Introduction to (homogeneous) cosmology

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global outline

funny fluids

- **fundamental notions, the FLRW universe**
  - metric, scale factor, redshift, distances
  - Einstein eqn's, evolution of the universe
  - a bit of dark energy

real particles

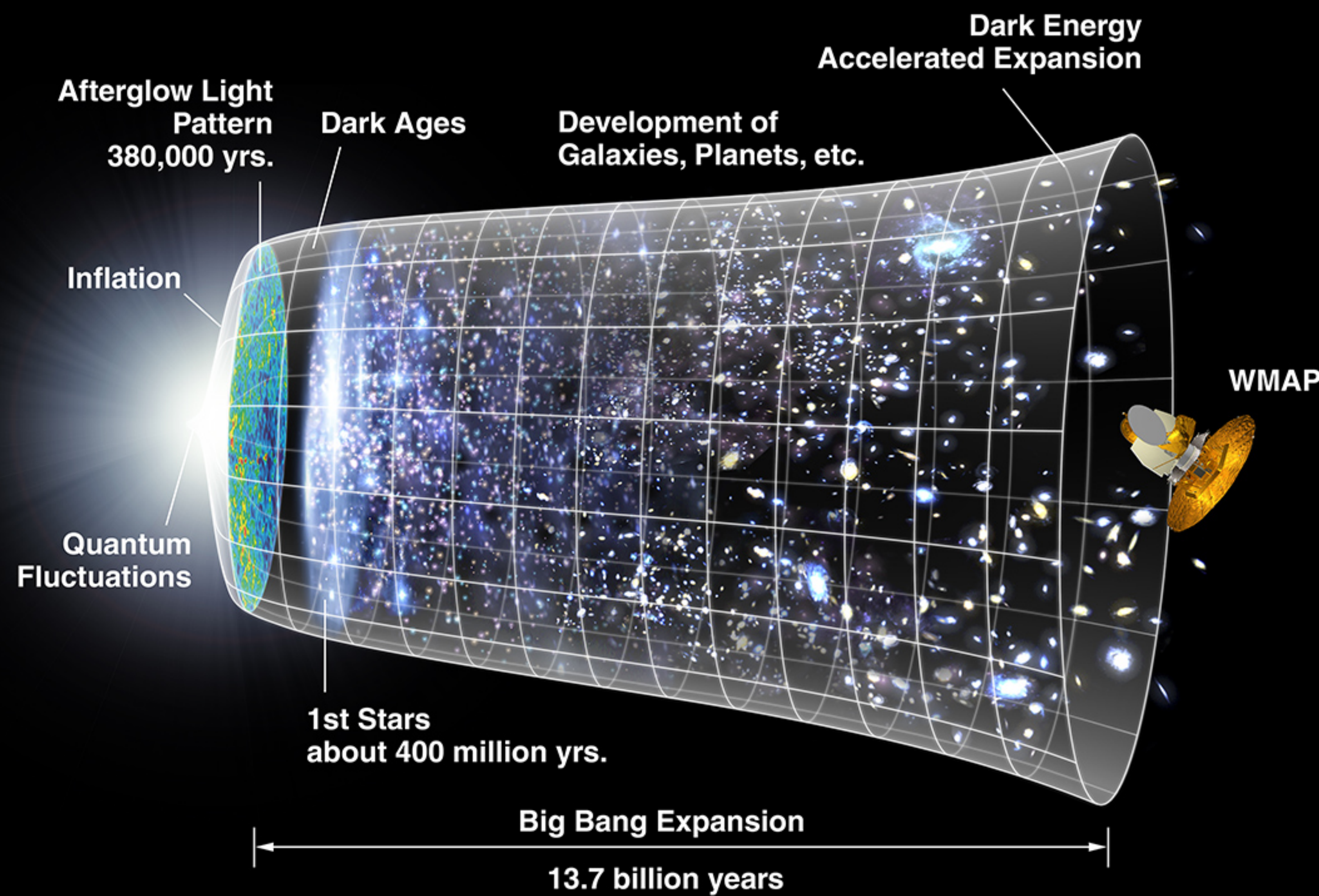
- **thermal history**
  - relativistic and non-relativistic particles
  - decouplings (neutrinos, photons, WIMP's)
  - BBN basics

crazy stuff

- **the early universe**
  - problems of standard cosmology
  - inflation, generation of perturbations, non-Gaussianity
- **perturbations, more on dark energy**
  - perturbation evolution, matter and CMB power spectra
  - more on dark energy and modified gravity models

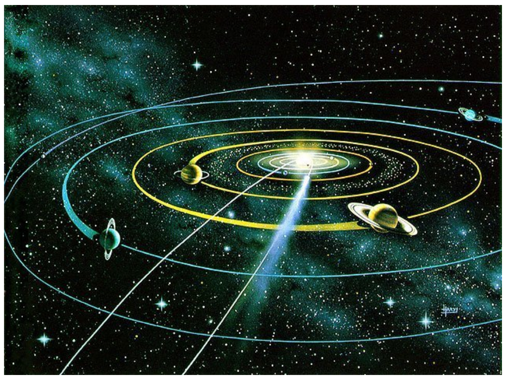
amazing observations

Brief history of the Universe



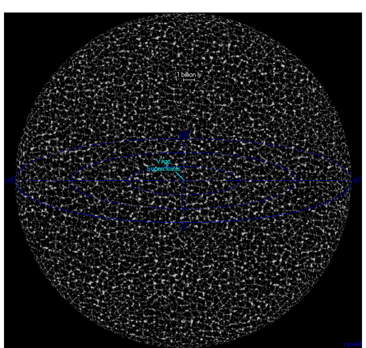
orders of magnitude

cosmology also goes right down to the Planck scale…
but you know this direction!



solar system:
size: billions of km (10^9 km)
$1AU = 1.5 \times 10^8$ km
Pluto ~ 40 AU

galaxies:
size ~ 10 kpc
1pc ≈ 3 light years = 3×10^{13} km
billions of stars (sizes vary!)



(observable) universe
size ~ 10 Gpc (~ 10^{23} km vs $l_P \sim 10^{-38}$ km)
~ 10^{11} galaxies

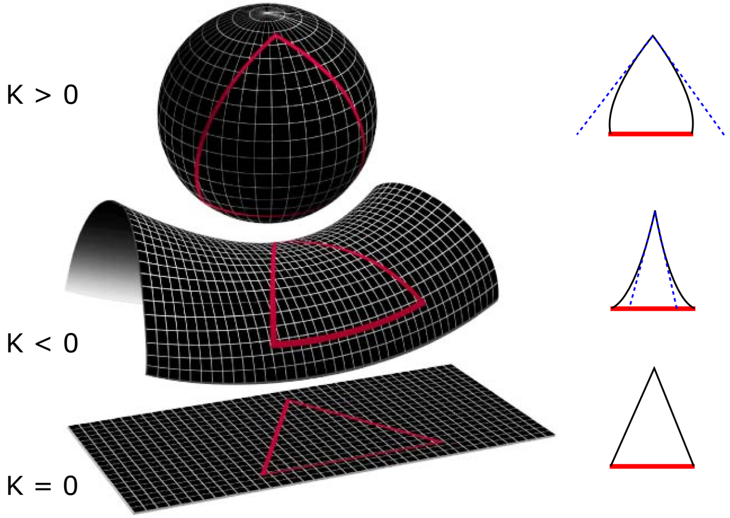
Outline for today

- **metric structure: cosmography**
  - the metric
  - expansion of the universe, redshift and Hubble’s law
  - cosmological distances and the age of the universe
- **content and evolution of the universe**
  - Einstein equations and the Bianchi identity
  - the critical density and the Ω’s
  - the evolution of the universe
- **very brief observational excursion**
- **dark energy and the future of the universe**
- **distance duality & inhomogeneous universes**

The metric

- **Cosmological principle**: all observers in the universe are equivalent
- implication: the universe is homogeneous and isotropic
- at least spatially -> **FLRW**
- theorem (differential geometry): spatial sections have constant curvature K(t)

$$ds^2 = dt^2 - \left(\frac{dR^2}{1 - KR^2} + R^2 d\Omega \right)$$



(at least for simply connected spaces)

The scale factor

- Maximal symmetry for spatial sections imposes an even stronger constraint: setting R(t) = a(t) r, the line element has the form

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - \kappa r^2} + r^2 d\Omega \right)$$

where k = ±1 or 0 is a constant

- For this metric, the curves (r,θ,ϕ)=const are geodesics for a 4-velocity u=(1,0,0,0) since $\Gamma^{\mu}{}_{00}$=0 [exercises] -> **comoving coordinates**

(geodesic eqn: $\ddot{X}^{\mu} + \Gamma^{\mu}_{\alpha\beta} \dot{X}^{\alpha} \dot{X}^{\beta} = 0$)

The cosmological redshift

Let us consider two galaxies at constant comoving separation ***d***, then for light moving from one to the other($ds^2=0$):

$$d = \int_{t_1}^{t_0} dt/a(t) = \int_{t_1+\delta t_1}^{t_0+\delta t_0} dt/a(t),$$

if δt_1 is e.g. the cycle time at emission, then to keep ***d*** constant we need that for δt_0 :

$$\frac{\delta t_0}{a(t_0)} = \frac{\delta t_1}{a(t_1)} \Rightarrow \frac{\delta t_1}{\delta t_0} = \frac{\nu_0}{\nu_1} = \frac{a(t_1)}{a(t_0)} = \frac{\lambda_1}{\lambda_0}$$

we observe therefore a redshift:

$$z \equiv (\lambda_0 - \lambda_1)/\lambda_1 \Rightarrow \boxed{1 + z = \frac{a(t_0)}{a(t_1)}}$$

The Hubble law

for two galaxies at a fixed **comoving** distance r_0:

physical distance $x(t) = a(t)r_0$

-> apparent motion:

$$\frac{dx}{dt} = \dot{a}r_0 = \frac{\dot{a}}{a}x \equiv H_0 x$$

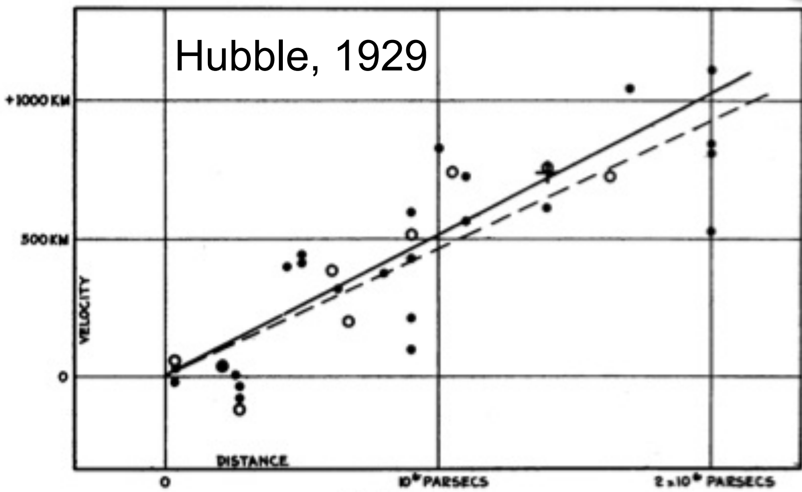


FIGURE 1

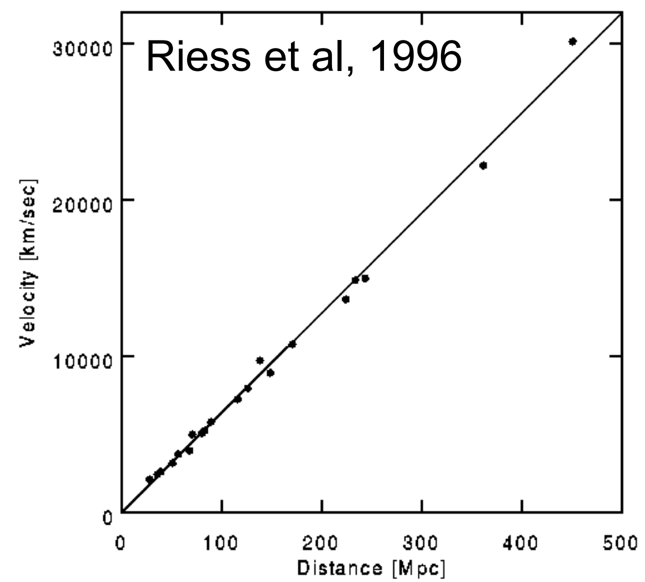
Velocity-Distance Relation among Extra-Galactic Nebulae.

Radial velocities, corrected for solar motion, are plotted against distances estimated from involved stars and mean luminosities of nebulae in a cluster. The black discs and full line represent the solution for solar motion using the nebulae individually; the circles and broken line represent the solution combining the nebulae into groups; the cross represents the mean velocity corresponding to the mean distance of 22 nebulae whose distances could not be estimated individually.

Utilisant les 42 nébuleuses figurant dans les listes de Hubble et de Strömberg [1], et tenant compte de la vitesse propre du soleil (300 Km. dans la direction $\alpha = 315°$, $\delta = 62°$), on trouve une distance moyenne de 0,95 millions de parsecs et une vitesse radiale de 600 Km./sec, soit 625 Km./sec à 10^6 parsecs [2].

Lemaitre, 1927 (!)

current value:
72±8 km/s/Mpc



cosmological distances

simpler to transform the distance variable r to χ:

$$r = S_\kappa(\chi) = \begin{cases} \sin\chi & \kappa = +1 \\ \chi & \kappa = 0 \\ \sinh\chi & \kappa = -1 \end{cases}$$

$$\Rightarrow ds^2 = dt^2 - a^2(t)\left(d\chi^2 + S_\kappa(\chi)^2 d\Omega\right)$$

$$\Rightarrow dV = a_0^2 S_\kappa(\chi)^2 d\Omega d\chi$$ volume element today

we can now *define* a «metric» distance:

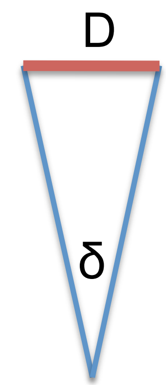
$$\boxed{d_m(\chi) = a_0 S_\kappa(\chi)}$$

$$\chi = \int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_{a_1}^{a_0} \frac{da}{a\dot{a}} = \frac{1}{a_0}\int_0^{z_1} \frac{dz}{H(z)}$$

cosmological distances

but physical distances need to be measurable!

1) angular diameter distance: object of physical size D observed under angle δ, but photons were emitted at time $t_1 < t_0$:



$$D = a(t_1)S_\kappa(\chi)\delta = \frac{a(t_1)}{a_0}a_0 S_\kappa(\chi)\delta \equiv d_A\delta$$

$$\boxed{d_A = \frac{1}{1+z}d_m}$$

2) luminosity distance: consider observed flux F for an object with known intrinsic luminosity L («standard candle»)

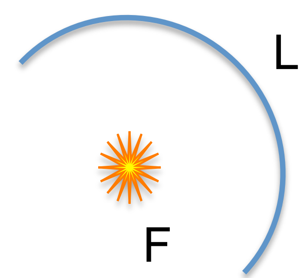
$$F \equiv \frac{L}{4\pi d_L^2}$$

surface: $4\pi d_m^2$

source emitting one photon per second:

1) redshift
2) increased time between arrivals

$$\boxed{d_L = (1+z)d_m}$$



age of the universe

computing the age of the universe is very straightforward:

$$t_0 = \int_0^{t_0} dt = \int_0^{a_0} \frac{da}{\dot{a}} = \int_0^{a_0} \frac{da}{aH(a)} = \int_0^{\infty} \frac{dz}{H(z)(1+z)}$$

but we need to know the evolution of the scale factor a(t). This in turn depends on the contents of the universe…

cue Einstein: $\underbrace{G_{\mu\nu}}_{\text{geometry}} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N \underbrace{T_{\mu\nu}}_{\text{content}}$

what is in the universe?

- homogeneous and isotropic metric: matter does also have to be distributed in this way
- in **some** coordinate system the energy momentum tensor has the form:

$$T^i_0 = 0, \quad T^1_1 = T^2_2 = T^3_3$$

  and the components depend only on time

$$\boxed{T^\nu_\mu = \mathrm{diag}\left(\rho(t), -p(t), -p(t), -p(t)\right)}$$

- the pressure determines the nature of the fluid, p = w ρ:
  - w = 0 : pressureless ‘dust’, ‘matter’
  - w = 1/3 : radiation
  - what is w for $T_{\mu\nu} = \Lambda g_{\mu\nu}$?

the conservation equation

- Bianchi identity (geometric identity for $G_{\mu\nu}$):

$$T^{\mu\nu}_{;\mu} = 0 = G^{\mu\nu}_{;\mu}$$

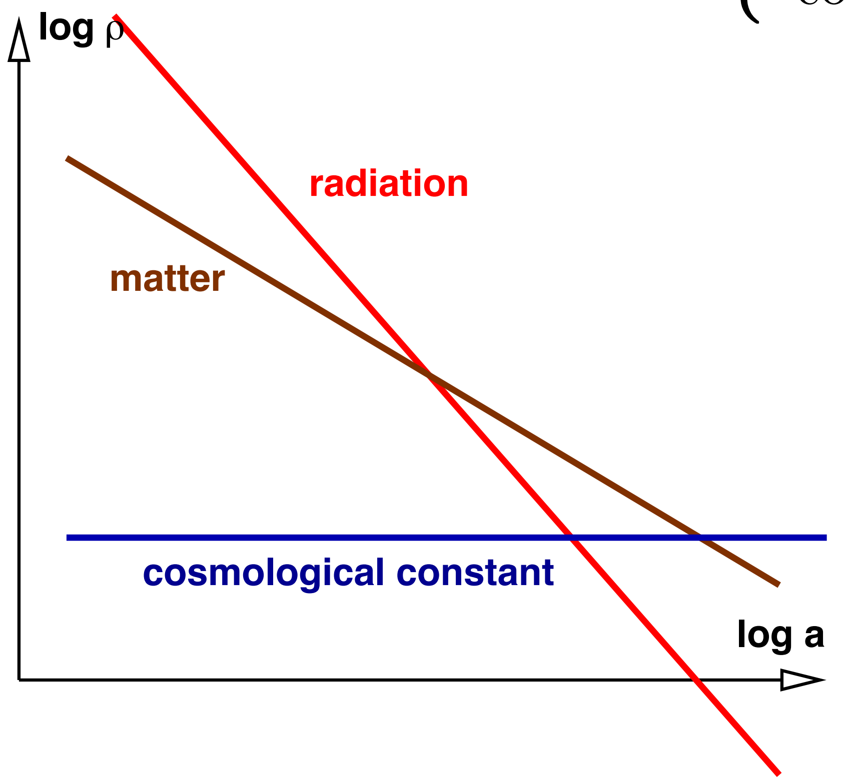
$$T^{\nu}_{0;\nu} = \dot{\rho} + \Gamma^{i}_{i0}(\rho + p) = \boxed{\dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)\underbrace{(\rho + p)}_{(1+w)\rho} = 0}$$

Questions (5 minutes, in groups):

- for a constant w, what is the evolution of ρ(a)?
  (eliminate the variable t from the equation)
- for the three cases w = 0, 1/3, -1, what is ρ(a)?
- does the result make sense?

evolution of the energy densities

$$\rho \propto a(t)^{-3(1+w)} \propto \begin{cases} a(t)^{-3} & \text{for } w = 0 \quad \text{(matter)} \\ a(t)^{-4} & \text{for } w = 1/3 \quad \text{(radiation)} \\ \text{const.} & \text{for } w = -1 \quad \text{(vacuum energy)} \end{cases}$$



dust/matter: dilution through expansion of space

radiation: additional redshift

at early times, the energy density in the universe should have been dominated by radiation

Einstein equations

- we now have all necessary ingredients to compute the Einstein equations:
  - metric
  - energy-momentum tensor

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$

$$R_{\mu\nu} \equiv R^{\alpha}_{\mu\alpha\nu} \qquad R \equiv g^{\mu\nu} R_{\mu\nu}$$

$$R^{\alpha}_{\beta\mu\nu} = \Gamma^{\alpha}_{\nu\beta,\mu} - \Gamma^{\alpha}_{\mu\beta,\nu} + \Gamma^{\delta}_{\nu\beta}\Gamma^{\alpha}_{\mu\delta} - \Gamma^{\delta}_{\mu\beta}\Gamma^{\alpha}_{\nu\delta}$$

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \left(g_{\beta\mu,\nu} + g_{\beta\nu,\mu} - g_{\mu\nu,\beta} \right)$$

you will compute it in the exercises… ☺

Friedmann equations

you should find:

$$R_{00} = -3\frac{\ddot{a}}{a} \qquad R_{ij} = -\left[\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\kappa}{a^2}\right] g_{ij}$$

$$R = -6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2}\right]$$

the space-time curvature is non-zero even for k=0!

0-0 component:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = \frac{8\pi G_N}{3}\rho$$

sum of ρ from all types of energy

i-i component:

$$2\left(\frac{\ddot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = -8\pi G_N p$$

Friedmann equations II

three comments:

- you can combine the two equations to find

  $$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G_N}{3}\left(\rho + 3p\right)$$

  -> the expansion is accelerating if $p < -\rho/3$
- the two Einstein equations and the conservation equation are not independent
- there are 3 unknown quantities (ρ, p and a) but only two equations, so one quantity needs to be given (normally p) – as well as the constant k.

the critical density

Friedmann eq. $\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = \frac{8\pi G_N}{3}\rho$

$$H \equiv \left(\frac{\dot{a}}{a}\right) \qquad \frac{\kappa}{a^2 H^2} = \frac{8\pi G_N \rho}{3H^2} - 1 \equiv \frac{\rho}{\rho_c} - 1 \equiv \Omega - 1$$

$$\Omega(t) > 1 \quad \Rightarrow \quad \kappa > 0 \Rightarrow \textbf{closed} \text{ universe}$$

$$\Omega(t) = 1 \quad \Rightarrow \quad \kappa = 0 \Rightarrow \textbf{flat} \text{ universe}$$

$$\Omega(t) < 1 \quad \Rightarrow \quad \kappa < 0 \Rightarrow \textbf{open} \text{ universe}$$

and: $\frac{d}{dt}\underbrace{\left(\frac{\Omega - 1}{\kappa}\right)}_{|\Omega - 1| \quad (\kappa \neq 0)} = \frac{d}{dt}\frac{1}{\dot{a}^2} = -2\frac{\ddot{a}}{\dot{a}^3}$

>0 for expanding universe filled with dust or radiation (and k ≠ 0)
-> the universe becomes “less flat”
-> strange (why?) -> Thursday

‘Ω form’ of Friedmann eq.

Friedmann eq. $$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = \frac{8\pi G_N}{3}\rho$$

evolution of ρ for the «usual» 4 constituents:

- radiation: a^{-4}
- dust: a^{-3}
- curvature: a^{-2} ($H^2 + k/a^2 \sim \rho$)
- cosmological constant: a^0

notation:

$$\Omega_X = \left.\frac{\rho_X}{\rho_c}\right|_{t_0}$$

$$H^2 = H_0^2 \left[\frac{8\pi G}{3H_0^2}\rho_0 \left(\frac{a}{a_0}\right)^{-n} + \ldots + \frac{\kappa}{H_0^2 a_0^2}\left(\frac{a}{a_0}\right)^{-2}\right]$$

$$H^2 = H_0^2 \left[\Omega_r \left(\frac{a}{a_0}\right)^{-4} + \Omega_m \left(\frac{a}{a_0}\right)^{-3} + \Omega_\Lambda + \Omega_\kappa \left(\frac{a}{a_0}\right)^{-2}\right]$$

$$\Omega_r + \Omega_m + \Omega_\Lambda + \Omega_\kappa = 1$$

evolution of the universe I

evolution of ρ for the «usual» 4 constituents:

- radiation: a^{-4}
- dust: a^{-3}
- curvature: a^{-2} ($H^2 + k/a^2 \sim \rho$)
- cosmological constant: a^0

notation:

$$\Omega_X = \left.\frac{\rho_X}{\rho_c}\right|_{t_0}$$

the primordial universe is dominated by radiation – sufficiently early we can neglect the rest!

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \Omega_r \left(\frac{a_0}{a}\right)^4 \Rightarrow a\dot{a} = \text{const.} \Rightarrow a da \propto dt$$

$$a(t) \propto t^{1/2}$$

evolution of the universe II

later, the radiation can be neglected (and we set Λ=0)

$$\dot{a}^2 = -\kappa + \frac{C}{a}, \qquad C = H_0 \Omega_0 a_0^3 > 0$$

1) $\kappa = 0 \rightarrow \sqrt{a}da = \sqrt{C}dt \text{ et } a(0) = 0 \Rightarrow \boxed{a(t) \propto t^{2/3}}$

2) $\kappa = 1 \rightarrow \dot{a}^2 = \frac{C}{a} - 1$

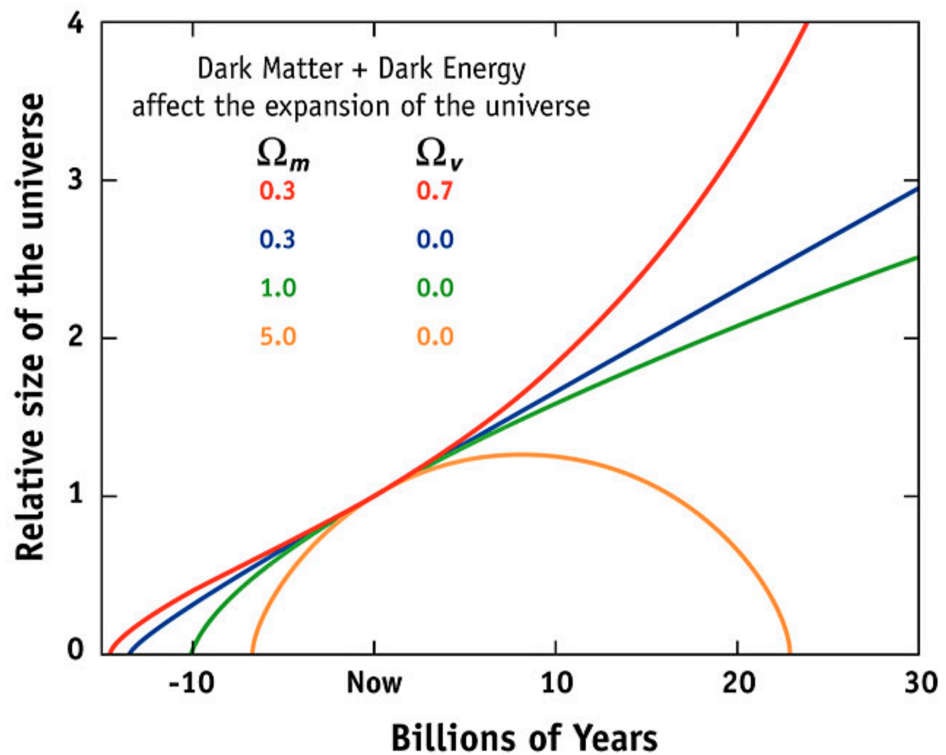
- for a=C we have that $\dot{a} = 0$ and the universe changes from expansion to contraction/collapse
- the substitution $a = C\sin^2\theta$ leads to $2C\sin^2\theta d\theta = dt$
- after integration: $t = C(\theta - \cos\theta\sin\theta)$
- we have a "big crunch" for θ=π et t=πC

evolution of the universe III

3) $\kappa = -1 \rightarrow \quad \dot{a}^2 = \frac{C}{a} + 1$

- always $\dot{a} \geq 0$ and the universe expands forever
- the substitution $a = C \sinh^2 \chi$ leads to $2C \sinh(\chi)^2 d\chi = dt$
- after integration:

$$t = C(\cosh\chi \sinh\chi - \chi)$$



finally we set $\Omega_\Lambda = 1$

$$\Omega_r = \Omega_m = \Omega_\kappa = 0$$

$$\dot{a}^2 = \frac{1}{3}\Lambda a^2$$

$$\boxed{a(t) = \exp(Ht),} \quad H = \sqrt{\frac{\Lambda}{3}}$$

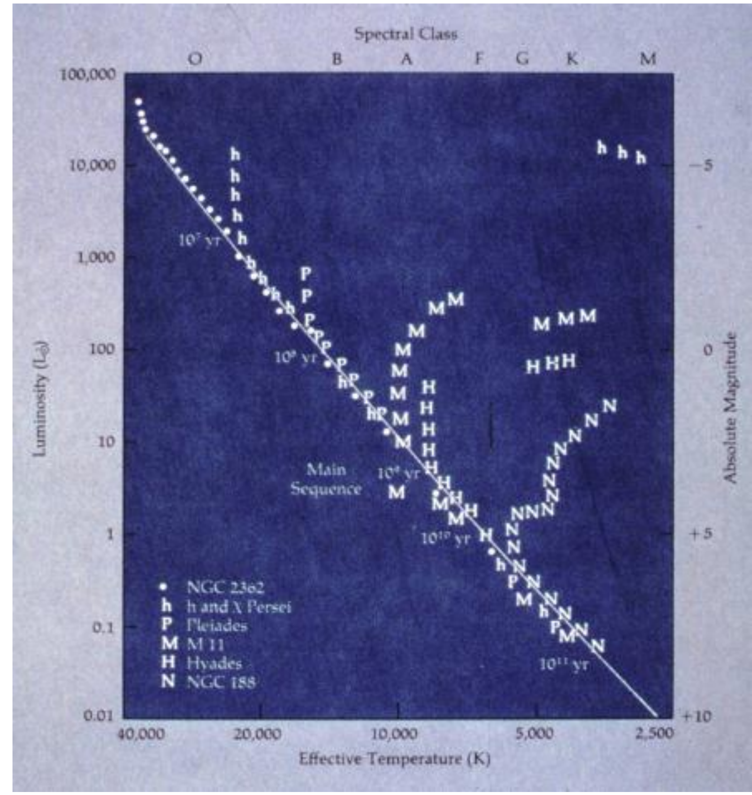
age of the universe revisited

we had: $$t_0 = \int_0^\infty \frac{dz}{H(z)(1+z)}$$

but for a matter-dominated universe:

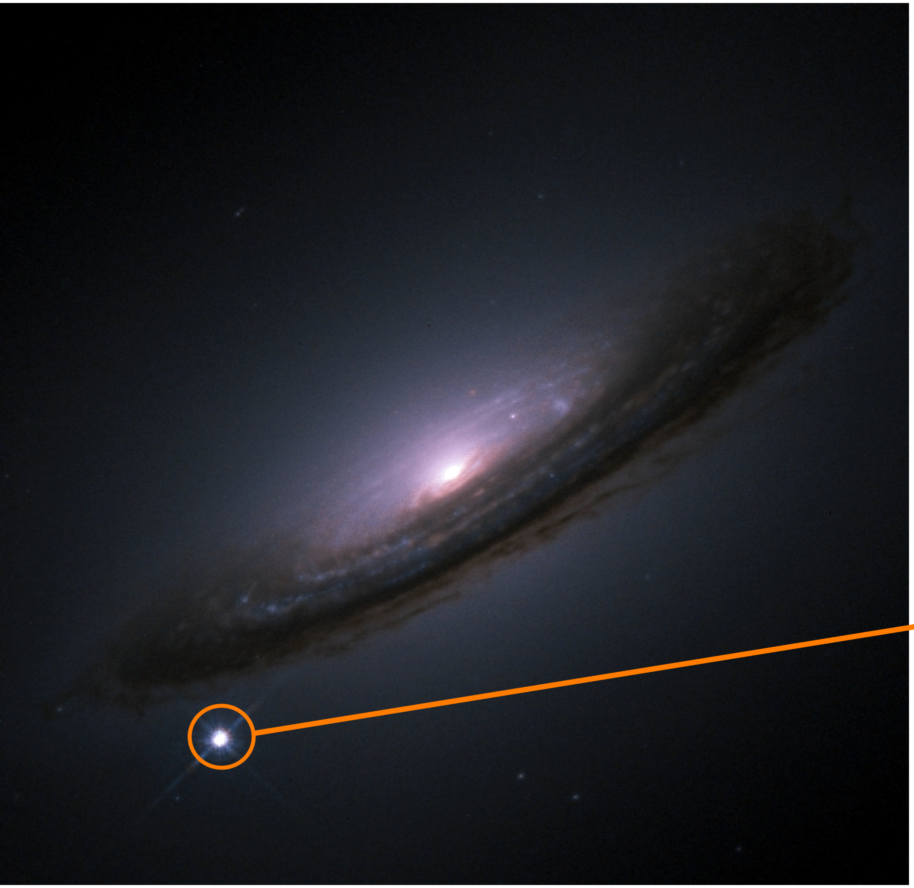
$$H = H_0 \left(\frac{a}{a_0}\right)^{-3/2} = H_0(1+z)^{3/2}$$

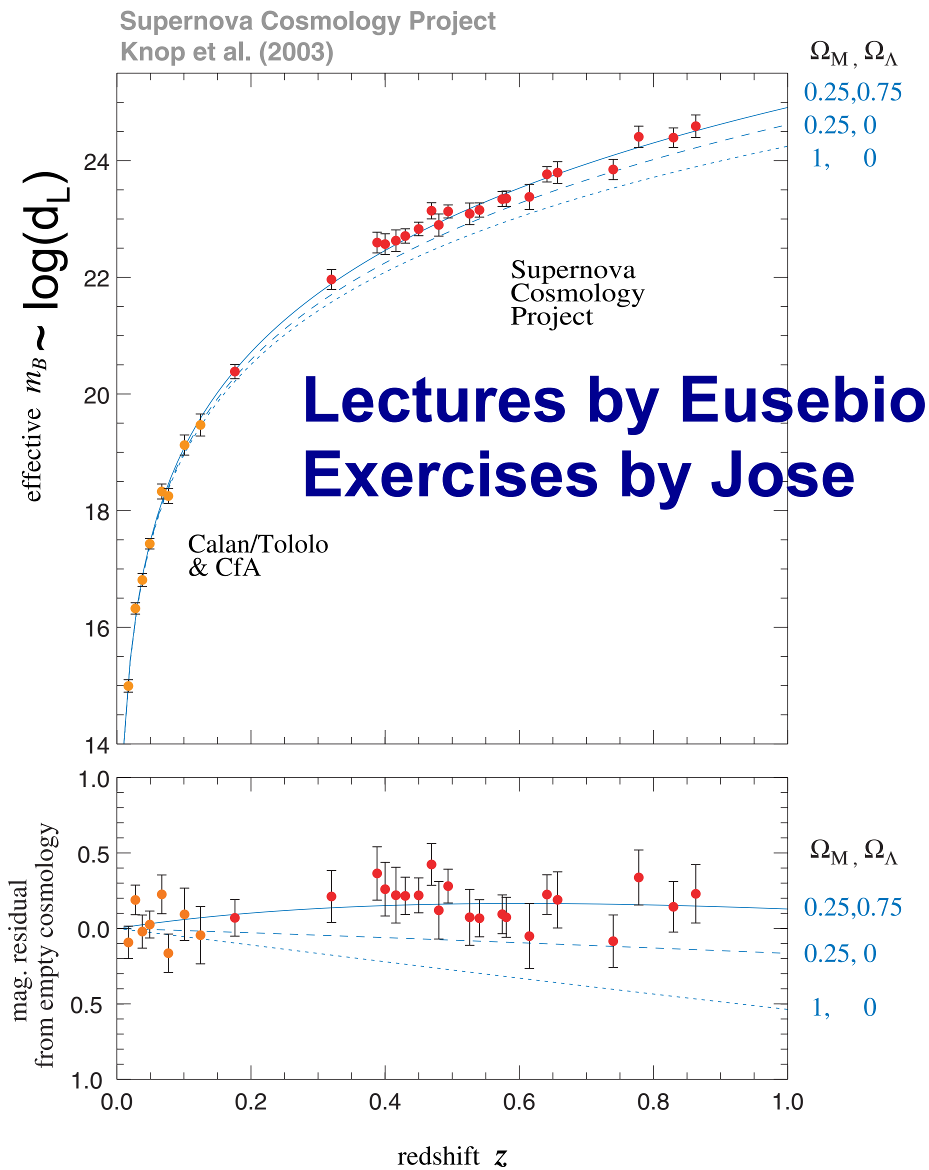
$$H_0 t_0 = \int_0^\infty \frac{dz}{(1+z)^{5/2}} = \int_1^\infty \frac{du}{u^{5/2}} = -\frac{2}{3}\left.\frac{1}{u^{3/2}}\right|_1^\infty = \frac{2}{3}$$



1/H_0 ~ 9.8 Gyr/[H_0/100 km/s/Mpc] ~ 13.6 Gyr -> t_0 ~ 9 Gyr but oldest globular star clusters are older: 11-18 Gyr …??!!

distances revisited





Lectures by Eusebio
Exercises by Jose

The luminosity distance depends on the contents of the universe through the expansion rate!

summary

- **Methods**
  - FLRW metric for homogeneous and isotropic space
  - perfect fluid energy-momentum tensor
  - Einstein eq (Friedmann eq) and Bianchi identity
  - photon and particle geodesics
- **Results**
  - comoving coordinate system, redshift, Hubble law
  - expansion history of universe linked to contents
  - distances and redshift give constraints on expansion history and therefore on the contents
- **and now for some other things …**

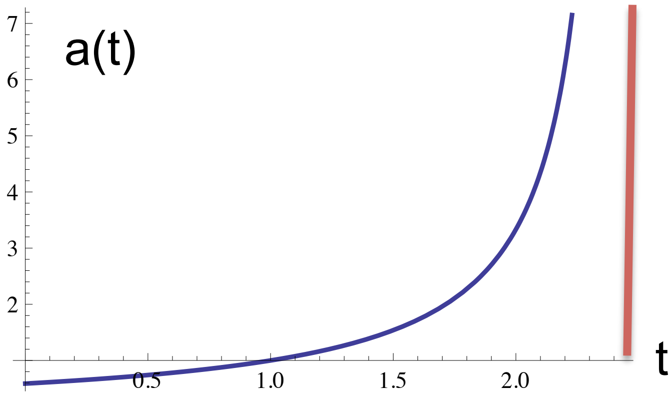
phantom DE and the big rip

- what happens if p/ρ = w < -1?
- (apart from classical and quantum instabilities)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left(\Omega_m a^{-3} + (1-\Omega_m)a^{-3(1+w)}\right) \to H_0^2(1-\Omega_m)a^{-3(1+w)}$$

- solve for a(t) with $a(t_0)=1$; let's pick w = -5/3 (similar for any w<-1)

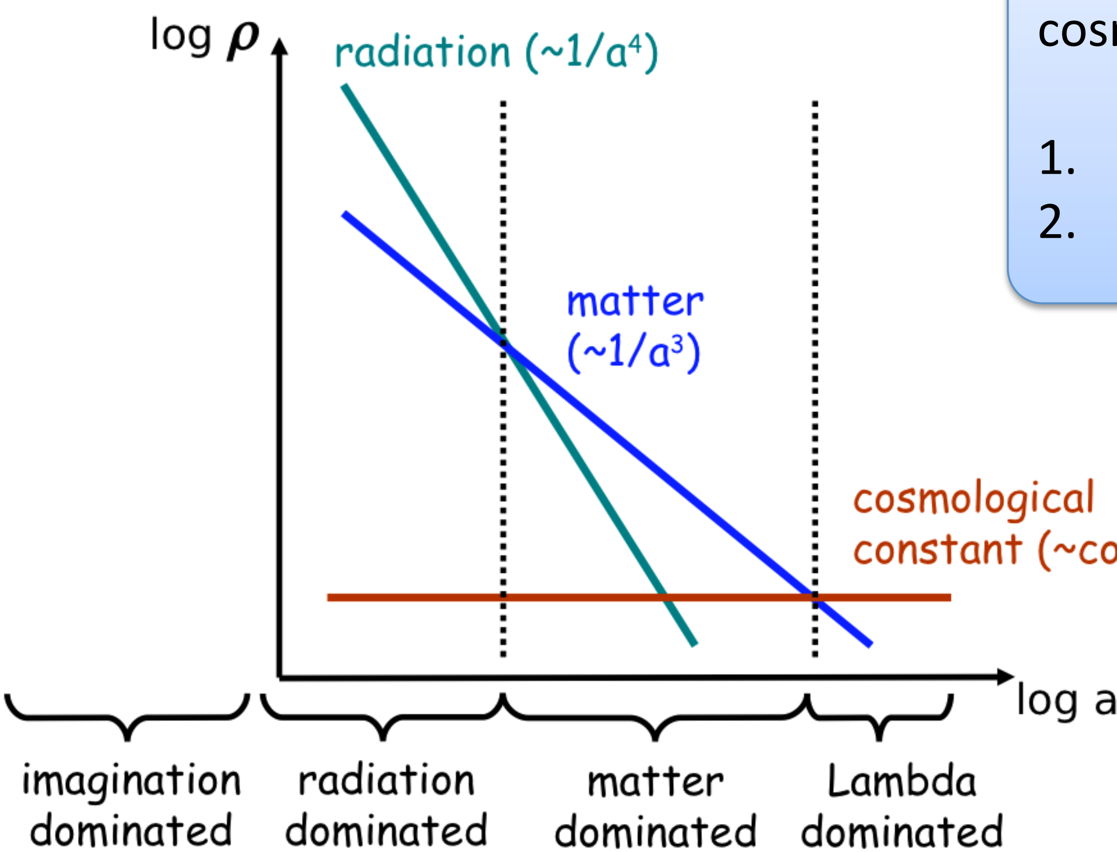
$$\to a(t) = \left(1 - H_0\sqrt{1-\Omega_m}(t-t_0)\right)^{-1}$$



infinite expansion at finite age!
-> big rip

What's the problem with Λ ?

Evolution of the Universe:

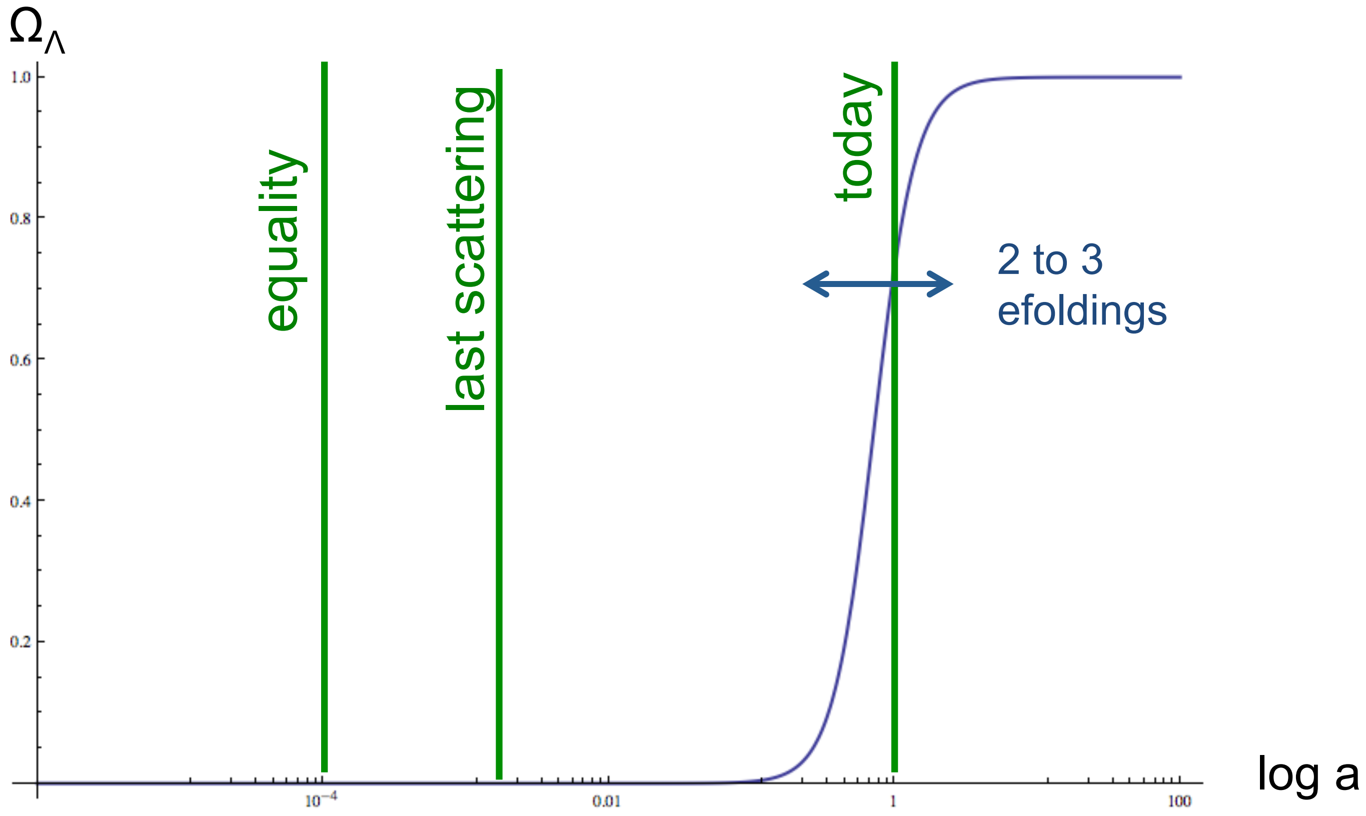


Classical problems of the cosmological constant:

1. Value: why so small? Natural?
2. Coincidence: Why now?

the coincidence problem

- why are we just now observing $\Omega_\Lambda \approx \Omega_m$?
- past: $\Omega_m \approx 1$, future: $\Omega_\Lambda \approx 1$



the naturalness problem

energy scale of observed Λ is ~ 2x10^{-3} eV
zero point fluctuations of a heavier particle of mass m:

cutoff scale

$$\int_0^{\Lambda} \frac{1}{2} \frac{d^3p}{(2\pi)^3} \sqrt{p^2+m^2} \simeq \frac{1}{16\pi^2} \left[\underbrace{\Lambda^4 + m^2\Lambda^2}_{\text{can in principle be absorbed into renormalization of observables}} - \underbrace{\frac{1}{4} m^4 \log\left(\frac{\Lambda^2}{m^2}\right)}_{\text{“running” term: this term is measureable for masses and couplings! Why not for cosmological constant?!}} \right]$$

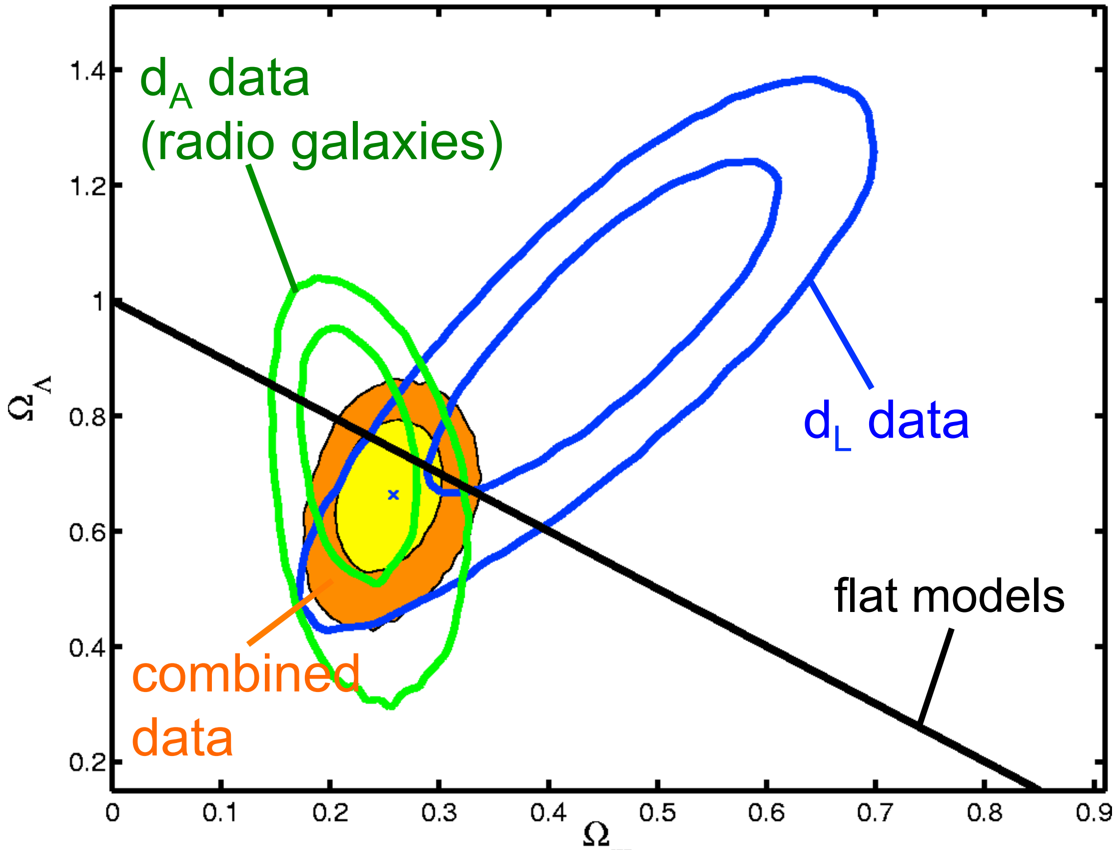
already the electron should contribute at m_e >> eV
(and the muon, and all other known particles!)

Possible explanations

1. It is a cosmological constant, and there is no problem ('anthropic principle', 'string landscape')
2. The (supernova) data is wrong
3. We are making a mistake with GR (aka 'backreaction') or the Copernican principle is violated ('LTB')
4. It is something evolving, e.g. a scalar field ('dark energy')
5. GR is wrong and needs to be modified ('modified gravity')

distance duality

- We found: $d_A = \frac{1}{1+z} d_m \quad d_L = (1+z) d_m \Rightarrow \boxed{d_L = (1+z)^2 d_A}$
- actually a very general relation, holds in all metric theories



• constrain photon loss, grey dust, axion-photon osc., etc

• very different systematics

-> no evidence of SN-Ia results being wrong!

(yes, there is newer data: BAO)

(in future maybe also gamma rays, gravitational waves from BH-BH mergers, and more)

LTB and Backreaction

Two large classes of models:

- Inhomogeneous cosmology: Copernican Principle is wrong, Universe is not homogeneous (and we live in a special place).

- Backreaction: GR is a nonlinear theory, so averaging is non-trivial. The evolution of the 'averaged' FLRW case may not be the same as the average of the true Universe.

Lemaitre-Tolman-Bondi

We live in the center of the world!

LTB metric: generalisation of FLRW to spherical symmetry, with new degrees of freedom

-> can choose a radial density profile, e.g. a huge void, to match one chosen quantity

☺ can mimic distance data (need to go out very far)

☺ demonstrates large effect from inhomogeneities

☹ unclear if all data can be mimicked (ISW, kSZ)

☹ mechanism to create such huge voids?

☹ fine-tuning to live in centre, ca $1:(1000)^3$ iirc

testing the geometry directly

Is it possible to test the geometry directly?

Yes! Clarkson et al (2008) -> in FLRW (integrate along ds=0):

$$H_0 D(z) = \frac{1}{\sqrt{-\Omega_k}} \sin\left(\sqrt{-\Omega_k} \int_0^z \frac{H_0}{H(u)} du\right)$$

$$\Rightarrow H_0 D'(z) = \frac{H_0}{H(z)} \cos\left(\sqrt{-\Omega_k} \int_0^z \frac{H_0}{H(u)} du\right)$$

$$\rightarrow \left(HD'\right)^2 - 1 = \sin^2(\cdots) = -\Omega_k \left(H_0 D\right)^2$$

It is possible to reconstruct the curvature by comparing a distance measurement (which depends on the geometry) with a radial measurement of H(z) without dependence on the geometry.

Backreaction

normal approach: separation into “background” and “perturbations”

$$g_{\mu\nu}(t,x) = \bar{g}_{\mu\nu}(t) + h_{\mu\nu}(t,x)$$

$$\rho(t,x) = \bar{\rho}(t) + \delta\rho(t,x)$$

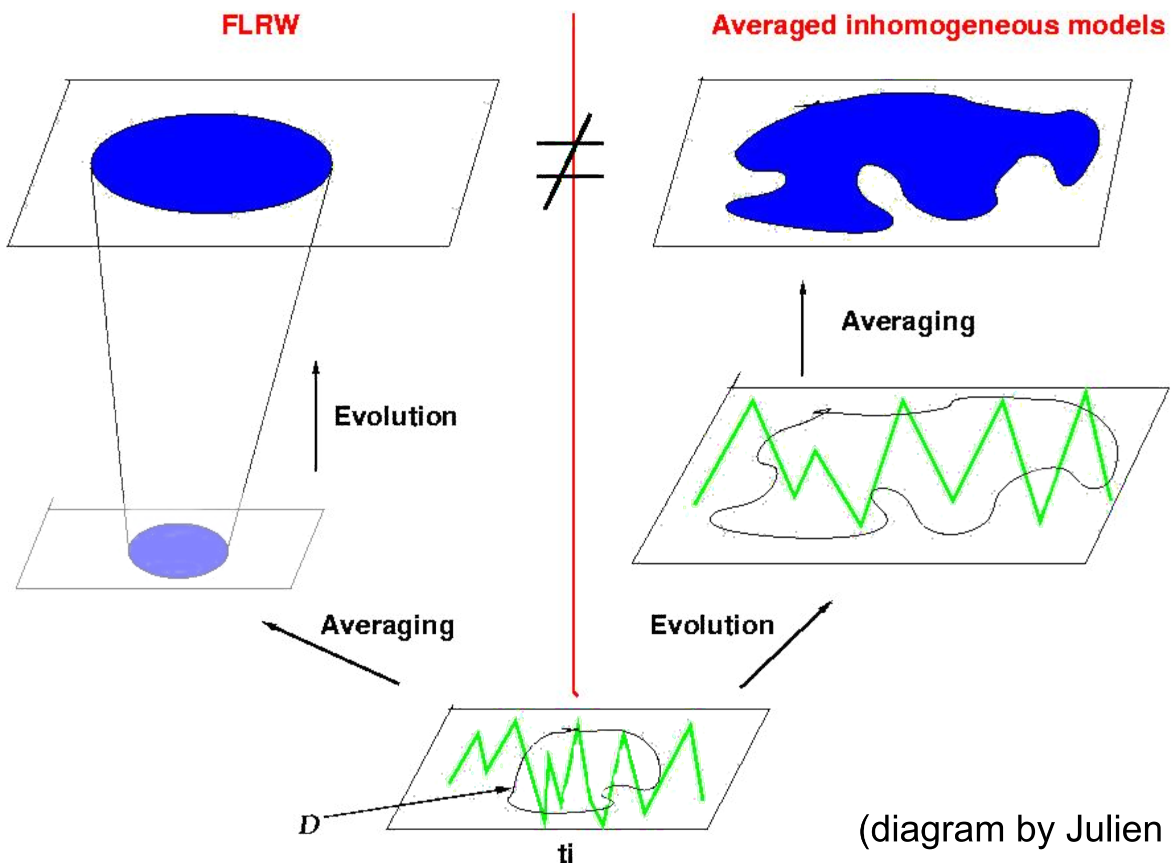
but which is the “correct” background, and why should it evolve as if it was a solution of Einsteins equations? The averaging required for the background does not commute with derivatives or quadratic expressions,

$$\partial_t \langle \phi \rangle \neq \langle \partial_t \phi \rangle \qquad \langle \theta^2 \rangle \neq \langle \theta \rangle^2$$

-> can derive set of averaged equations, taking into account that some operations not not commute: “Buchert equations”

average and evolution

the average of the evolved universe is in general not the evolution of the averaged universe!



(diagram by Julien Larena)

Buchert equations

- Einstein eqs, irrotational dust, 3+1 split (as defined by freely-falling observers)
- averaging over spatial domain D
- $a_D \sim V_D^{1/3}$ [<-> enforce isotropic & homogen. coord. sys.]
- set of effective, averaged, local eqs.:

$$\frac{\dot{a}_D}{a_D} = \frac{8\pi G}{3}\langle\rho\rangle_D - \frac{1}{6}\left(\mathscr{Q} + \langle\mathscr{R}\rangle_D\right) \quad 3\frac{\ddot{a}_D}{a_D} = -4\pi G\langle\rho\rangle_D + \mathscr{Q}$$

$$\mathscr{Q} = \frac{2}{3}\left\langle\left(\theta - \langle\theta\rangle_D\right)^2\right\rangle_D - \langle\sigma_{ij}\sigma^{ij}\rangle_D$$

if this is positive then
it looks like dark energy!

(θ expansion rate, σ shear, from expansion tensor Θ)

- $\langle\rho\rangle \sim a^{-3}$
- looks like Friedmann eqs., but with extra contribution!

Backreaction

- ☺ is certainly present at some level
- ☺ could possibly explain (apparent) acceleration without dark energy or modifications of gravity
- ☺ then also solves coincidence problem

- ☹ amplitude unknown (too small? [*])
- ☹ scaling unknown (shear vs variance of expansion)
- ☹ link with observations difficult

[*] Poisson eq: $-\left(\frac{k}{Ha}\right)^2 \phi = \frac{3}{2}\delta$ (k = aH : horizon size)

=> Φ never becomes large, only δ ! (but this is not a sufficient argument)

Resources (tiny subset!)

- Books & lecture notes
  - Scott Dodelson, “Modern Cosmology”, AP 2003
  - Ruth Durrer, “The Cosmic Microwave Background”, CUP 2008
  - Lots of reviews (e.g. Euclid theory group, arXiv:1206.1225)
  - Wayne Hu’s webpage, background.uchicago.edu
  - my (old) lecture notes, http://theory.physics.unige.ch/~kunz/lectures/cosmo_II_2005.pdf
- codes
  - Boltzmann codes: CAMB (camb.info), CLASS (class-code.net), etc
  - cosmoMC (with many likelihoods), cosmologist.info/cosmomc/
  - icosmo, icosmos, Fisher4Cast, etc
- lots of cosmological data sets are publicly available!
  - WMAP (and others): Lambda archive, lambda.gsfc.nasa.gov
  - supernova data (e.g. supernova.lbl.gov/Union/)
  - …