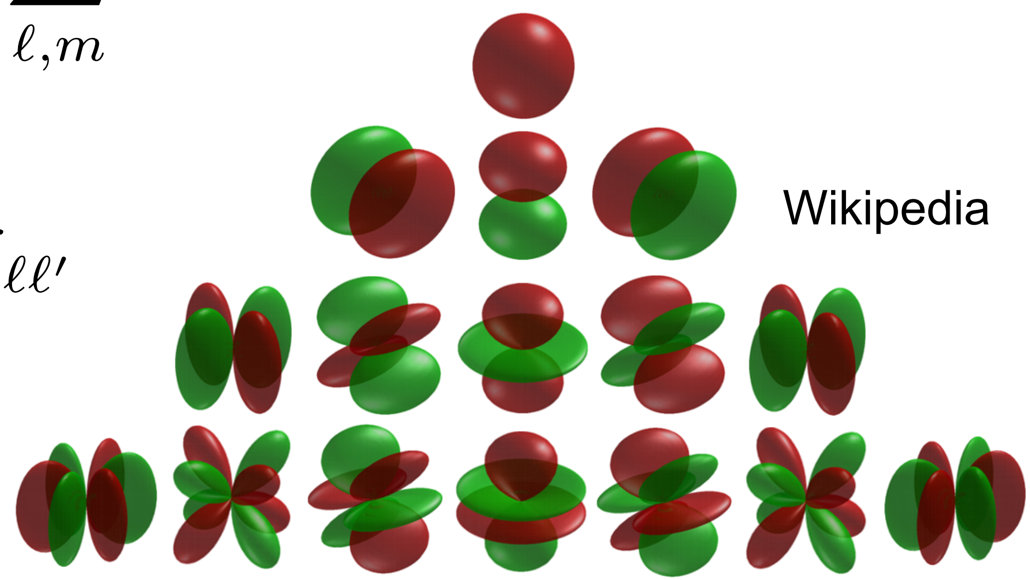
Fourier-analysis on sky sphere: instead of e^{ikt} the basis functions are spherical harmonics $Y_{lm}(n)$

$$\delta T(n) = T(n) - T_0 = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(n)$$

statistical isotropy:

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = C_\ell \delta_{mm'} \delta_{\ell\ell'}$$

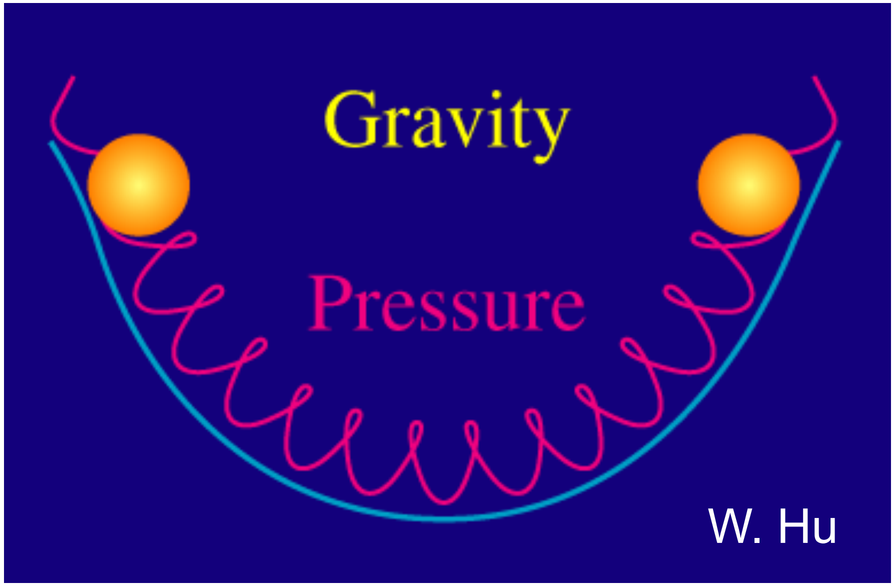
power-spectrum
~ δT^2



Wikipedia

perturbation evolution

The overdensities in the baryon-photon fluid collapse under the influence of gravity, until the pressure is strong enough to resist. Then the plasma starts to oscillate, until recombination.



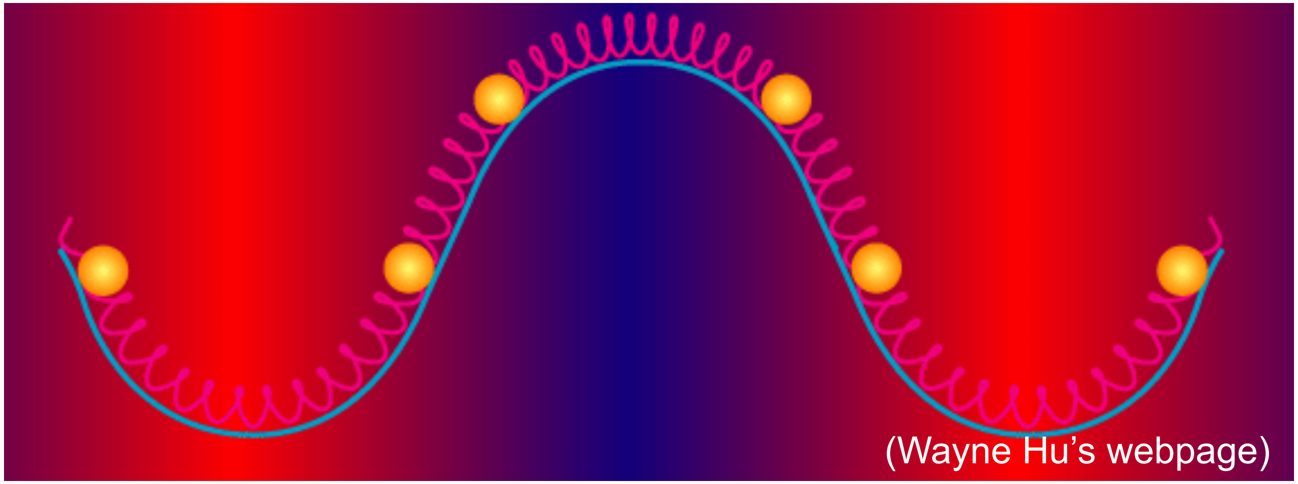
We therefore see (mostly) the oscillation pattern at t_{rec}!

The largest scale that had just time to collapse will create the first peak, the scale that collapsed and re-expanded the second peak, etc.
-> angular diameter distance to z=1100!

density and temperature

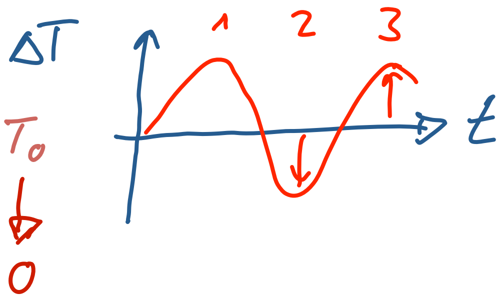
Why do we see the density fluctuations as temperature variations?

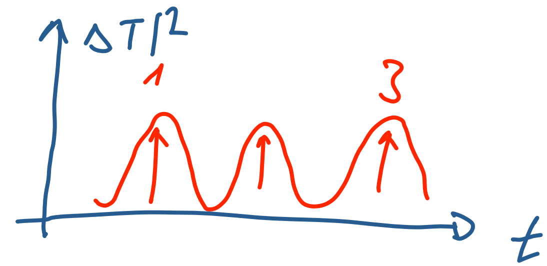
Stefan-Boltzmann: $\rho_\gamma \sim \sigma T^4$ -> $\delta_\gamma = \frac{\delta\rho_\gamma}{\rho_\gamma} \approx 4\frac{\delta T}{T}$



In addition, line-of-sight motion of the "last-scattering" electrons leads to red-/blue shifts $\sim V_b$, out of phase with δ_γ

peak height

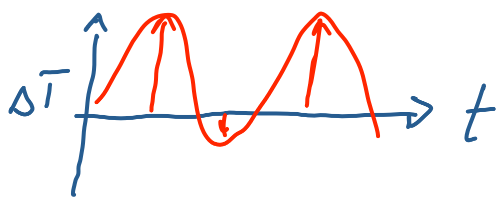


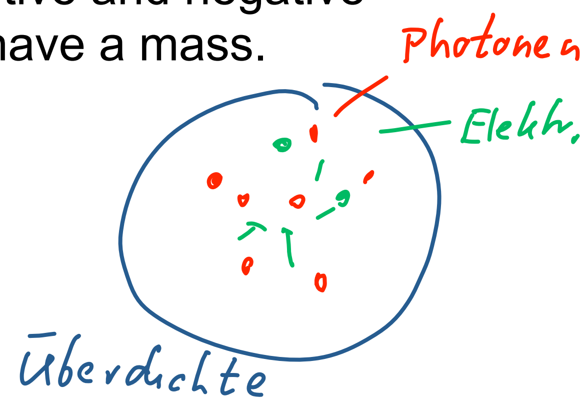


A pure radiation „fluid“ would oscillate with equal positive and negative amplitude. But the electrons that are dragged along have a mass.

-> stronger compression (peaks # 1,3,…)

-> reduced rarification (peaks # 2,4,…).

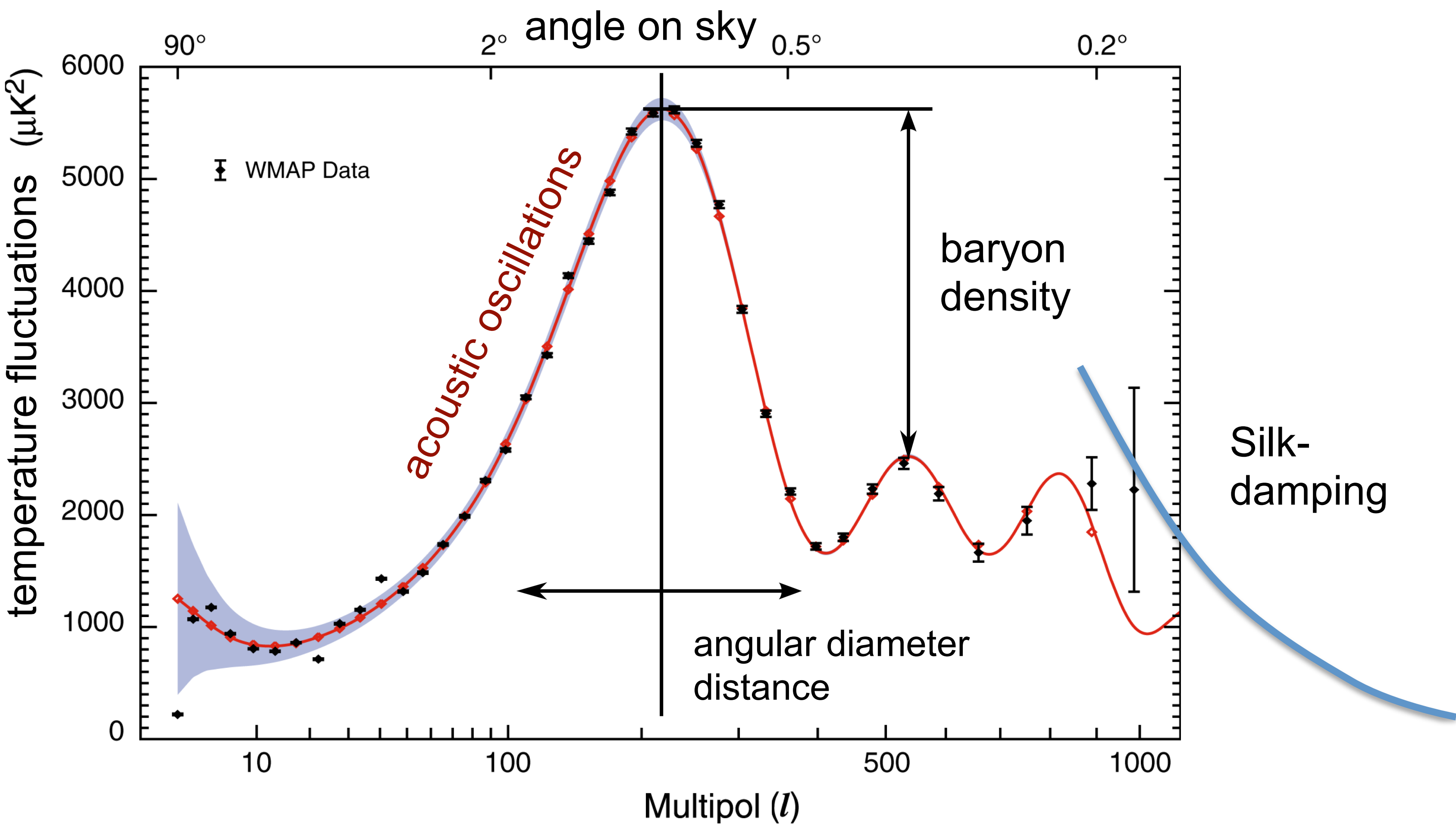




The relative height of the first two peaks thus measures the amount of baryons!

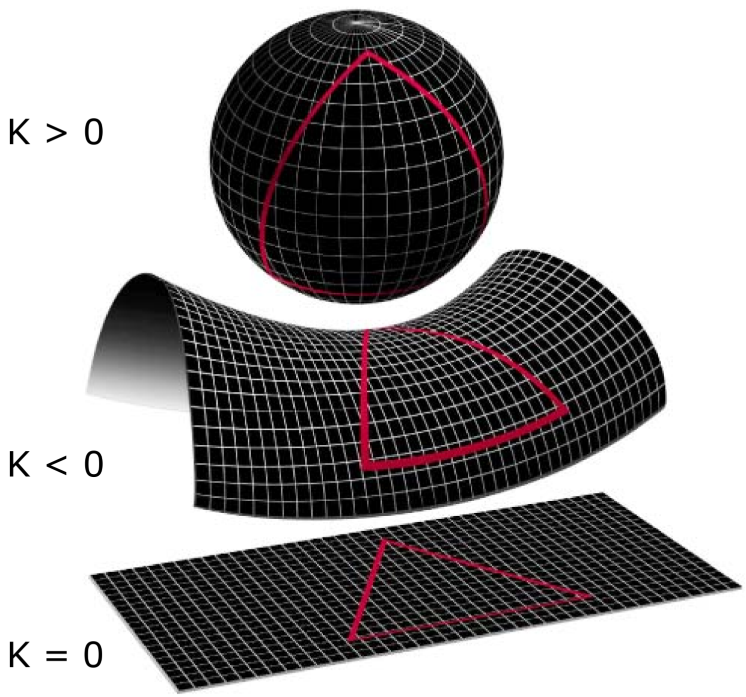
Dark matter doesn‘t feel the radiation pressure and undergoes gravitational collapse. The radiation feels the DM potential wells, which changes the amplitude of the maxima overall.

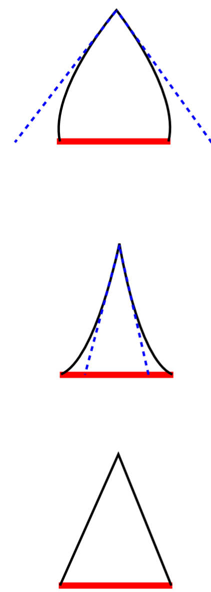
the CMB power spectrum



CMB physics is mostly linear -> very clean probe!

CMB and curvature

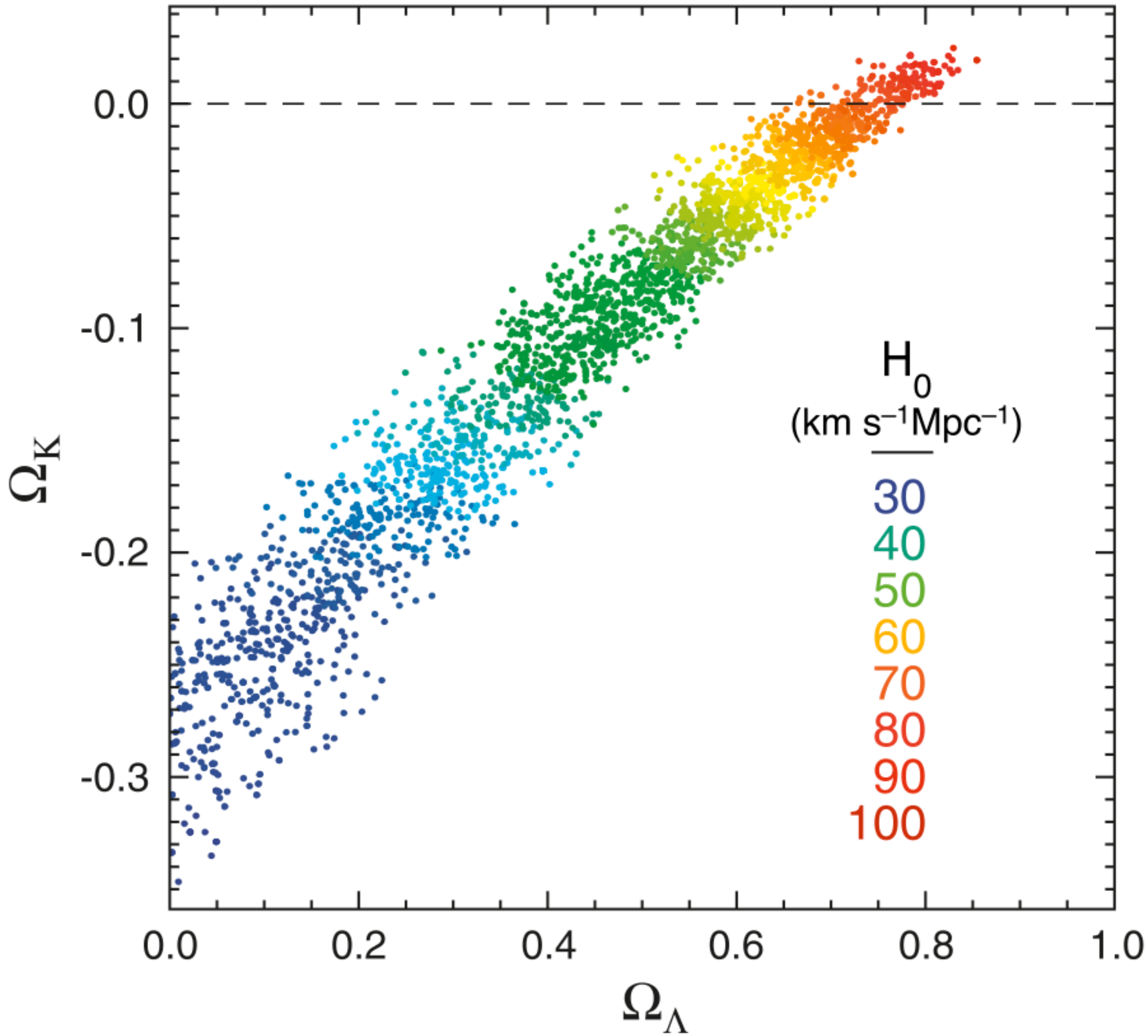




The WMAP satellite provides ~ 0.3% measurement of the angular scale of the first peak!

-> measurement of the geometry of the universe

geometric degeneracy



gravitational lensing of CMB

Light is deflected by gravitational perturbations along photon path.

Also true for CMB
-> shifts power around in C_l
-> introduces non-Gaussianity
-> changes polarisation
=> can be estimated!

-> Probe of large-scale structure evolution, can break geometric degeneracy!

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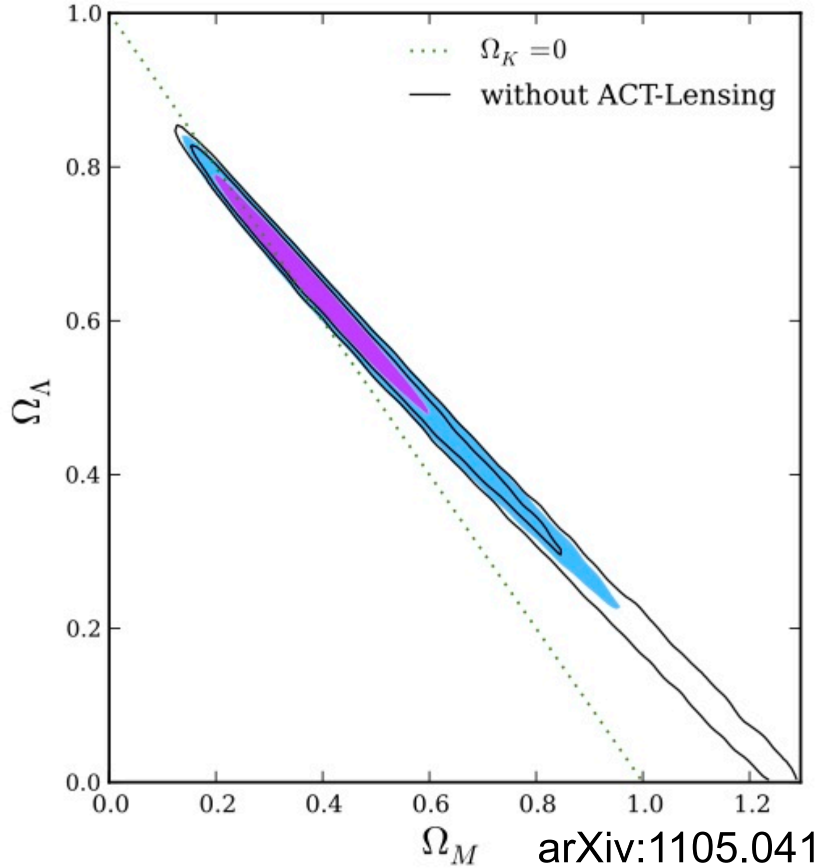
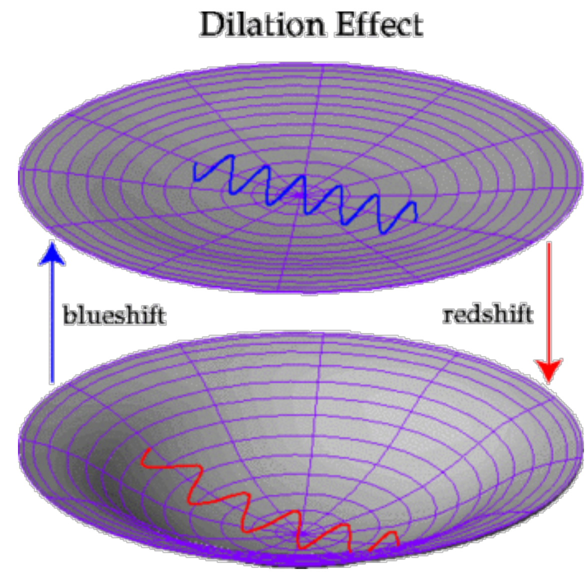


arXiv:1105.0419

(integrated) Sachs-Wolfe eff.

Impact of gravitational potential on CMB:

$$\frac{\delta T}{T} \sim (\Phi - \Psi)|_{\mathrm{dec}} + \int_{t_{\mathrm{dec}}}^{t_0} \left(\dot{\Phi} - \dot{\Psi}\right) dt$$



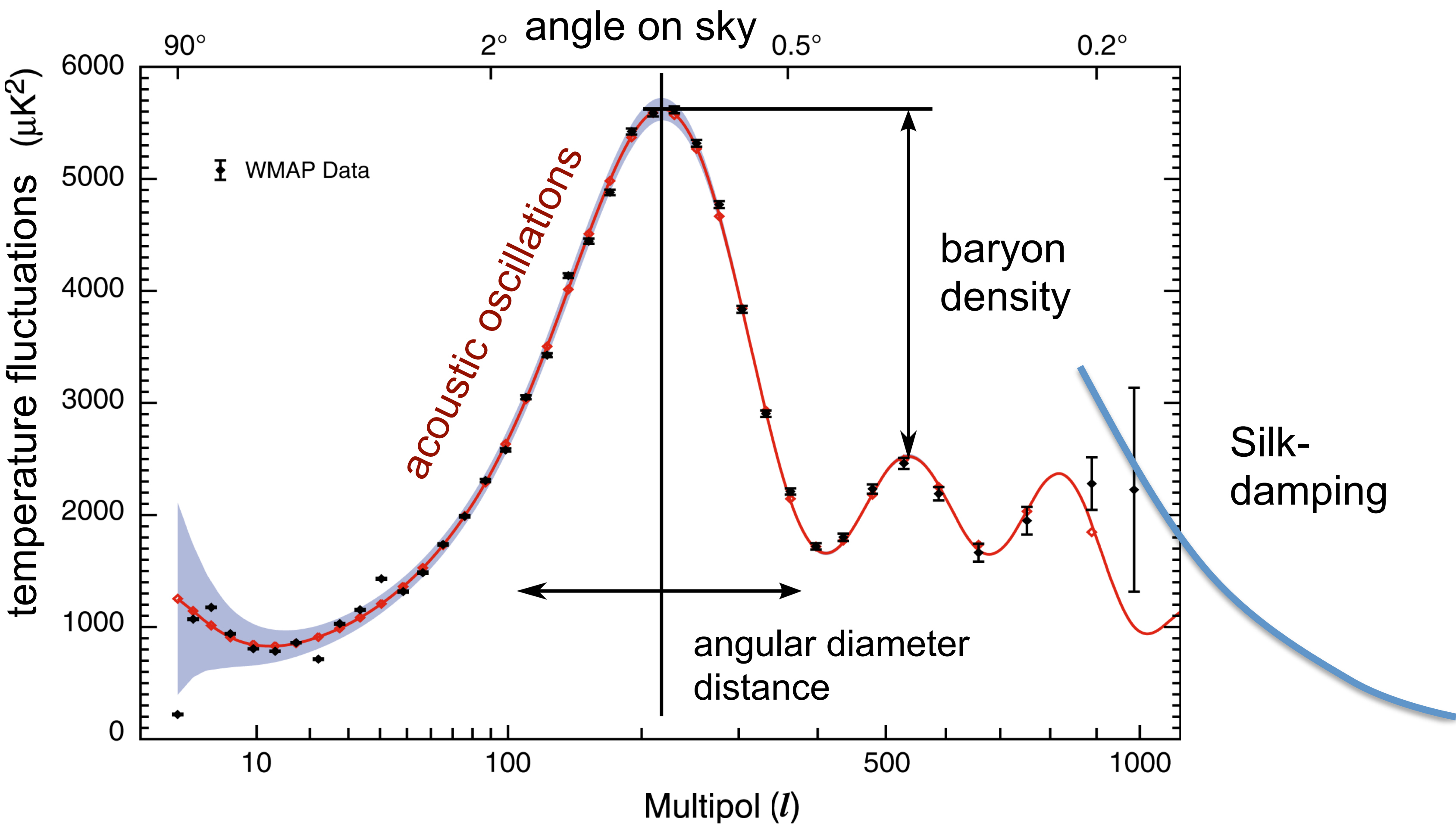
First term: SW -> ~ constant contribution

Second term: ISW -> depends on evolution of the gravitational potential along photon path!

Poisson eq. in matter dom. $\nabla^2 \Phi = 4\pi G a^2 \rho_m \delta_m$, $\rho_m \sim a^{-3}$, $\delta_m \sim a$

No ISW effect in a pure matter dominated universe.
But when dark energy begins accelerating the expansion: Φ, Ψ decay
-> ISW provides direct test of accelerated expansion
-> cosmic variance: large uncertainties … about 3σ when correlating with large scale structure

the CMB power spectrum



CMB physics is mostly linear -> very clean probe!

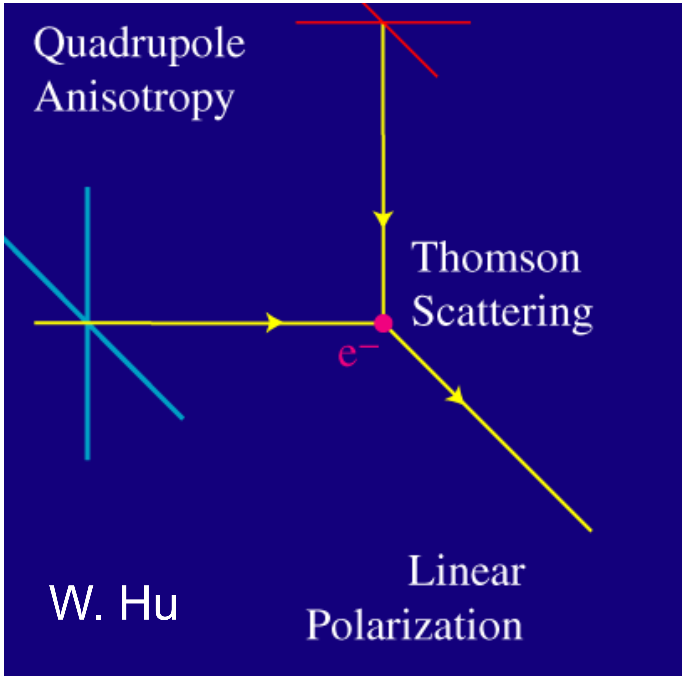
polarization

Scattering of light depends on polarisation angle -> last scattering polarizes light depending on local quadrupole.

-> also reionization probe (scattering again)

Scalar (density) perturbations do not lead to vorticity in polarization pattern ("B-modes")

BUT gravitational waves (tensor perturbations) do! (as does lensing)



"B-mode" polarization is a probe of exotic (exciting) physics!

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the power of the CMB

6 parameters (plus A_{SZ})
eg. $\{H_0, \Omega_b, \Omega_m, n_s, A_s, \tau\}$

- flat ΛCDM model
- WMAP 7yr data

WMAP Cosmological Parameters
Model: lcdm+sz+lens
Data: wmap7

$10^2\Omega_b h^2$	$2.258^{+0.057}_{-0.056}$	$1-n_s$	0.037 ± 0.014
$1-n_s$	$0.0079 < 1-n_s < 0.0642$ (95% CL)	$A_{\rm BAO}(z=0.35)$	$0.463^{+0.021}_{-0.020}$
C_{220}	5763^{+38}_{-40}	$d_A(z_{\rm eq})$	14281^{+158}_{-161} Mpc
$d_A(z_*)$	14116^{+160}_{-163} Mpc	$\Delta^2_{\mathcal{R}}$	$(2.43 \pm 0.11) \times 10^{-9}$
h	0.710 ± 0.025	H_0	71.0 ± 2.5 km/s/Mpc
$k_{\rm eq}$	$0.00974^{+0.00041}_{-0.00040}$	$\ell_{\rm eq}$	137.5 ± 4.3
ℓ_*	302.44 ± 0.80	n_s	0.963 ± 0.014
Ω_b	0.0449 ± 0.0028	$\Omega_b h^2$	$0.02258^{+0.00057}_{-0.00056}$
Ω_c	0.222 ± 0.026	$\Omega_c h^2$	0.1109 ± 0.0056
Ω_Λ	0.734 ± 0.029	Ω_m	0.266 ± 0.029
$\Omega_m h^2$	$0.1334^{+0.0056}_{-0.0055}$	$r_{\rm hor}(z_{\rm dec})$	285.5 ± 3.0 Mpc
$r_s(z_d)$	153.2 ± 1.7 Mpc	$r_s(z_d)/D_v(z=0.2)$	$0.1922^{+0.0072}_{-0.0073}$
$r_s(z_d)/D_v(z=0.35)$	$0.1153^{+0.0038}_{-0.0039}$	$r_s(z_*)$	$146.6^{+1.5}_{-1.6}$ Mpc
R	1.719 ± 0.019	σ_8	0.801 ± 0.030
$A_{\rm SZ}$	$0.97^{+0.68}_{-0.97}$	t_0	13.75 ± 0.13 Gyr
τ	0.088 ± 0.015	θ_*	0.010388 ± 0.000027
θ_*	$0.5952 \pm 0.0016\ ^\circ$	t_*	379164^{+5187}_{-5243} yr
$z_{\rm dec}$	1088.2 ± 1.2	z_d	1020.3 ± 1.4
$z_{\rm eq}$	3196^{+134}_{-133}	$z_{\rm reion}$	10.5 ± 1.2
z_*	$1090.79^{+0.94}_{-0.92}$		

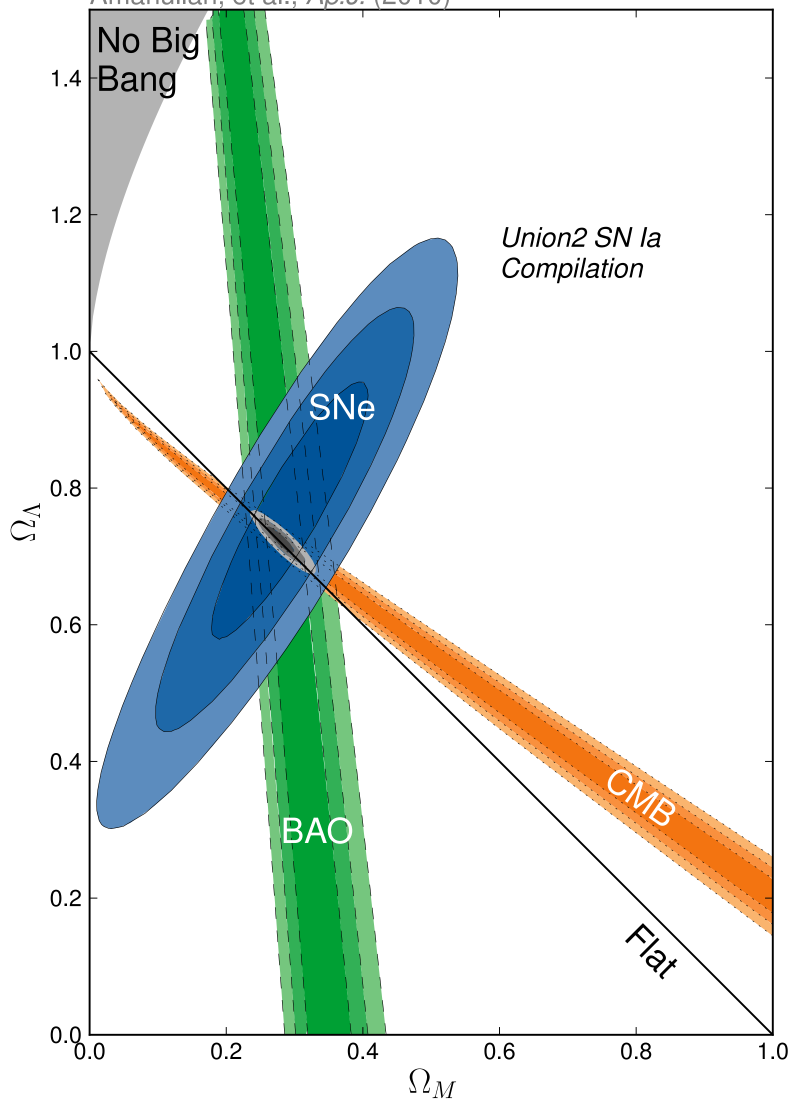
CMB summary

- CMB: left-over radiation from initial hot state, "photo of the big-bang"
- Basically we are seeing sound-waves from … what? Inflation?
- Key cosmological observable due to theoretical cleanness, measures many parameters directly
- Even more when combined with other observations (or things like lensing, SZ, ...)
- Large-scale polarisation pretty much rules out any "causal" late-time source of perturbations!
- Lots of exciting stuff: Polarisation (grav. waves), non-Gaussianity (origin of perturbations), …

HOT TOPIC

Dark Energy

Supernova Cosmology Project
Amanullah, et al., *Ap.J.* (2010)



Physics Nobel prize 2011:

"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"

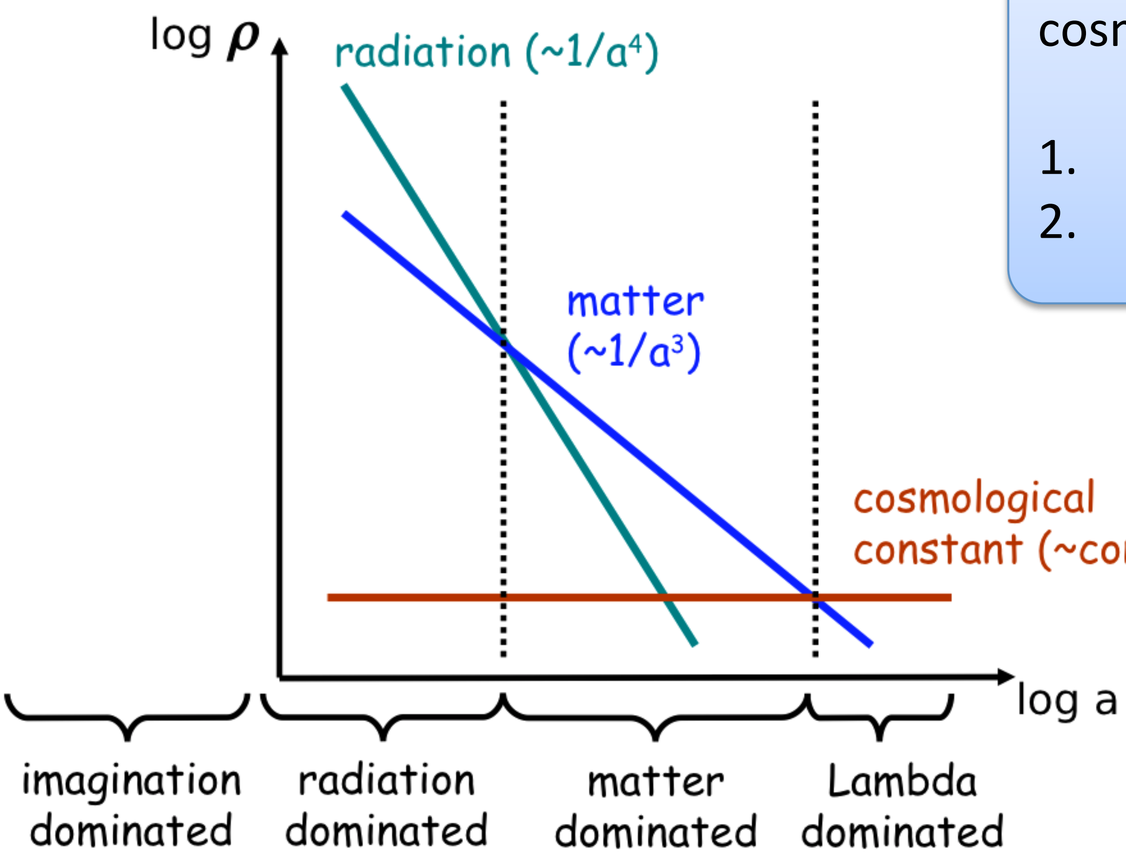
accelerating expansion: $w < -1/3$

- we know that for Λ: $w = -1$
- data is consistent with Λ

why look elsewhere?

What's the problem with Λ?

Evolution of the Universe:

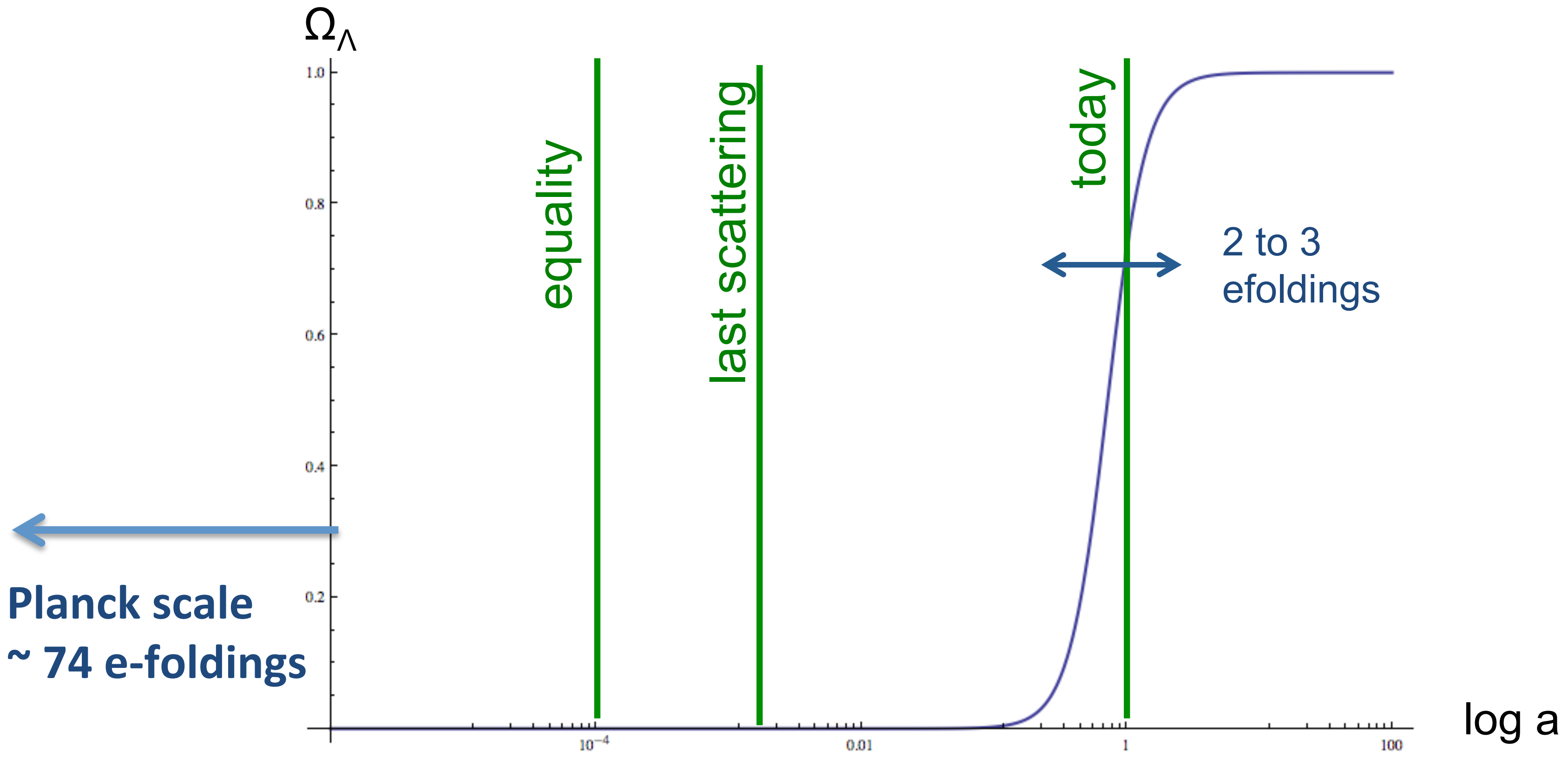


Classical problems of the cosmological constant:

1. Value: why so small? Natural?
2. Coincidence: Why now?

the coincidence problem

- why are we just now observing $\Omega_\Lambda \approx \Omega_m$?
- past: $\Omega_m \approx 1$, future: $\Omega_\Lambda \approx 1$



the naturalness problem

energy scale of observed Λ is ~ 2×10^{-3} eV
zero point fluctuations of a heavier particle of mass m:

cutoff scale

$$\int_0^{\Lambda} \frac{1}{2} \frac{d^3p}{(2\pi)^3} \sqrt{p^2 + m^2} \simeq \frac{1}{16\pi^2} \left[\Lambda^4 + m^2\Lambda^2 - \frac{1}{4} m^4 \log\left(\frac{\Lambda^2}{m^2}\right) \right]$$

can in principle be absorbed into renormalization of observables

"running" term: this term is measureable for masses and couplings! Why not for cosmological constant?!

already the electron should contribute at m_e >> eV
(and the muon, and all other known particles!)

Possible explanations

1. It is a cosmological constant, and there is no problem ('anthropic principle', 'string landscape')
2. The (supernova) data is wrong
3. We are making a mistake with GR (aka 'backreaction') or the Copernican principle is violated ('LTB')
4. It is something evolving, e.g. a scalar field ('dark energy')
5. GR is wrong and needs to be modified ('modified gravity')

scalar fields in cosmology

GR + scalar field:

$$S = S_g + S_\phi = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right)$$

gravity e.o.m. (Einstein eq.):

$$\frac{\delta S[g_{\mu\nu},\phi]}{\delta g^{\mu\nu}} = 0 \qquad G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

entries in scalar field EM tensor (FLRW metric)

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

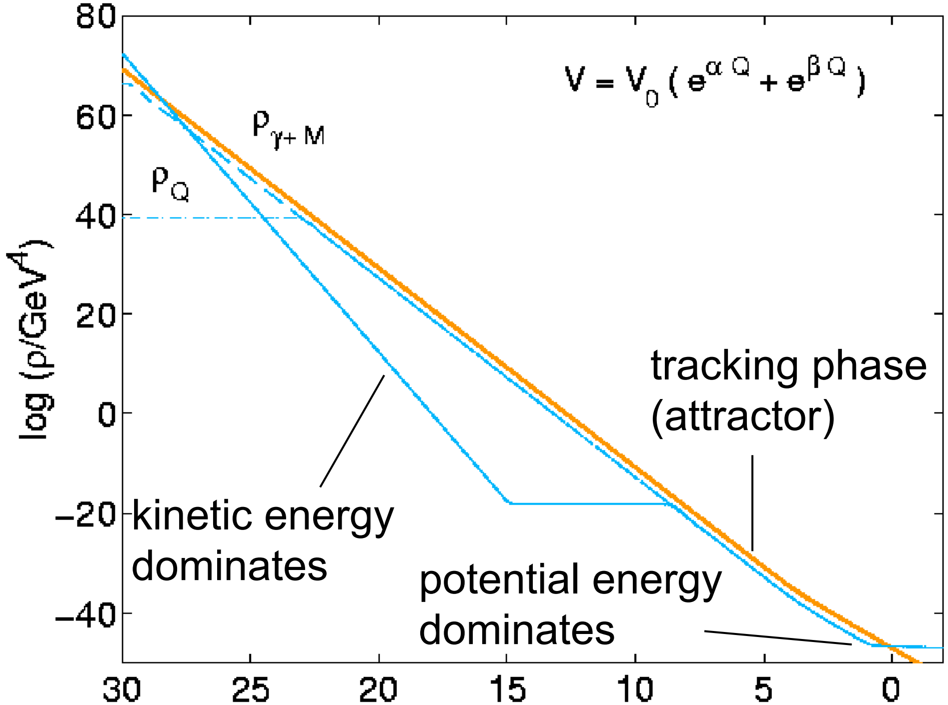
scalar field e.o.m. :

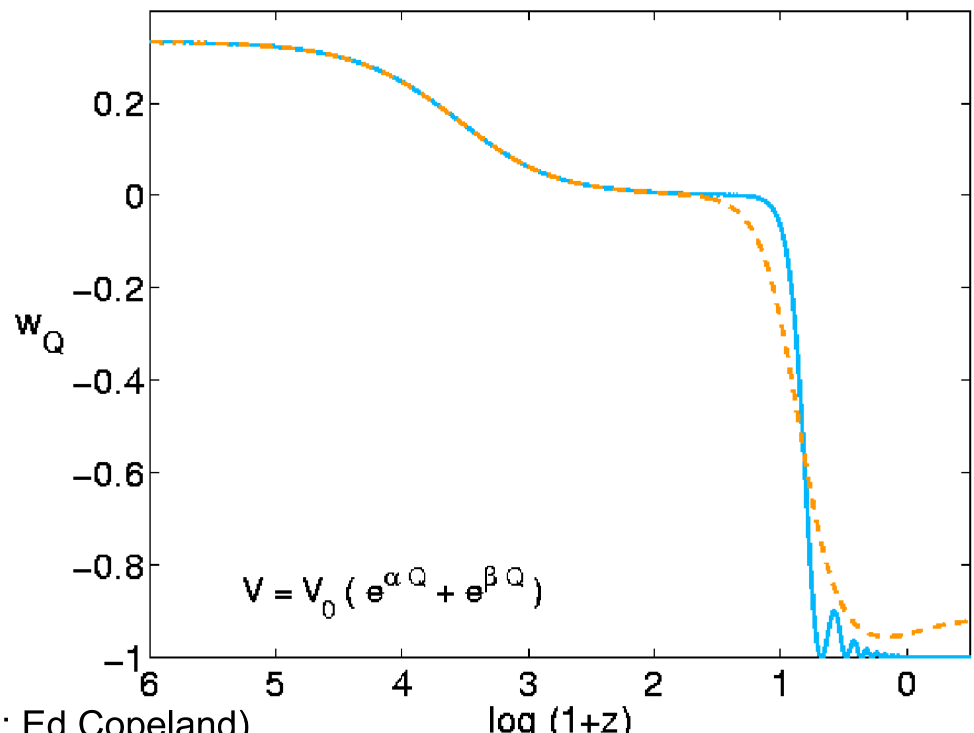
$$\frac{\delta S[g_{\mu\nu},\phi]}{\delta\phi} = 0 \qquad \ddot{\phi} + 3H\dot{\phi} + dV(\phi)/d\phi = 0$$

- this is the general method to compute Einstein eq., EM tensor and field e.o.m. from any action
- w=p/ρ for scalar fields can vary, as a function of V(ϕ)

4. evolving dark energy

- Inflation: accelerated expansion with help of scalar field
- Dark Energy: accelerated expansion with help of scalar field
- If $w=p/\rho$ can change, then initial dark energy density can be much higher -> solves one problem of Λ
- extra bonus: tracking behaviour





(figures: Ed Copeland)

Quintessential problems

- no solution to coincidence problem (need to e.g. put a bump into the potential at the right place)
- Still need to get somehow $\Lambda = 0$
- potential needs to be very flat
- need to avoid corrections to potential
- need to avoid couplings to baryons
- no obvious candidates for scalar field
- but nonetheless quintessence is the 'standard evolving dark energy model'

(***there are many other scalar field models*** – e.g. 'k-essence' and 'growing neutrino' models offer potential solutions to coincidence problem.)

phenomenological DE

No obvious scalar field candidates

-> we can ask reverse question: what model do we need to agree with data?

-> relationship $V(\phi(t))$ <-> $w(t)$ <-> $H(t)$

-> we can always reconstruct a potential that would give us a certain $w(z)$! Actually, we don't even need to do this explicitly, as we can directly compute the behaviour of the perturbations (later)

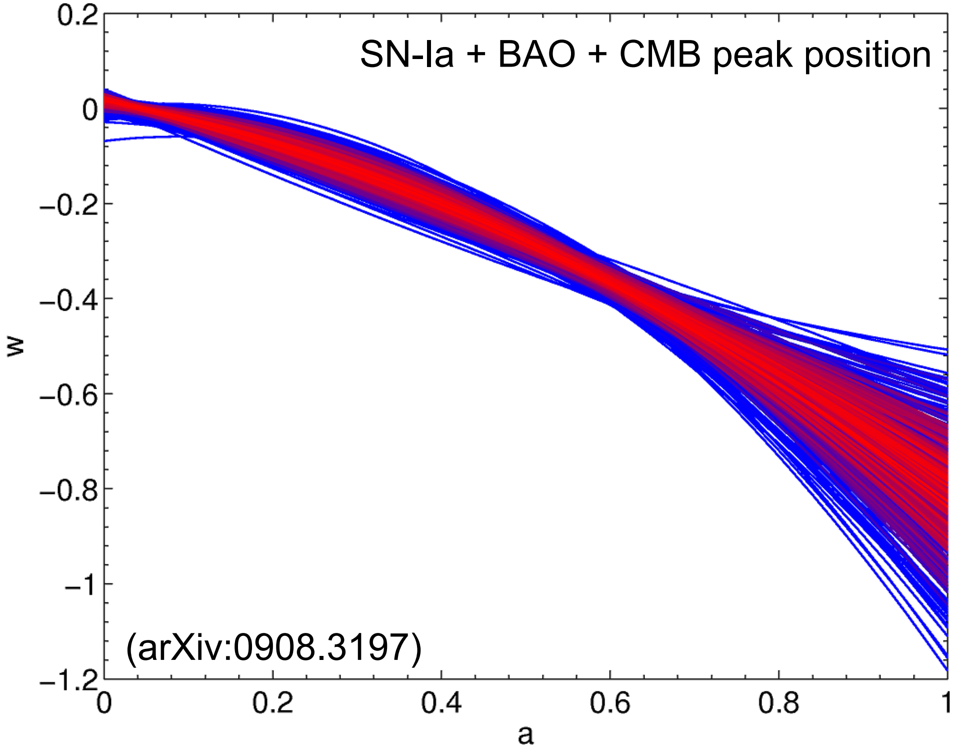
-> 'MCMC' method: pick a $w(z)$, compute observables, compare to data (does it fit?), repeat

evolving total w(z)

flat universe: $H^2 = \frac{8\pi G}{3}\rho$ $\quad \dot{\rho} + 3H(\rho + p) = 0$ $\quad p = w\rho$

$$\int \frac{d\rho}{\rho} = 3\int (1+w)\frac{da}{a}$$

$$H^2 = H_0^2 \exp\left\{\int_0^z \frac{3(1+w)}{(1+z')}dz'\right\} \qquad d_L = (1+z)\int_0^z \frac{du}{H(u)}$$



example:
$w(a) = w_0 + w_1\, a + w_2\, a^2$

best $\chi^2 = 309.8$
ΛCDM: $\chi^2 = 311.9$
w const: $\chi^2 = 391.3$

What is w_{DE}? Beware:
- MUST leave Ω_m free
- need DE model (split not unique in general), e.g. scalar field

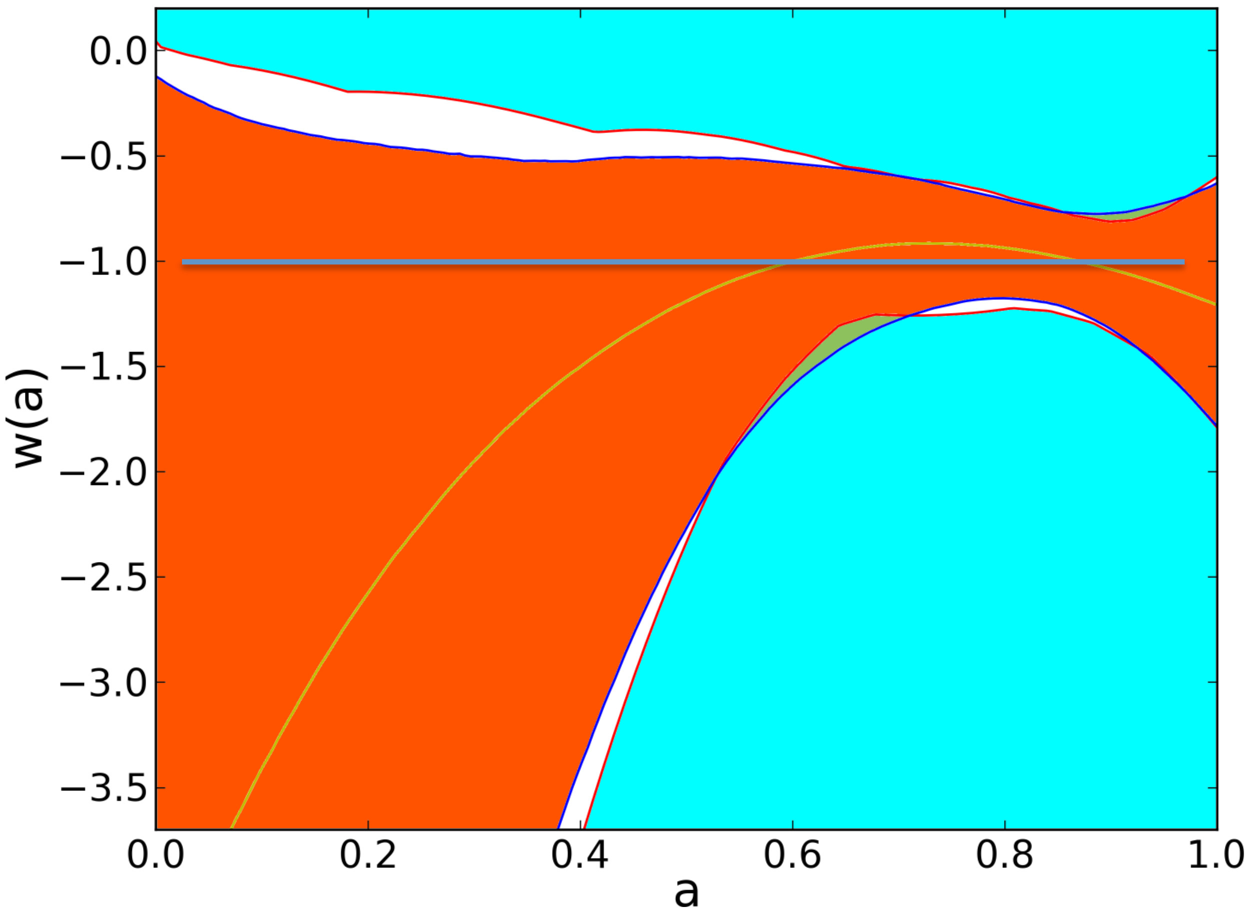
parametrisations of w

- vast literature
- generally, inverse methods difficult and noisy
- forward methods better: parametrise w(a)
  - $w = w_0$ constant
  - $w = w_0 + (1-a)\, w_a$ (especially forecasts, DETF FoM)
  - general series expansions in a or z
  - w = f(a), with f(a) e.g. a transition
  - w in bins
  - w as expansion in some other functional basis
- balance between stiffness of expansion and size of error bars -> regularisations, PCA, Gaussian processes, …

w of quintessence models

Play same game, but now using effective quintessence model (with some tricks to cross w=-1) including perturbations, and CMB+SN-Ia data.

Parameters: $\{\Omega_m, \Omega_b h^2, H_0, \tau, n_s, A_s, w_0, w_1, w_2, w_3\}$ (cubic expansion of w(a))



- 95% limits
- w=-1 is a good fit
- best constraints at low z
- ca 10%-15% error on w at ‘best’ redshift
- not very strong dependence on parametrisation

Is it just Λ?

- remember the problems
- also: inflation

5. modified gravity models

4D generalisation of GR:

⇨ Scalar/(V)/Tensor : natural generalisation, strong limits from solar system, effects can be screened

⇨ f(R) : modify action: R + f(R) (e.g. R-μ^4/R), consistency constraints and problems with matter dominated era

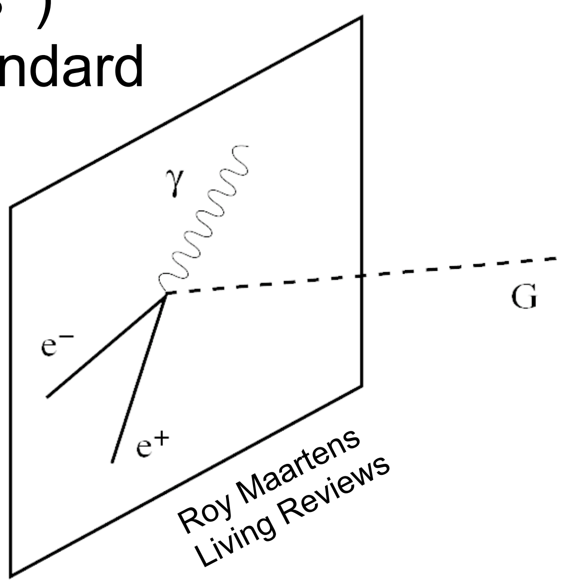
⇨ Galileons / extra symmetries -> 'Horndeski'-type theories (most general scalar-tensor theories w/ 2nd-order e.o.m.) **HOT TOPIC**

⇨ massive gravitons / degravitation (~ related to DGP, galileons)

Higher-dimensional gravity (aka "braneworlds")

gravity (closed strings) propagates freely, standard model (open strings) fixed to branes

⇨ DGP : sum of 5D and 4D gravity action



- instabilities, ghosts, finetuning
- solar-system tests
- dependence on background

non-cosmological probes

- fifth force (weak, long-range) from couplings of standard model to new fields

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-> screening mechanisms (Chameleon, Vainshtein, …)

- new particles with strange couplings and/or mass hierarchies (KK)
- varying "fundamental constants" and other violations of the equivalence principle
- perihelion shifts / solar system constraints (including double pulsar timings, etc)
- modifications to stellar structure models
- short-distance gravity modified (now well below 0.1mm)

cosmological DE/MG probes

What can we actually measure?

two kinds of equations:

$$G_{\mu\nu} = -8\pi G T_{\mu\nu} \qquad T^{\nu}_{\mu;\nu} = 0$$

$$g_{\mu\nu} \rightarrow G_{\mu\nu} = -8\pi G T_{\mu\nu}$$

determine metric coeffs from $T_{\mu\nu}$

$$g_{\mu\nu} \rightarrow \nabla_{\nu} T^{\nu}_{\mu} = 0 \rightarrow G_{\mu\nu} = -8\pi G T_{\mu\nu}$$

determine evolution of $T_{\mu\nu}$ from metric and "physics"

Dark Phenomenology

modified "Einstein" eq:
(projection to 3+1D)

$$X_{\mu\nu} = -8\pi G T_{\mu\nu}$$

$$G_{\mu\nu} = -8\pi G T_{\mu\nu} - Y_{\mu\nu} \qquad Y_{\mu\nu} \equiv X_{\mu\nu} - G_{\mu\nu}$$

$Y_{\mu\nu}$ can be seen as an effective DE energy-momentum tensor.

Is it conserved?

Yes, since $T_{\mu\nu}$ is conserved, and since $G_{\mu\nu}$ obeys the Bianchi identities!

Cosmology can measure effective DE EMT

parametrisations

- could parametrise (effective) dark energy with anisotropic stress σ and pressure perturbation δp
- or directly deviations in metric potentials, e.g.

$$-k^2\phi = 4\pi G a^2 Q \rho_m \Delta_m \qquad \psi = (1+\eta)\phi$$

- in both cases two new functions of space and time -> much worse than w(z)!
- can either restrict form (e.g. just sub- and super-horizon behaviour) or course binning and PCA
- BUT: at least in principle we know what to look for! (And results can then be compared with theoretical predictions)

some model predictions

scalar field: $$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) \right)$$

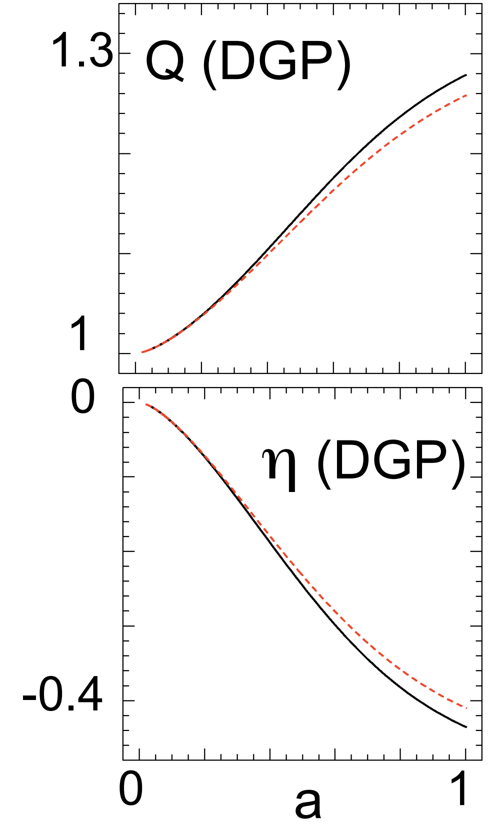
One degree of freedom: $V(\phi)$ <-> $w(z)$ therefore
other variables fixed: $c_s^2 = 1$, $\sigma = 0$
-> $\eta = 0$, $Q(k >> H_0) = 1$, $Q(k \sim H_0) \sim 1.1$

(naïve) DGP: compute in 5D, project result to 4D

Lue, Starkmann 04
Koyama, Maartens 06

$$\eta = \frac{2}{3\beta - 1} \qquad Q = 1 - \frac{1}{3\beta}$$

implies large
DE perturb.



Scalar-Tensor: Boisseau, Esposito-Farese, Polarski, Starobinski 2000, Acquaviva, Baccigalupi, Perrotta 04

$$\mathcal{L} = F(\varphi)R - \partial_\mu \varphi \partial^\mu \varphi - 2V(\phi) + 16\pi G^* \mathcal{L}_{\text{matter}}$$

$$\eta = \frac{F'^2}{F + F'^2} \qquad Q = \frac{G^*}{F G_0} \frac{2(F + F'^2)}{2F + 3F'^2}$$

f(R): $S_g = \int d^4x \sqrt{-g} f(R)$ similar to scalar-tensor

current “MG” constraints

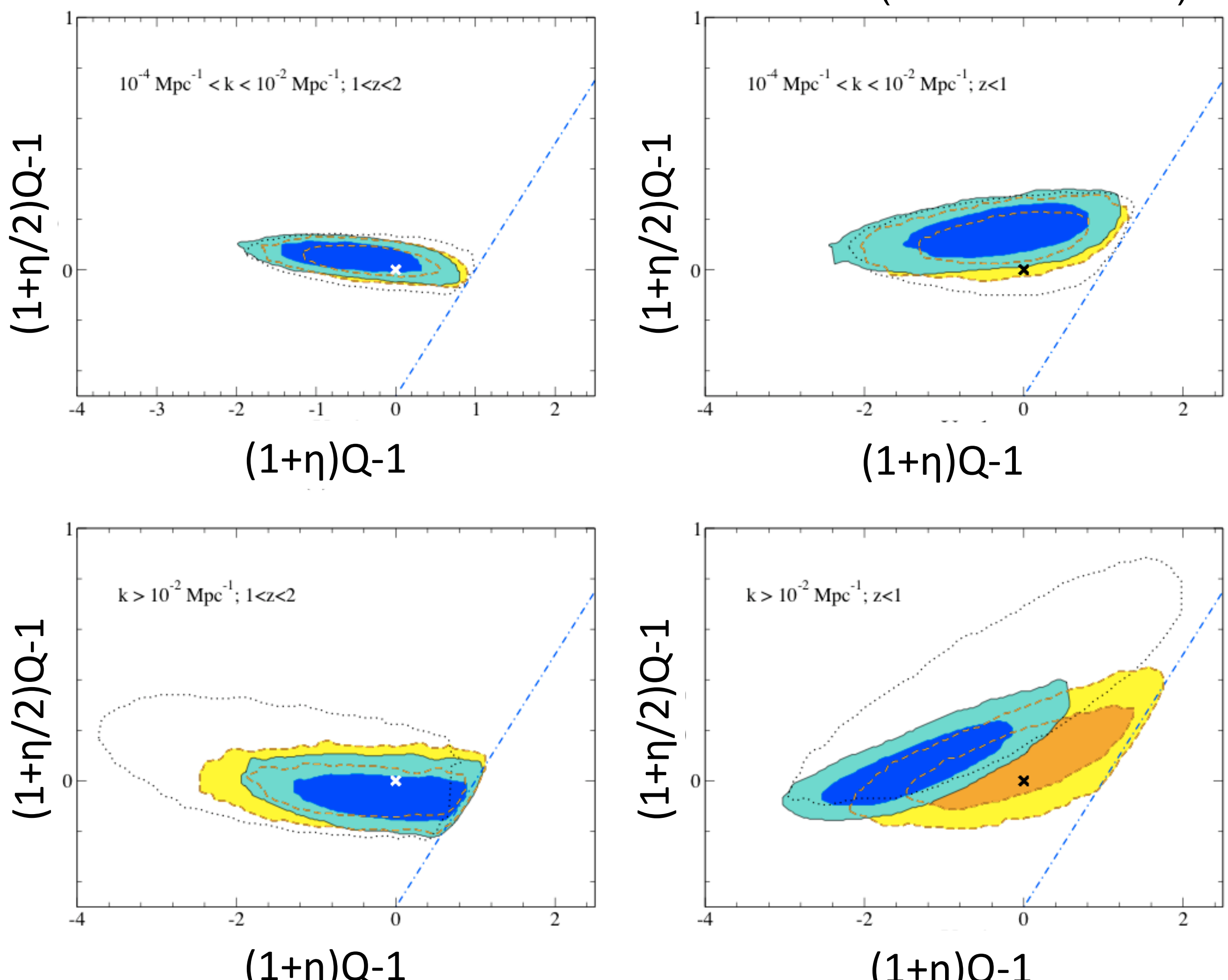
HOT TOPIC

(arxiv:1003:0001)

- 2x2 grid in k and z
- CMB + SN-Ia + WL + P(k)

- weak constraints
- WL data not very reliable (blue vs yellow)
- no deviation from GR

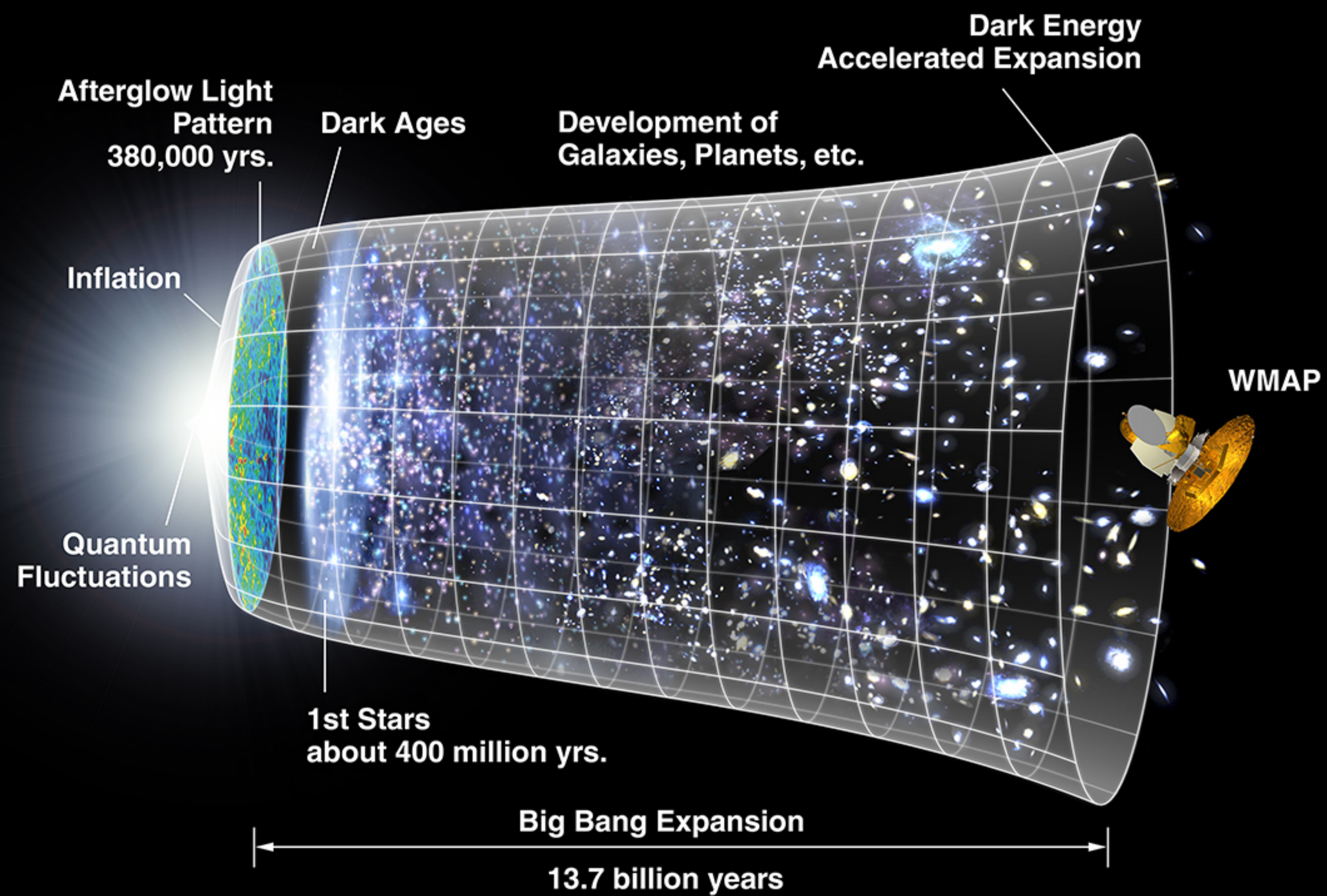
- future data will improve constraints by at least one order of magnitude



DE/MG summary

- The data clearly sees something incompatible with standard cosmology w/o DE.
- We have no model that we really like.
- Might still be due to mis-understanding of GR.
- Dark energy models need fine-tuning.
- Modified gravity models need screening.
- New d.o.f. necessary, usually look like scalars anyway! (-> difficult to distinguish MG – DE)
- The perturbation evolution contains much more information than w(a).
- But the data is perfectly in agreement with Λ

Brief history of the Universe



Resources (tiny subset!)

- Books & lecture notes
  - Scott Dodelson, "Modern Cosmology", AP 2003
  - Ruth Durrer, "The Cosmic Microwave Background", CUP 2008
  - Lots of reviews (e.g. Euclid theory group, arXiv:1206.1225)
  - Wayne Hu's webpage, background.uchicago.edu
  - my (old) lecture notes, http://theory.physics.unige.ch/~kunz/lectures/cosmo_II_2005.pdf
- codes
  - Boltzmann codes: CAMB (camb.info), CLASS (class-code.net), etc
  - cosmoMC (with many likelihoods), cosmologist.info/cosmomc/
  - icosmo, icosmos, Fisher4Cast, etc
- lots of cosmological data sets are publicly available!
  - WMAP (and others): Lambda archive, lambda.gsfc.nasa.gov
  - supernova data (e.g. supernova.lbl.gov/Union/)
  - …

Ze final words

There are known knowns.

These are things we know that we know.

There are known unknowns.

That is to say, there are things that we know we don't know.

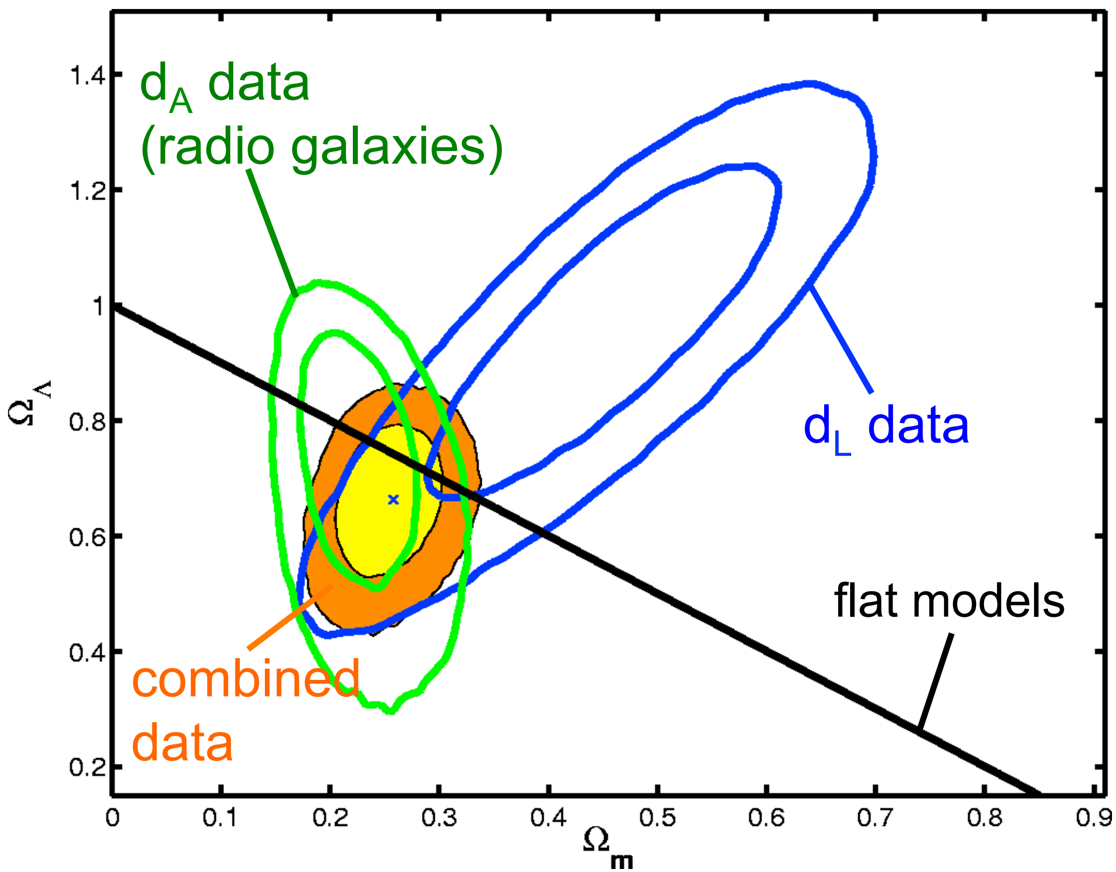
But there are also unknown unknowns.

There are things we don't know we don't know.

(Don, famous poet of early 21st century)

distance duality

- We found: $d_A = \frac{1}{1+z} d_m \quad d_L = (1+z) d_m \Rightarrow d_L = (1+z)^2 d_A$
- actually a very general relation, holds in all metric theories



- constrain photon loss, grey dust, axion-photon osc., etc
- very different systematics

-> no evidence of SN-Ia results being wrong!

(yes, there is newer data: BAO)

(in future maybe also gamma rays, gravitational waves from BH-BH mergers, and more)

LTB and Backreaction

Two large classes of models:

- Inhomogeneous cosmology: Copernican Principle is wrong, Universe is not homogeneous (and we live in a special place).

- Backreaction: GR is a nonlinear theory, so averaging is non-trivial. The evolution of the 'averaged' FLRW case may not be the same as the average of the true Universe.

Lemaitre-Tolman-Bondi

We live in the center of the world!

LTB metric: generalisation of FLRW to spherical symmetry, with new degrees of freedom

-> can choose a radial density profile, e.g. a huge void, to match one chosen quantity

☺ can mimic distance data (need to go out very far)

☺ demonstrates large effect from inhomogeneities

☹ unclear if all data can be mimicked (ISW, kSZ)

☹ mechanism to create such huge voids?

☹ fine-tuning to live in centre, ca $1{:}(1000)^3$ iirc

testing the geometry directly

Is it possible to test the geometry directly?

Yes! Clarkson et al (2008) -> in FLRW (integrate along ds=0):

$$H_0 D(z) = \frac{1}{\sqrt{-\Omega_k}} \sin\left(\sqrt{-\Omega_k} \int_0^z \frac{H_0}{H(u)} du\right)$$

$$\Rightarrow H_0 D'(z) = \frac{H_0}{H(z)} \cos\left(\sqrt{-\Omega_k} \int_0^z \frac{H_0}{H(u)} du\right)$$

$$\rightarrow \left(HD'\right)^2 - 1 = \sin^2(\cdots) = -\Omega_k \left(H_0 D\right)^2$$

It is possible to reconstruct the curvature by comparing a distance measurement (which depends on the geometry) with a radial measurement of H(z) without dependence on the geometry.

Backreaction

normal approach: separation into “background” and “perturbations”

$$g_{\mu\nu}(t,x) = \bar{g}_{\mu\nu}(t) + h_{\mu\nu}(t,x)$$

$$\rho(t,x) = \bar{\rho}(t) + \delta\rho(t,x)$$

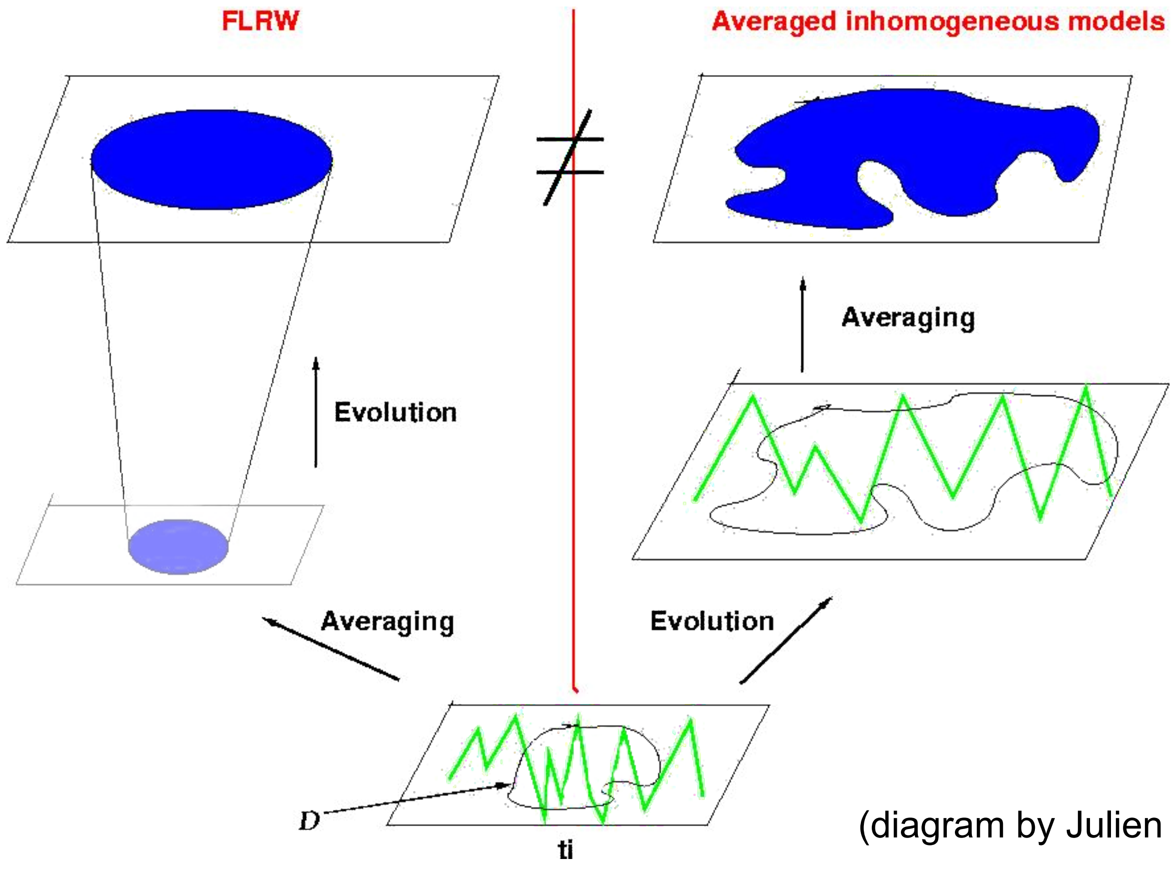
but which is the “correct” background, and why should it evolve as if it was a solution of Einsteins equations? The averaging required for the background does not commute with derivatives or quadratic expressions,

$$\partial_t \langle \phi \rangle \neq \langle \partial_t \phi \rangle \qquad \langle \theta^2 \rangle \neq \langle \theta \rangle^2$$

-> can derive set of averaged equations, taking into account that some operations not not commute: “Buchert equations”

average and evolution

the average of the evolved universe is in general not the evolution of the averaged universe!



(diagram by Julien Larena)

Buchert equations

- Einstein eqs, irrotational dust, 3+1 split (as defined by freely-falling observers)
- averaging over spatial domain D
- $a_D \sim V_D^{1/3}$ [<-> enforce isotropic & homogen. coord. sys.]
- set of effective, averaged, local eqs.:

$$\frac{\dot{a}_D}{a_D} = \frac{8\pi G}{3}\langle\rho\rangle_D - \frac{1}{6}\left(\mathcal{Q} + \langle\mathcal{R}\rangle_D\right) \quad 3\frac{\ddot{a}_D}{a_D} = -4\pi G\langle\rho\rangle_D + \mathcal{Q}$$

$$\mathcal{Q} = \frac{2}{3}\left\langle\left(\theta - \langle\theta\rangle_D\right)^2\right\rangle_D - \langle\sigma_{ij}\sigma^{ij}\rangle_D$$

if this is positive then
it looks like dark energy!

(θ expansion rate, σ shear, from expansion tensor Θ)

- $\langle\rho\rangle \sim a^{-3}$
- looks like Friedmann eqs., but with extra contribution!

Backreaction

- ☺ is certainly present at some level
- ☺ could possibly explain (apparent) acceleration without dark energy or modifications of gravity
- ☺ then also solves coincidence problem

- ☹ amplitude unknown (too small? [*])
- ☹ scaling unknown (shear vs variance of expansion)
- ☹ link with observations difficult

[*] Poisson eq: $-\left(\frac{k}{Ha}\right)^2 \phi = \frac{3}{2}\delta$ (k = aH : horizon size)

=> Φ never becomes large, only δ ! (but this is not a sufficient argument)