EFFECTIVE FIELD THEORIES

Ulf-G. Meißner, Univ. Bonn & FZ Jülich

Supported by DFG, SFB/TR-16

and by DFG, SFB/TR-110

and by EU, I3HP EPOS

and by BMBF 06BN7008

and by HGF VIQCD VH-VI-417

STRUCTURE of the LECTURES

I) Introduction

II) Effective Field Theories: Learning by Examples

III) The Paradigm Shift in Quantum Field Theory

IV) Structure of Effective Field Theories

V) A detailed Look at a Model EFT

VI) Chiral QCD Dynamics

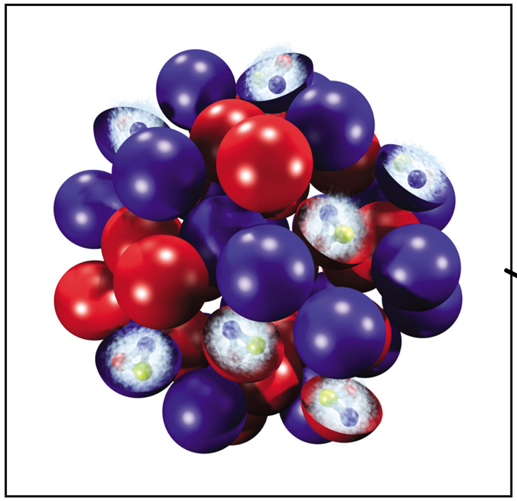
VII) Testing Chiral Dynamcis in Hadron-Hadron Scattering

- more emphasis on the foundations rather than on specific calculations

Introduction

BASIC IDEAS: RESOLUTION MATTERS

- Dynamics at long distances does not depend on what goes on at short distances
- Equivalently, low energy interactions do not care about the details of high energy interactions
- Or: you don't need to understand nuclear physics to build a bridge



BASIC IDEAS: ORGANISATION

- This is quite true, but how to make the idea precise and quantitative?
- necessary & sufficient ingredients to construct an **E**ffective **F**ield **T**heory:

  ⋆ *scale separation* – what is low, what is high?

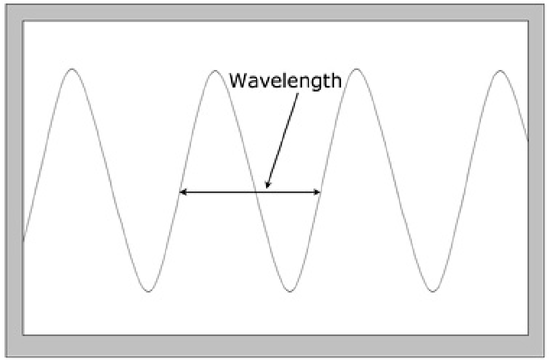
  ⋆ *active degrees of freedom* – what are the building blocks?

  ⋆ *symmetries* – how are the interactions constrained by symmetries?

  ⋆ *power counting* – how to organize the expansion in low over high?

- a note on units for a quantum particle ($\hbar = c = 1$)

$$p \sim \frac{1}{\lambda}, \quad E = p \quad \text{or} \quad E = \frac{p^2}{2m}$$



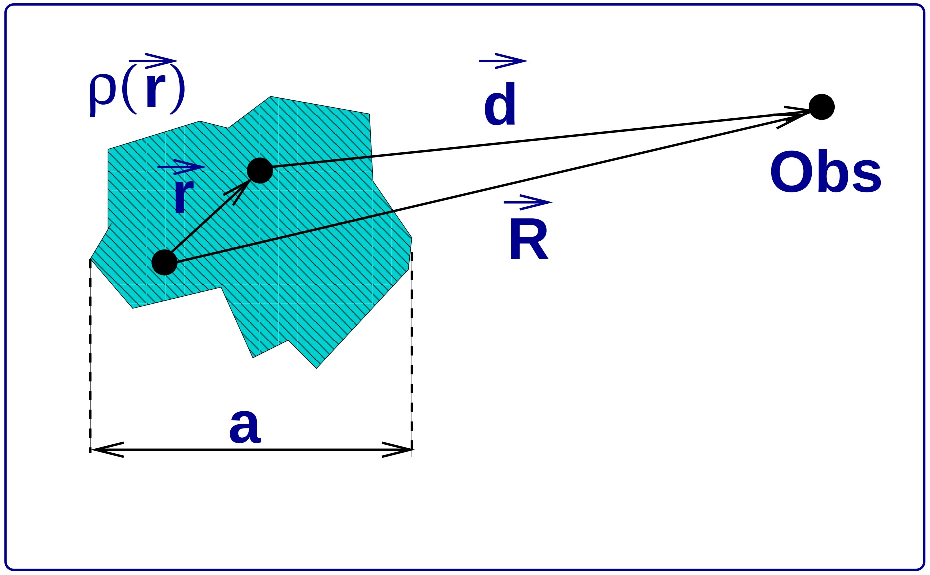
$\longrightarrow$ long wavelength $\leftrightarrow$ low momentum

Effective Field Theory: Learning by Example

EXAMPLE 1: MULTIPOLE EXPANSION

- Multipole expansion for electric potentials [not quite a quantum field theory]

$$
\begin{aligned}
V &\approx \int \frac{\rho(\vec{r})}{d} d^3r \\
&= \int \frac{\rho(\vec{r})}{\sqrt{R^2 + 2rR\cos\theta + r^2}} d^3r \\
&= \sum_{n=0}^{\infty} \frac{1}{R^{n+1}} \int r^n P_n(\cos\theta)\rho(\vec{r}) d^3r \\
&= q\frac{1}{R} + p\frac{1}{R^2} + Q\frac{1}{R^3} + \ldots
\end{aligned}
$$



- the sum converges quickly for $a \ll R$

- long-distance (low-energy) probes are only sensitive to bulk properties:

  charge q, dipole moment p, $\ldots$

- aside: “don’t be a slave of indices, make them your slaves” [Howard Georgi]

EXAMPLE 2: WHY THE SKY IS BLUE

- Light-atom scattering involves very different scales:

$$\lambda_{\rm light} \sim 5000\ \text{Å} \gg a_{\rm atom} \sim \text{ a few Å} \sim \text{ a few } a_0$$

$\Rightarrow$ photons are insensitive to the atomic structure

$$\stackrel{\text{gauge inv.},P,T}{\Longrightarrow} \quad \boxed{H_{\rm eff} = \chi^\star \left[-\tfrac{1}{2}c_E\vec{E}^2 - \tfrac{1}{2}c_B\vec{B}^2\right]\chi}$$

(χ = atomic wave function)

- fixing the constants: $\frac{\text{field energy}}{\text{volume}} \sim \frac{1}{2}(\vec{E}^2+\vec{B}^2) \Rightarrow \boxed{c_E = k_E\, a_0^3,\ c_B = k_B\, a_0^3}$

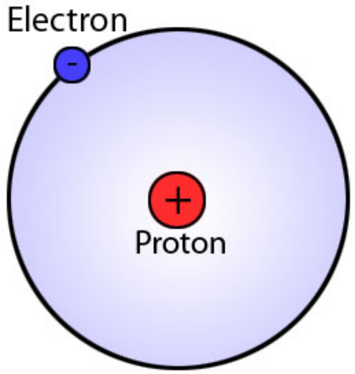
- If k_E and k_B are natural, i.e.of order one, and with $|\vec{E}| \sim \omega$ and $|\vec{B}| \sim |\vec{k}| \sim \omega$:

$$\boxed{\frac{d\sigma}{d\Omega} = |\langle f|H_{\rm eff}|i\rangle|^2 \sim \omega^4\, a_0^6 \left(1+\frac{\omega^2}{\Delta E^2}\right)}$$

ΔE = corr. due to atomic excitations

EXAMPLE 3: THE HYDROGEN ATOM

- text-book example of a quantum bound state of an electron and a proton



- lowest order: we need the mass & charge of the electron & charge of the proton & the static Coulomb interaction:

$$E = E_0 = -\frac{m_e \alpha^2}{2n^2}\ , \quad \alpha = \frac{e^2}{4\pi}$$

but this is not the *exact* answer, how can we improve on it?

we can get an approximate answer and improve on it $\longrightarrow$ difference to maths!

- beyond leading order: $E = E_0\left[1 + \mathcal{O}\left(\alpha, \frac{m_e}{m_p}\right)\right]$ $\boxed{\longrightarrow \text{systematic expansion}}$

  – corrections from the em interaction fine-structure from $\vec{L}\cdot\vec{S} \sim \alpha^4$ etc.

  – corrections from the proton structure $m_p \longrightarrow$ reduced mass $\mu = \frac{m_e m_p}{m_e + m_p}$

  $\mu_p \longrightarrow$ hyperfine interaction

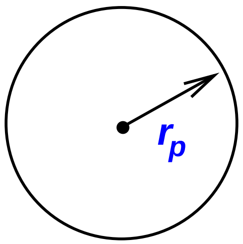
EXAMPLE 3 cont'd: DIMENSIONAL ANALYSIS

- calculate the influence of the proton **size** r_p on the hydrogen energy levels

- natural scales: length $a_0 = 1/(m_e\alpha) \sim 0.5$ Å

  time $1/\mathrm{Ryd} = 2/(m_e\alpha^2)\,, \quad 1\mathrm{Ryd} = 13.6$ eV

- proton charge radius: $F_1(q^2) = 1 + q^2 F_1'(0) + \ldots$



$$F_1'(0) \simeq \frac{1}{m_p^2}\,, \quad q \sim \frac{1}{a_0} = m_e\alpha \;\to\; \left(\frac{m_e\alpha}{m_p}\right)^2 \sim 10^{-11}$$

- contribution of $\mathcal{O}(40\,\mathrm{kHz})$ to the energy levels

- $1/m_p = 0.2$ fm. Actual proton size $\simeq 0.85$ fm

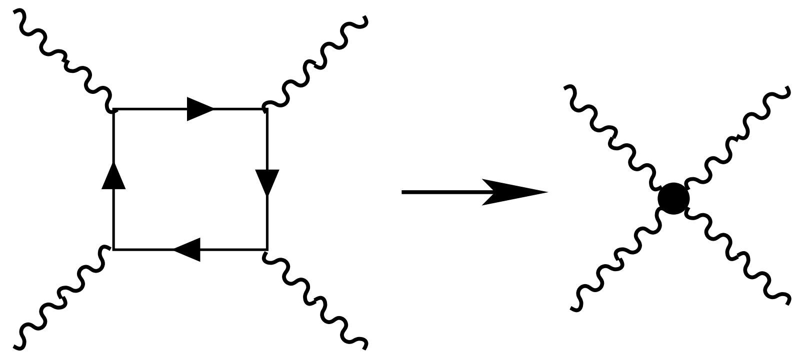
  $\longrightarrow$ net contribution to $1S$ about 1000 kHz

- Proton size measurable in *muonic hydrogen* $(m_\mu/m_e \sim 200)$ Pohl et al. (2010)

what a pleasure: can do calculations without knowing the underlying theory

EXAMPLE 4: LIGHT-BY-LIGHT SCATTERING

Euler, Heisenberg, Kockel 1936



- energy scales: photon energy ω,
  electron mass m_e

- consider $\omega \ll m_e$

- fermions as massive dofs integrated out: $\mathcal{L}_{\rm QED}[\psi, \bar\psi, A_\mu] \to \mathcal{L}_{\rm eff}[A_\mu]$

$$\boxed{\mathcal{L}_{\rm eff} = \frac{1}{2}(\vec{E}^2 - \vec{B}^2) + \frac{e^4}{360\pi^2 m_e^4}\left[(\vec{E}^2 - \vec{B}^2)^2 + 7(\vec{E}\cdot\vec{B})^2\right] + \ldots}$$

- invariants: $F_{\mu\nu}F^{\mu\nu} \sim \vec{E}^2 - \vec{B}^2$, $F_{\mu\nu}\tilde{F}^{\mu\nu} \sim (\vec{E}\cdot\vec{B})^2$

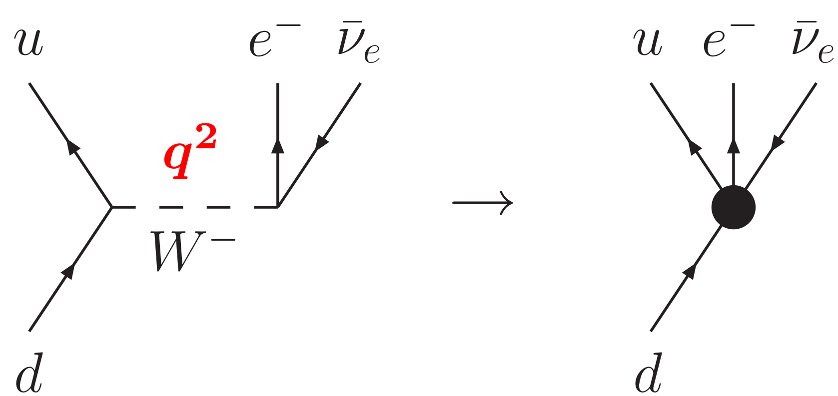
- energy expansion: $(\omega/m_e)^{2n}$ small parameter

- leads to the Xsection: $\sigma(\omega) = \frac{1}{16\pi^2}\frac{973}{10125\pi}\frac{e^8}{m_e^2}(\omega/m_e)^6$

- measurements with PW lasers Bregant et al, Phys. Rev. D **78** (2008) 032006

EXAMPLE 5: FERMI THEORY

- Weak decays
  - mediated by the charged W bosons, $M_W \simeq 80$ GeV
  - energy release in neutron β-decay $\simeq 1$ MeV $\quad [n \to p e^- \nu_e]$
  - energy release in kaon decays $\simeq$ a few 100 MeV $\quad [K \to \pi\, \ell\, \nu]$

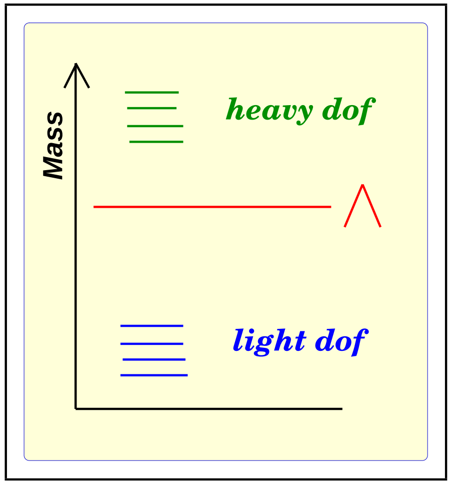


$$\frac{e^2}{8 \sin\theta_W} \times \frac{1}{M_W^2 - q^2} \xrightarrow{q^2 \ll M_W^2} \frac{e^2}{8 M_W^2 \sin\theta_W} \left\{ 1 + \frac{q^2}{M_W^2} + \ldots \right\}$$

$$= \frac{G_F}{\sqrt{2}} + \mathcal{O}\left(\frac{q^2}{M_W^2} \right)$$

$\Rightarrow$ Fermi's current-current interaction

BRIEF SUMMARY of EFFECTIVE FIELD THEORY



- Separation of scales: low and high energy dynamics
  - ⋆ low-energy dynamics in terms of relevant dof's energies $\sim$ momenta $\sim Q$
  - ⋆ high-energy dynamics not resolved $\longrightarrow$ contact interactions

Weinberg 1979

- Small parameter(s) & power counting
  - ⋆ Standard QFT: trees + loops $\longrightarrow$ renormalization
  - ⋆ Expansion in powers of energy/momenta Q over the large scale Λ

$$\boxed{\mathcal{M} = \sum_\nu \left(\frac{Q}{\Lambda}\right)^\nu f(Q/\mu, g_i)}$$

μ – regularization scale

g_i – low–energy constants

- f is a function of $\mathcal{O}(1)$ – "naturalness"
- ν bounded from below

$\Rightarrow$ systematic and controlled expansion

NB: bound states require non-perturbative resummation

The Paradigm Shift in Quantum Field Theory

A NEW LOOK AT RENORMALIZATION

- Renormalization: method to tame the infinities in quantum field theories
- Renormalizable gauge field theories have led to some of the most stunning successes in physics: QED tested to better than $\mathbf{10^{-10}}$
- It has become clear that no theory works at **all** scales, e.g. the Standard Model must break down at the Plank scale (or even earlier)
- The basic idea about renormalization today is that the influences of higher energy processes are localisable in a few structural properties which can be captured by an adjustment of parameters.

> "In this picture, the presence of infinities in quantum field theory is neither a disaster, nor an asset. It is simply a reminder of a practical limitation – we do not know what happens at distances much smaller than those we can look at directly" (Georgi 1989)

THE PARADIGM OF EFFECTIVE FIELD THEORY

- constructing a **Q**uantum **F**ield **T**heory in 4 steps

1) construct the action $S[\ldots]$, respect *symmetries*

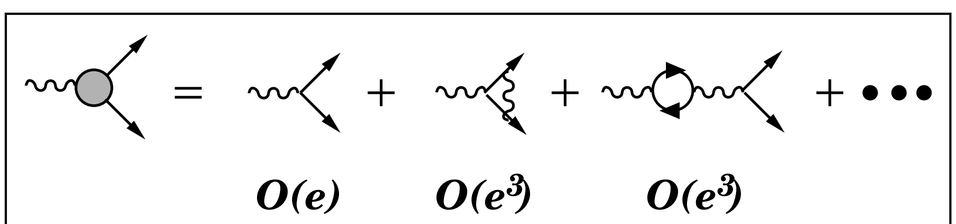
e.g. gauge invariance of QED $\psi \to \psi' = \mathrm{e}^{-i\alpha(x)}\psi,\ A_\mu \to A'_\mu = A_\mu - \partial_\mu \alpha(x)$

2) retain *renormalizable* interactions ($D \leq 4$)

keep $\underbrace{\bar\psi\gamma_\mu\psi A^\mu}_{D=4},\ \underbrace{F_{\mu\nu}F^{\mu\nu}}_{D=4},\ldots$ drop $\underbrace{\bar\psi\sigma_{\mu\nu}\psi F^{\mu\nu}}_{D=5},\ \underbrace{(F_{\mu\nu}F^{\mu\nu})^2}_{D=8},\ldots$

3) *quantize*: calculate scattering processes in perturbation theory:

tree + *loop* graphs



THE PARADIGM OF EFFECTIVE FIELD THEORY cont'd

4) fix the *parameters* from *data*, make *predictions*

e.g. $$\mu_e = -\frac{e g_e \vec{s}_e}{2m_e}\,, \quad g_e = 2\left[1 + \frac{e^2}{8\pi^2} + \mathcal{O}(e^4)\right]$$

- constructing an **E**ffective **F**ield **T**heory

  steps 1,3,4: logically necessary

  step 2: renormalizability = physics at all scales

another consistent & predictive paradigm:

keep rules 1,3,4, but instead use

2*) work at *low* energies & *expand* in powers of the *energy*

– separation of scales

– only a finite number of operators plays a role

– familiar concept → examples just discussed

EFT: FUNDAMENTAL THEOREM

- Weinberg's conjecture: Physica **A96** (1979) 327

> Quantum Field Theory has no content besides unitarity, analyticity, cluster decomposition, and symmetries.

$$\Downarrow$$

To calculate the S-matrix for any theory below some scale, simply use the most general effective Lagrangian consistent with these principles in terms of the appropriate asymptotic states.

Structure of Effective Field Theories

STRUCTURE of EFTs

- Energy expansion [derivative/momentum/. . .]

  dimensional analysis:

  (a) derivatives $\to$ powers of $\boldsymbol{q}$ [small scale]

  (b) be Λ the hard [limiting] scale

  $\to$ any derivative $\partial \sim q/\Lambda$

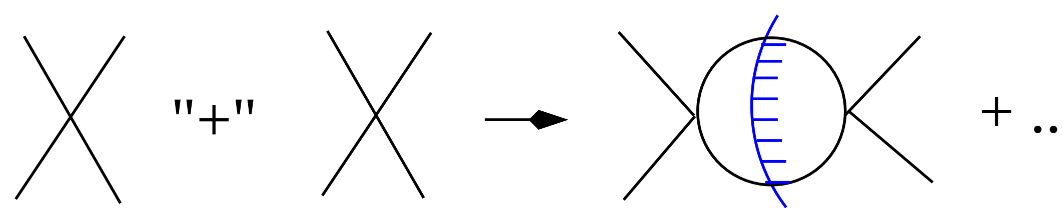
  $\to$ N derivative vertex $\sim q^N/\Lambda^N$

  $\to$ for $E[q] \ll \Lambda$, terms w/ more derivatives are suppressed

- Energy expansion = Loop expansion

  interactions generate loops

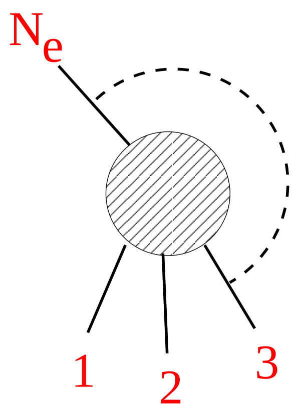
  loops generate imaginary parts



$\Rightarrow$ all this is contained in the *power counting*, which assigns a dimension [not the canonical one] to each diagram

POWER COUNTING THEOREM

- Consider the S-matrix for N_e interacting bosons



$$S = \delta(p_1 + p_2 + \ldots + p_{N_e})\mathcal{M}$$

$$[\mathcal{M}] = 4 - N_e$$ pion field $\sim 1/2\omega$

$$\boxed{\mathcal{M} = \mathcal{M}(q, g, \mu) = q^D\, f(q/\mu, g)}$$ couplings g, renorm. scale μ

$$\Rightarrow D = 4 - N_e - \Delta$$

$\downarrow$ N_d vertices of dim. $4-d$ (d = # of derivatives)
N_i internal propagators

$$\Delta = -\sum_d N_d(d-4) - 2N_i - N_e$$

topology: $N_L = N_i - \sum_d N_d + 1$ number of loops N_L

$\downarrow$

$$\boxed{\boldsymbol{D = 2 + \sum_d N_d(d-2) + 2N_L}}$$

– tree graphs ($N_L = 0$) dominant

– loop graphs suppressed by powers of q^2

POWER COUNTING – ALTERNATIVE DERIVATION

- Consider $\mathcal{L}_{\rm eff} = \sum_d \mathcal{L}^{(d)}$, d bounded from below

- for interacting Goldstone bosons, $d \geq 2$ and $iD(q) = \dfrac{1}{q^2 - M^2}$

- consider an L-loop diagram with I internal lines and V_d vertices of order d

$$Amp \propto \int (d^4q)^L \, \frac{1}{(q^2)^I} \prod_d (q^d)^{V_d}$$

- let $Amp \sim q^\nu \to \nu = 4L - 2I + \sum\limits_d dV_d$

- topology: $L = I - \sum_d V_d + 1$

- eliminate I: $\to \boxed{\nu = 2 + 2L + \sum\limits_d V_d(d-2)}$ $\surd$

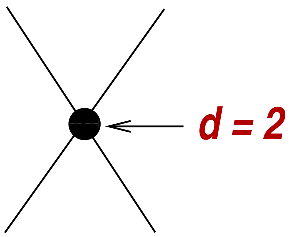
POWER COUNTING for PION-PION SCATTERING

- $\pi(p_1) + \pi(p_2) \to \pi(p_3) + \pi(p_4)$

  leading interaction $\sim \partial\pi\,\partial\pi \Rightarrow d = 2$

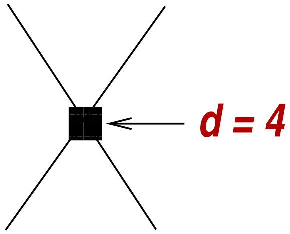
- leading order (LO)

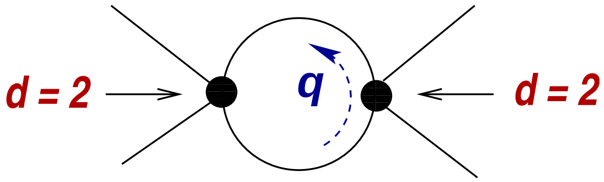
  $d = 2, N_L = 0 \Rightarrow D = 2$



- next-to-leading order (NLO)

  $a)\ d = 4, N_L = 0 \Rightarrow D = 4$ $\qquad$ $b)\ d = 2, N_L = 1 \Rightarrow D = 4$





$$\sim \int d^4q \frac{q_1 \cdot q_2 \;\; q_3 \cdot q_4}{(q^2 - M_\pi^2)(q^2 - M_\pi^2)} \sim \mathcal{O}(q^4)$$

LOW-ENERGY CONSTANTS (LECs)

- consider a covariant & parity-invariant theory of Goldstone bosons parameterized in some matrix-valued field $\boldsymbol{U}$

$$\mathcal{L}_{\rm eff} = g_2 {\rm Tr}(\partial_\mu U \partial^\mu U^\dagger) + g_4^{(1)} \left[{\rm Tr}(\partial_\mu U \partial^\mu U^\dagger)\right]^2 + g_4^{(2)} {\rm Tr}(\partial_\mu U \partial^\nu U^\dagger){\rm Tr}(\partial_\nu U \partial^\mu U^\dagger) + \ldots$$

- couplings = **low-energy constants** (LECs)

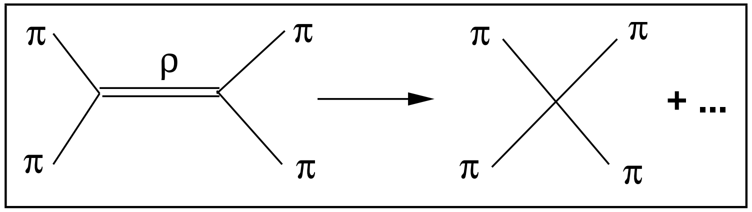
  $g_2 \neq 0$ spontaneous chiral symmetry breaking (cf also the $g_{>2}^{(i)}$)

  $g_4^{(1)}, g_4^{(2)}, \ldots$ must be fixed from data (or calculated from the underlying theory)

- calculations in EFT: fix the LECs from some processes, then make **predictions**

- LECs encode information about the high mass states that are integrated out

$$\frac{1}{M_\rho^2 - q^2} \stackrel{q^2 \ll M_\rho^2}{\longrightarrow} \frac{1}{M_\rho^2}\left(1 + \frac{q^2}{M_\rho^2} + \ldots\right)$$

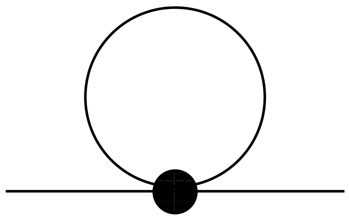


LOOPS and DIVERGENCES

- Loop diagrams generate imag. parts, but are mostly **divergent**

$\Rightarrow$ choose a mass-independent & symmetry-preserving regularization scheme
[like dimensional regularization]

Ex.:



$$
\begin{aligned}
&= -i\Delta_\pi(0) = \frac{-i}{(2\pi)^d}\int d^dp \frac{1}{M^2 - p^2 - i\varepsilon} \quad \text{[d space-time dim.]} \\
&= (2\pi)^{-d}\int d^dk \frac{1}{M^2 + k^2} \ \text{with} \ \ p_0 = ik_0 \ , \quad -p^2 = k_0^2 + \vec{k}^{\,2} \\
&= (2\pi)^{-d}\int d^dk \int_0^\infty d\lambda \exp(-\lambda(M^2 + k^2)) \\
&= (2\pi)^{-d}\int_0^\infty d\lambda \exp(-\lambda M^2) \underbrace{\int d^dk \exp(-\lambda k^2)}_{(\pi/\lambda)^{d/2}} \\
&= (4\pi)^{-d} M^{d-2}\,\Gamma\left(1 - \frac{d}{2}\right) \qquad \text{has a pole at } d = 4
\end{aligned}
$$

$\Rightarrow$ absorb in LECs: $\boxed{\boldsymbol{g_i \to g_i^{\rm ren} + \beta_i \frac{1}{d-4}}}$ always possible!

INTERMEDIATE SUMMARY

- Effective field theories explore scale separation in physical systems

  low-energy physics treated explicitely

  high-energy modes integrated out $\longrightarrow$ contact interactions

  $\longrightarrow$ low-energy constants

- Interactions generate loops, loops restore unitarity
- Power counting: systematic ordering of all graphs, loops are suppressed
- Loop graphs are generally divergent $\longrightarrow$ order-by-order renormalization

A detailed Look at a Model EFT

Study many aspects of EFTs in a simple model

The MODEL at TREE LEVEL I

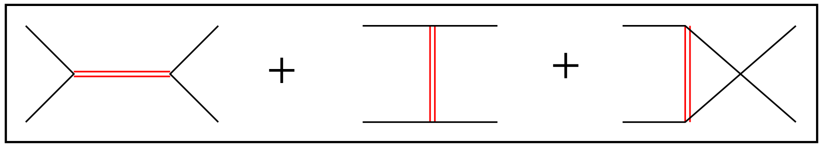
- Consider a model with light/heavy fields (ϕ/Φ) with mass m/M:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\Phi)^2 - \frac{m^2}{2}\phi^2 - \frac{M^2}{2}\Phi^2 - \frac{g}{2}\phi^2\Phi$$

$\Rightarrow$ Integrate out the heavy fields Φ

- consider $\phi\phi \to \phi\phi$ at tree level:

$$T_{\rm tree} = \frac{g^2}{M^2 - s} + \frac{g^2}{M^2 - t} + \frac{g^2}{M^2 - u}$$



$$s = (p_1 + p_2)^2,\ t = (p_1 - p_3)^2,\ u = (p_1 - p_4)^2,\quad s + t + u = 4m^2$$

- Let $M \to \infty$: $\quad T_{\rm tree} = \dfrac{3g^2}{M^2} + \dfrac{4g^2m^2}{M^4} + \dfrac{g^2}{M^6}(s^2 + t^2 + u^2) + \cdots$
- At low energies, subsequent terms are suppressed by powers of E^2/M^2
- Task: find an effective Lagrangian $\mathcal{L}_{\rm eff}$ w/ light fields only,
  that reproduces the above scattering amplitude

The MODEL at TREE LEVEL II

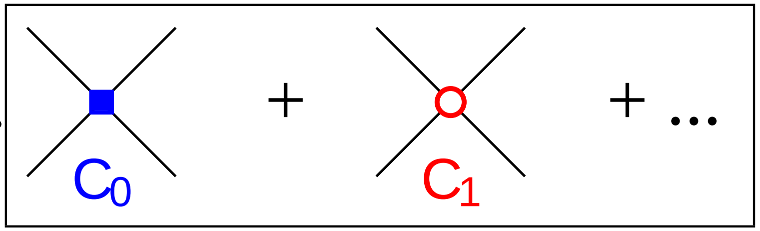
- The effective Lagrangian must contain an infinite tower of terms:

$$\mathcal{L}_{\rm eff} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 + C_0\phi^4 + C_1\phi^2\nabla^2\phi^2 + C_2\phi^2(\nabla^2)^2\phi^2 + \cdots$$

$\Rightarrow$ with the LECs $C_0, C_1, C_2, \ldots$

- Tree level scattering amplitude:

$$T^{\rm eff}_{\rm tree} = 24C_0 - 32m^2C_1 + 8C_2(s^2+t^2+u^2) + \cdots$$



- Matching: $T_{\rm tree} = T^{\rm eff}_{\rm tree}$

$$\Rightarrow 24C_0 - 32m^2C_1 = \frac{3g^2}{M^2} + \frac{4g^2m^2}{M^4}\,, \quad C_2 = \frac{g^2}{M^6}\,, \ldots$$

  – the LECs C_0 and C_1 can not be separated (model artefact)

  – EFT (non-ren.) equivalent to underlying th'y (super-ren.) at tree level, but beyond?

  – related problem: violation of the unitarity bound

UNITARITY BOUND

- Partial wave amplitudes:

$$T^{\rm eff}(s,\cos\theta) = 16\pi\sqrt{s}\sum_{l=0}^{\infty}(2l+1)T_l^{\rm eff}(s)P_l(\cos\theta)$$

- Unitarity relation: $\mathrm{Im}\,T_l^{\rm eff}(s) \geq p\,|T_l^{\rm eff}(s)|^2\,, \quad p = \sqrt{s/4-m^2}$

$\Rightarrow$ Unitarity bound: $|\mathrm{Re}\,T_l^{\rm eff}(s)| \leq 1/(2p)$

- S-wave: $\mathrm{Re}\,T^{\rm eff}_{0,\rm tree}(s) = 24\tilde{C}_0 + \frac{32}{3}\,C_2(s/4-m^2)^2 + \cdots\,, \quad \tilde{C}_0 = C_0 - \frac{4m^2}{3}\,C_1$

$\Rightarrow$ Saturates the unit.bound at: $s_{\rm M} = 4M^2\sqrt{\dfrac{16\pi-3\tilde{g}^2}{4\tilde{g}^2/3}} + O(1)\,, \quad \tilde{g} = \dfrac{g}{M}$

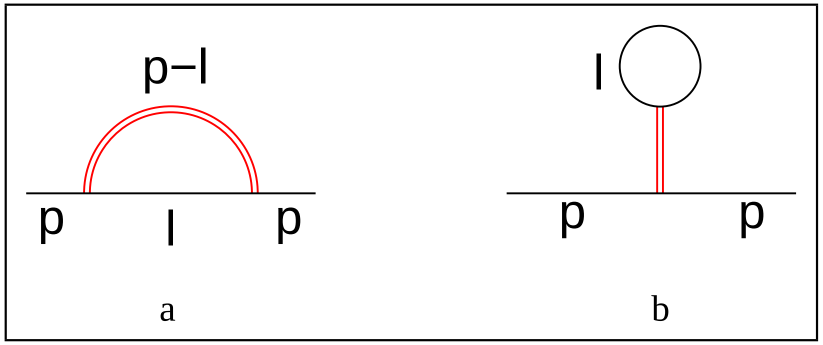
  – $s_{\rm M}$ is of order $M^2 \to$ EFT no longer applicable

  – loops are neccessary to restore unitarity

  – the underlying th'y provides an *ultra-violet (UV) completion* of the EFT at scales of order M

$\tilde{g}$ finite as $M \to \infty$

LIGHT PARTICLE MASS at ONE LOOP I



- Self-energy: $\Sigma(p^2) = \Sigma_a(p^2) + \Sigma_b(p^2)$

- utilize dim. reg.: $\displaystyle\int_l f(l) \doteq -i \int \frac{d^D l}{(2\pi)^D} f(l)$

- Infinities are poles as $D \to 4$: $\displaystyle L = \frac{\mu^{D-4}}{16\pi^2}\left(\frac{1}{D-4} - \frac{1}{2}\left(\Gamma'(1) + \ln 4\pi\right)\right)$

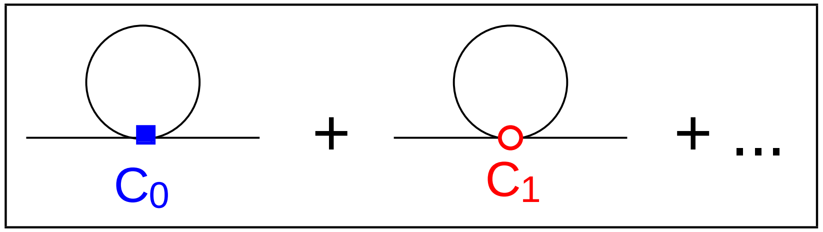
$\Rightarrow$ Self-energy:

$$\Sigma(p^2) = 2g^2 L - \frac{g^2}{16\pi^2}\left(-1 + \ln\frac{M^2}{\mu^2}\right) + \frac{g^2 m^2}{M^2} L$$
$$- \frac{g^2}{16\pi^2 M^2}\left(m^2 \ln\frac{M^2}{\mu^2} - \frac{3m^2}{2} \ln\frac{m^2}{\mu^2} - \frac{1}{2}(p^2 - m^2)\right) + O(M^{-4})$$

– $O(M^{-4})$ includes also terms of $O(M^{-4}\ln^k M^2)$

– let us try to reproduce this result within the EFT $\to$ loops?

LIGHT PARTICLE MASS at ONE LOOP II



- Calculation to $O(M^{-2})$:

$$\mathcal{L} = \tfrac{1}{2}\,(\partial\phi)^2 - \tfrac{m^2}{2}\,\phi^2 + C_0\,\phi^4 + O(M^{-4})$$

$$\Rightarrow \qquad \Sigma_{\rm eff}(p^2) = 12C_0 \int_l \frac{1}{m^2 - l^2} + O(M^{-4})$$

$$= 24C_0 m^2\, L_{\rm eff} - \frac{3C_0 m^2}{4\pi^2}\left(1 - \ln\frac{m^2}{\mu_{\rm eff}^2}\right) + O(M^{-4})$$

- Obviously not the same: $\Sigma_a + \Sigma_b \neq \Sigma_{\rm eff}$
- Must include mass and wave function renormalization counterterms:

$$\mathcal{L}_{\rm eff} \longrightarrow \mathcal{L}_{\rm eff} + \tfrac{A}{2}\,(\partial\phi)^2 - \tfrac{B}{2}\,\phi^2$$

$$A = \frac{g^2}{32\pi^2 M^2} + O(M^{-4})$$

$$B = g^2\left(2L_{\rm eff} + \frac{1}{16\pi^2}\left(\ln\frac{M^2}{\mu_{\rm eff}^2} - 1\right)\right) + \frac{g^2 m^2}{M^2}\left(2L_{\rm eff} + \frac{1}{16\pi^2}\left(\ln\frac{M^2}{\mu_{\rm eff}^2} - 1\right)\right) + O(M^{-4})$$

$\Rightarrow$ compare now the physical masses !

PHYSICAL MASS I

- Underlying th'y: $m^2 - m_\mathrm{P}^2 - (\Sigma_a(m_\mathrm{P}^2) + \Sigma_b(m_\mathrm{P}^2)) = 0$

$\Rightarrow$ Lowest order in g:

$$m_\mathrm{P}^2 = m_\mathrm{r}^2(\mu) + \frac{g^2}{16\pi^2}\left(-1 + \ln\frac{M^2}{\mu^2}\right) + \frac{g^2 m_\mathrm{r}^2(\mu)}{16\pi^2 M^2}\left(\ln\frac{M^2}{\mu^2} - \frac{3}{2}\ln\frac{m_\mathrm{r}^2(\mu)}{\mu^2}\right) + O(M^{-4})$$

- *running mass*: $m_\mathrm{r}^2(\mu) = m^2 + 2g^2 L - \frac{g^2 m^2}{M^2} L$ (no renorm. of g and M)

- EFT: $m^2 + B - (1+A)m_\mathrm{P}^2 - \Sigma_\mathrm{eff}(m_\mathrm{P}^2) = 0$

$\Rightarrow$ Lowest order in $1/M^2$:

$$m_\mathrm{P}^2 = m_\mathrm{r,eff}^2 + \frac{3g^2 m_\mathrm{r,eff}^2}{32\pi^2 M^2}\left(1 - \ln\frac{m_\mathrm{r,eff}^2(\mu_\mathrm{eff})}{\mu_\mathrm{eff}^2(\mu_\mathrm{eff})}\right) + O(M^{-4})$$

- *running mass*: $m_\mathrm{r,eff}^2(\mu_\mathrm{eff}) = m_\mathrm{eff}^2 - \frac{3g^2 m_\mathrm{eff}^2}{M^2} L_\mathrm{eff}$

- both running masses are *related* so that physical observables are the same

$\Rightarrow$ work this out (exercise)

RUNNING MASSES

- Running masses run different in both theories:

$$\mu \frac{dm_{\rm r}^2(\mu)}{d\mu} = \frac{g^2}{8\pi^2} - \frac{g^2 m_{\rm r}^2(\mu)}{16\pi^2 M^2}$$

$$\mu_{\rm eff} \frac{dm_{\rm r,eff}^2(\mu_{\rm eff})}{d\mu_{\rm eff}} = \frac{3g^2 m_{\rm r,eff}^2(\mu_{\rm eff})}{16\pi^2 M^2} + O(M^{-4})$$

$\longrightarrow$ $m_{\rm r,eff}(\mu_{\rm eff})$ does not depend on μ (observables!)

$\longrightarrow$ loops in general don't match, differences taken away by renormalization

$\longrightarrow$ both theories physically equivalent $\longrightarrow$ decoupling theorem

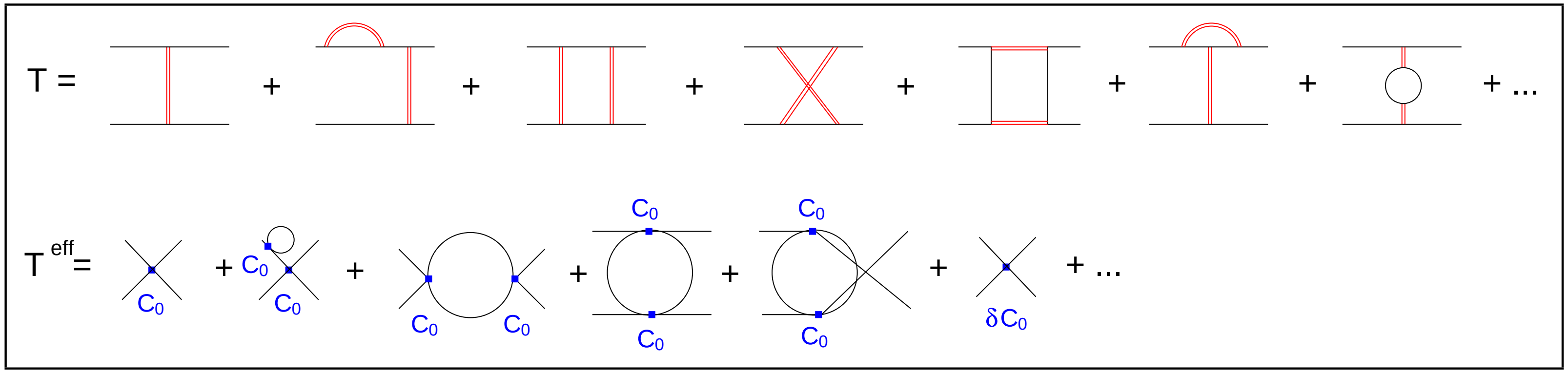
$\longrightarrow$ In $m_{\rm r,eff}(\mu_{\rm eff})$ all logs of the light mass cancel!

$\longrightarrow$ general phenomenon for decoupling EFTs: Parameters of the EFT encode the short-distance dynamics and thus depend on the light masses in polynomial form

$\longrightarrow$ running mass in the EFT not protected from large loop corrections (fine-tuning?)

MATCHING of the QUARTIC COUPLING at ONE LOOP

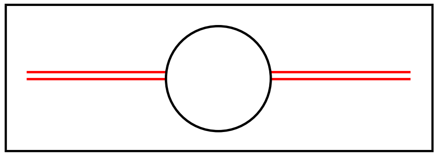
- $T = T^{\text{eff}}$:



$\Rightarrow$ Couplings diverge: $C_i = \nu_i L_{\text{eff}} + C_i^{\text{r}}(\mu_{\text{eff}})\,, \quad \mu_{\text{eff}} \dfrac{dC_i^{\text{r}}(\mu_{\text{eff}})}{d\mu_{\text{eff}}} = -\dfrac{\nu_i}{16\pi^2}$

- Lengthy calculation:

$$C_0 = \frac{9g^4}{8M_r^4} L_{\text{eff}} + \frac{g^2}{8M_r^2} - \frac{g^4}{64\pi^2 M_r^4}\left(3 + 2\ln\frac{\mu_{\text{eff}}^2}{M_r^2}\right) + O(M_r^{-6}) = \nu_0 L_{\text{eff}} + C_0^r(\mu_{\text{eff}})$$



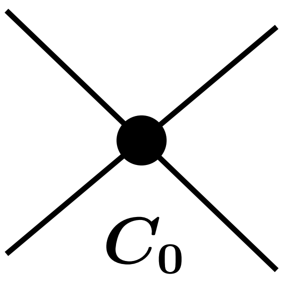
– the heavy mass gets renormalized $M \to M_r$

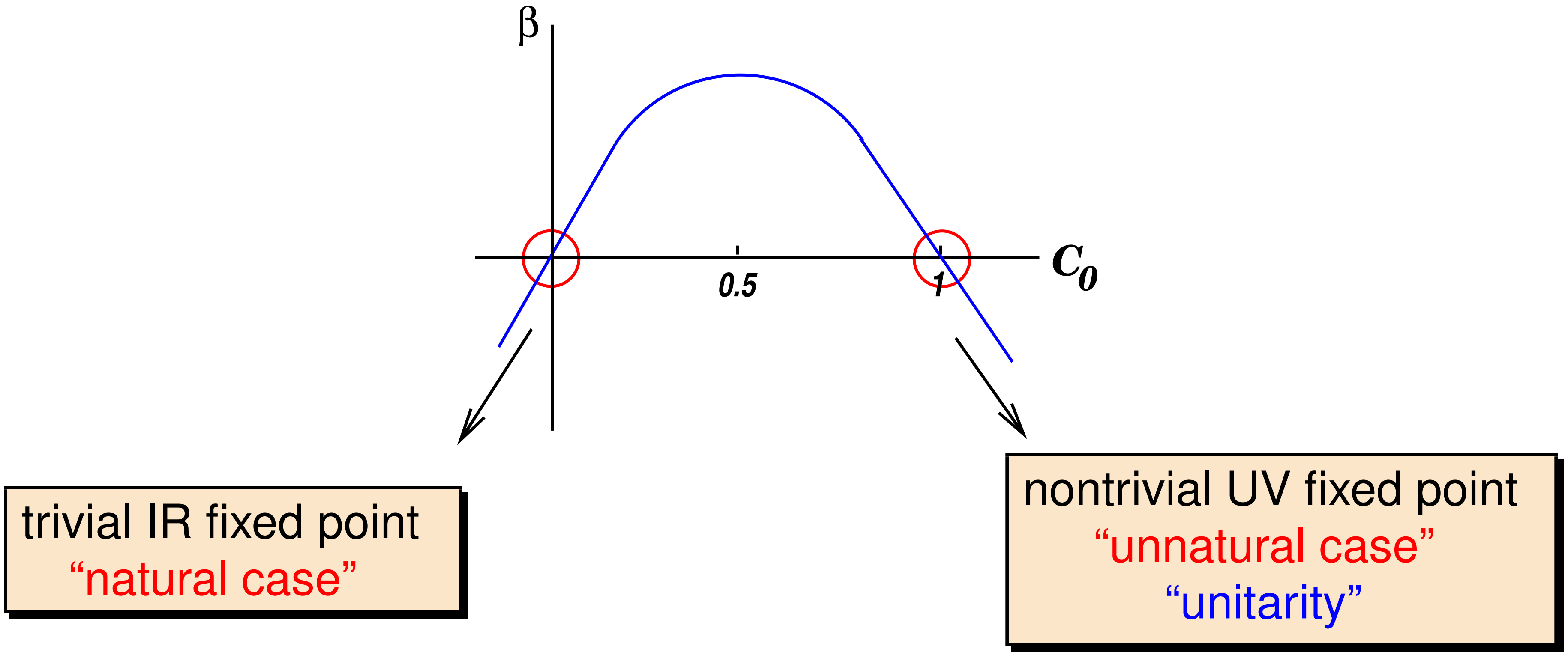
– RG equation for C_0^r: $\mu_{\text{eff}} \dfrac{dC_0^r}{d\mu_{\text{eff}}} = -\dfrac{9}{2\pi^2}\,(C_0^r)^2$

NUCLEAR PHYSICS from a RGE POINT of VIEW

- Renormalization group analysis of the low-energy NN interaction

$$\beta = \mu \frac{d}{d\mu} C_0 = -C_0 \left(C_0 - 1 \right)$$





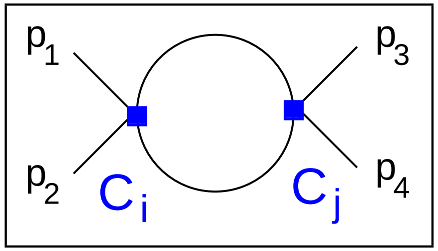
DEPENDENCE of the COUPLINGS on the HEAVY MASS

- Tree level $\rightarrow$dimensionality of the couplings as $M \to \infty$:

$$C_i \sim g^2/M^{2(i+1)} \propto M^{-2i}$$

- *Naturalness:* dimensionless coupling $\tilde{C}_i = C_i M^{2i}$ should be $O(1)$

- Beyond tree level - consider a typical one-loop graph



- In dim reg: $I_{ij} = \dfrac{\tilde{C}_i \tilde{C}_j}{M^{2(i+j)}} \underbrace{\int_l \frac{N(l;\{p_i\})}{(m^2 - l^2)(m^2 - (P-l)^2)}}_{m^{2(i+j)} \log(m/\mu_{\rm eff})}$

$\Rightarrow$ Irrelevant couplings (negative mass dimension) stay irrelevant

– can be easily generalized to all one and higher loop graphs

– also holds in mass-dependent reg. scheme (e.g. cut-off) $\longrightarrow$ exercise

RENORMALIZATION GROUP FLOW I

- Consider an EFT with $m \ll \Lambda \ll M$

$\Rightarrow$ varying Λ in some range $\Lambda_{\text{eff}} \leq \Lambda \leq M$ must leave the physics invariant

$\Rightarrow$ masses and couplings must run accordingly, e.g. ϕ^4-theory with only coupling C_0

$\Rightarrow$ Renormalization Group Equations (RGEs):

$$m^2(\Lambda) = m^2(M) + 12C_0 \int_l^M \frac{1}{m^2(M)+l^2}\bigg|_{\text{Eucl.}} - 12C_0 \int_l^\Lambda \frac{1}{m^2(M)+l^2}\bigg|_{\text{Eucl.}}$$

$$= m^2(M) + \tfrac{3C_0}{4\pi^2}\left(M^2 - \Lambda^2 - m^2(M)\ln\tfrac{M^2}{\Lambda^2} + \cdots\right)$$

$$\Lambda \tfrac{d}{d\Lambda}\, m^2(\Lambda) = -\tfrac{3C_0}{2\pi^2}\,\Lambda^2\left(1 + O\left(\tfrac{m^2}{\Lambda^2}\right)\right)$$

$\Rightarrow$ flow equations will generate all possible types of operators at lower scales

- Q: is that a problem?
- A: no, as we will show now!

RENORMALIZATION GROUP FLOW II

- Crucial statement:

> Even if the theories may look very different at the hard scale $\Lambda = M$, the differences vanish when going to lower scales $\Lambda = \Lambda_{\text{eff}}$

- Model with two couplings, one marginal (C_0) and one irrelevant (C_1):

$\Rightarrow$ RGEs:
$$\Lambda \frac{dC_0}{d\Lambda} = \beta_0(C_0, \Lambda^2 C_1)\,, \quad \Lambda \frac{dC_1}{d\Lambda} = \Lambda^{-2} \beta_1(C_0, \Lambda^2 C_1)$$

- Rescaling $\tilde{C}_0 = C_0$, $\tilde{C}_1 = \Lambda^2 C_1$ (dimensionless couplings)

$\Rightarrow$
$$\Lambda \frac{d\tilde{C}_0}{d\Lambda} = \beta_0(\tilde{C}_0, \tilde{C}_1)\,, \qquad \Lambda \frac{d\tilde{C}_1}{d\Lambda} - 2\tilde{C}_1 = \beta_1(\tilde{C}_0, \tilde{C}_1)$$

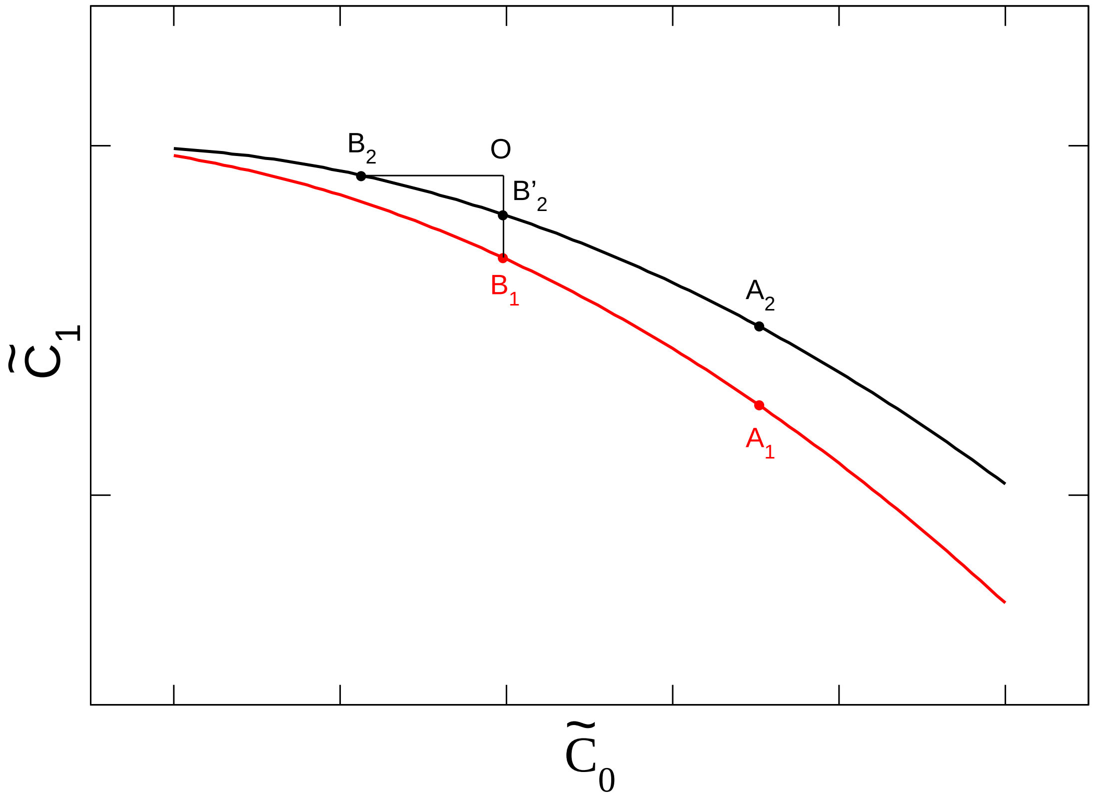
- RG flow equations, provided the values of $\tilde{C}_i(\Lambda)$ are fixed at some point:

$$\tilde{C}_i(\Lambda)\big|_{\Lambda=M} = \tilde{C}_i^{(0)}\,, \qquad i = 0, 1$$

- assume a solution $(\bar{C}_0, \bar{C}_1)$ at large scale $\Lambda = M \to$ analyze small dev's

RENORMALIZATION GROUP FLOW III

- solution: all trajectories approach each other in the IR (exercise: do the math)

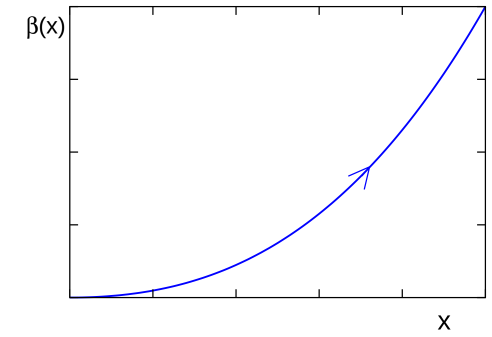


$\Rightarrow$ $\tilde{C}_1(\Lambda_{\text{eff}})$ can be expressed in terms of $\tilde{C}_0(\Lambda_{\text{eff}})$ up to terms of order $(\Lambda_{\text{eff}}^2/M^2)$.

$\Rightarrow$ physics at the large scale M gets decoupled from the one at the low scale Λ_{eff}

TRIVIALITY and the HIGGS MASS

- *Triviality:* if the β-function is strictly positive and grows faster than a linear function, the bare coupling $\tilde{C}_0(M) \to \infty$ at a finite M $\longrightarrow$ one can only remove the cut-off if the renormalized coupling vanishes, i.e. the theory is trivial



- Higgs sector in the SM: $\mathcal{L}_H = \frac{1}{2}\,\partial_\mu\Phi^\dagger\partial^\mu\Phi - \frac{1}{2}m_0^2\,\Phi^\dagger\Phi - \frac{1}{4}\tilde{C}_0\,(\Phi^\dagger\Phi)^2$

- RGE for $\tilde{C}_0$: $\displaystyle\frac{1}{\tilde{C}_0(\Lambda)} = \frac{1}{\tilde{C}_0(M)} + \frac{3}{2\pi^2}\ln\frac{M}{\Lambda} \geq \frac{3}{2\pi^2}\ln\frac{M}{\Lambda}$

- use tree level SM relations: $\displaystyle\left(\frac{M_H}{M_W}\right)^2 = \frac{8\tilde{C}_0(\Lambda)}{g^2}$

$\Rightarrow$ bound on the Higgs mass: Dashen, Neuberger (1983)

$$\frac{M_H}{M_W} \leq \frac{4\pi}{g\sqrt{3}}\,\frac{1}{(\ln(M/\Lambda))^{1/2}} \simeq \frac{900\text{ GeV}}{M_W}\,\frac{1}{(\ln(M/\Lambda))^{1/2}} \longrightarrow M_H \leq 900\text{ GeV}$$

INTERMEDIATE SUMMARY

- Decoupling EFTs: Appelquist, Carrazone (1975)
  - effects of the heavy fields are power-suppressed or appear in the renormalization of the light field couplings
  - as $M_H \to \infty$, heavy fields decouple & shifts become unobservable
  - RGEs / RG flow: powerful tool to analyze decoupling EFTs
  - Examples:
    - QED at $E \ll m_e \longrightarrow$ Euler-Heisenberg Lagrangian
    - weak int. at $E \ll M_W \longrightarrow$ Fermi's four-fermion Lagrangian
    - SM at $E \ll 1$ TeV $\longrightarrow \mathcal{L}_{\text{eff}} = SU(3)_C \times SU(2)_L \times U(1)_Y$

INTERMEDIATE SUMMARY cont'd

- Non-decoupling EFTs:
  - during the transition $\mathcal{L} \longrightarrow \mathcal{L}_{\text{eff}}$, phase transition via spontaneous symmetry breaking w/ generation of (pseudo-) Goldstone bosons with masses $M_{\text{GB}} \ll \Lambda_{\text{SSB}}$
  - SSB entails relations between MEs w/ different no. of GBs

    $\longrightarrow$ $D < 4$ or $D \geq 4$ becomes meaningless

    $\longrightarrow$ $\mathcal{L}_{\text{eff}}$ is intrinsically non-renormalizable
  - Examples:
    - SM w/o Higgs $\longrightarrow$ GBs = longitudinal comp. of the V-bosons
    - SM below $\Lambda_{\chi SB} \simeq 1$ GeV $\longrightarrow$ QCD chiral dynamics

QCD chiral dynamics

INTRO: CHIRAL SYMMETRY

- Massless fermions have chiral symmetry:

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi$$

- left/right-decomposition:

$$\psi = \frac{1}{2}(1-\gamma_5)\psi + \frac{1}{2}(1+\gamma_5)\psi = P_L\psi + P_R\psi = \psi_L + \psi_R$$

- projectors:

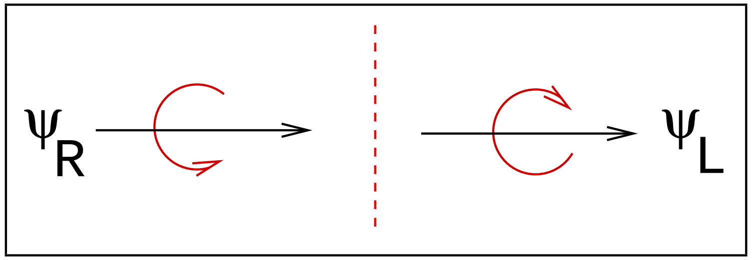
$$P_L^2 = P_L, \; P_R^2 = P_R, \; P_L \cdot P_R = 0, \; P_L + P_R = \mathbb{1}$$

- helicity eigenstates:

$$\frac{1}{2}\hat{h}\psi_{L,R} = \pm\frac{1}{2}\psi_{L,R} \quad \hat{h} = \vec{\sigma}\cdot\vec{p}/|\vec{p}\,|$$

- L/R fields do **not** interact $\longrightarrow$ conserved L/R currents

$$\mathcal{L} = i\bar{\psi}_L\gamma_\mu\partial^\mu\psi_L + i\bar{\psi}_R\gamma_\mu\partial^\mu\psi_R$$



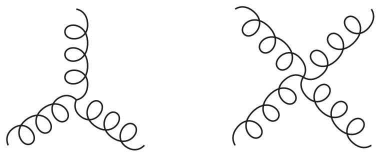
- mass terms break chiral symmetry: $\bar{\psi}\mathcal{M}\psi = \bar{\psi}_R\mathcal{M}\psi_L + \bar{\psi}_L\mathcal{M}\psi_R$

QCD LAGRANGIAN

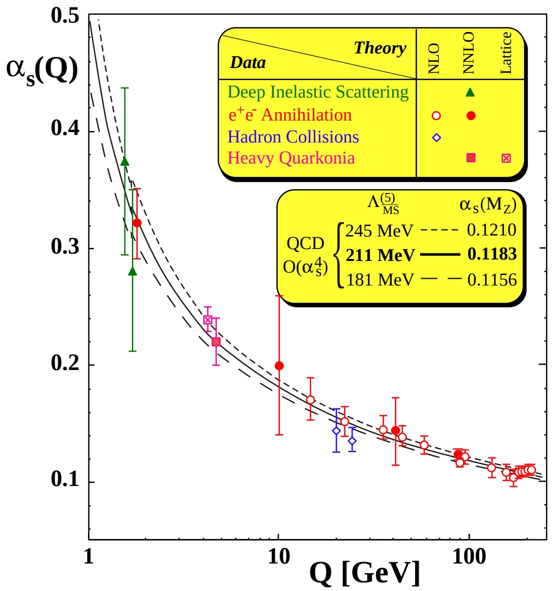
- $$\boxed{\mathcal{L}_{\rm QCD} = -\frac{1}{2}{\rm Tr}\, G^a_{\mu\nu} G^{\mu\nu,a} + \sum_f \bar{q}_f (i \not{D} - \mathcal{M}) q_f + \ldots}$$

$$D_\mu = \partial_\mu + i g A^a_\mu \lambda^a/2\,, \quad G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g[A^b_\mu, A^c_\nu], \quad f = (u,d,s,c,b,t)$$

- local color gauge invariance SU(3)$_C$
- non-linear couplings:



- running of $\alpha_s = g^2/4\pi$



- **light** (u,d,s) and heavy (c,b,t) quark flavors:

$$\boxed{m_{\rm light} \ll \Lambda_{\rm had}\,, m_{\rm heavy} \gg \Lambda_{\rm had}}$$

- quarks and gluons are confined within hadrons

CHIRAL SYMMETRY of QCD

- Three flavor QCD:

$$\boxed{\mathcal{L}_{\rm QCD} = \mathcal{L}^0_{\rm QCD} - \bar{q}\mathcal{M}q}\;,\quad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}\;,\quad \mathcal{M} = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

- $\mathcal{L}^0_{\rm QCD}$ is invariant under **chiral** $SU(3)_L \times SU(3)_R$ (split off U(1)'s)

$$\mathcal{L}^0_{\rm QCD}(G_{\mu\nu}, q', D_\mu q') = \mathcal{L}^0_{\rm QCD}(G_{\mu\nu}, q, D_\mu q)$$

$$q' = RP_R q + LP_L q = Rq_R + Lq_L \quad R, L \in SU(3)_{R,L}$$

- conserved L/R-handed [vector/axial-vector] Noether currents:

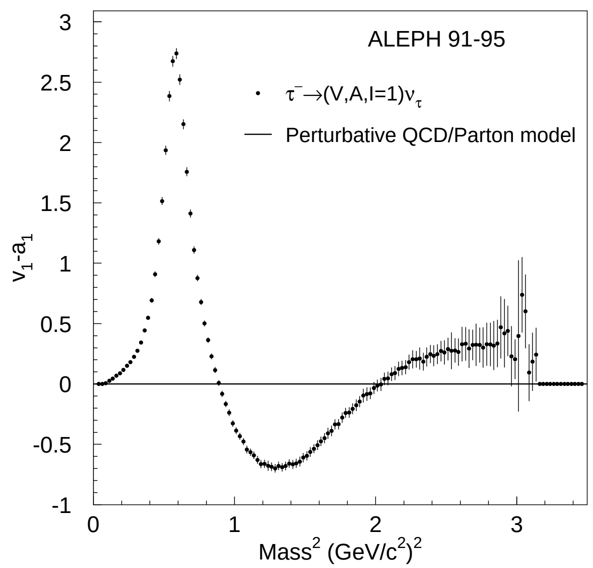
$$J^{\mu,a}_{L,R} = \bar{q}_{L,R}\gamma^\mu \frac{\lambda^a}{2} q_{L,R}\;,\quad a = 1,\ldots,8$$

$$\partial_\mu J^{\mu,a}_{L,R} = 0 \quad [\text{or } \; V^\mu = J^\mu_L + J^\mu_R\;,\;\; A^\mu = J^\mu_L - J^\mu_R]$$

- Is this symmety reflected in the vacuum structure/hadron spectrum?

THE FATE of QCD's CHIRAL SYMMETRY

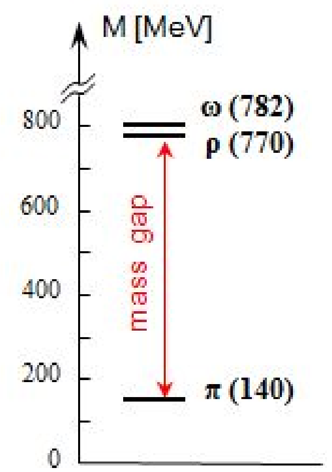
- the chiral symmetry is not "visible" (spontaneously broken)
  - no parity doublets
  - $\langle 0|AA|0\rangle \neq \langle 0|VV|0\rangle$
  - scalar condensate $\bar{q}q$ acquires v.e.v.
  - Vafa-Witten theorem
  - (almost) massless pseudoscalar bosons



- the chiral symmetry is realized in the Nambu-Goldstone mode
  - weakly interacting massless pseudoscalar excitations
  - approximate symmetry (small quark masses)

    $\longrightarrow \pi, K, \eta$ as Pseudo-Goldstone Bosons
  - calls for an effective field theory

  $\Rightarrow$ Chiral Perturbation Theory



THE FATE of QCD's CHIRAL SYMMETRY II

- Wigner mode $Q_5^a|0\rangle = Q^a|0\rangle = 0 \; (a = 1,\ldots,8)$?
- parity doublets: $dQ_5^a/dt = 0 \to [H, Q_5^a] = 0$

  single particle state: $H|\psi_p\rangle = E_p|\psi_p\rangle$

  axial rotation: $H(\mathrm{e}^{iQ_5^a}|\psi_p\rangle) = \mathrm{e}^{iQ_5^a}H|\psi_p\rangle = \underbrace{E_p(\mathrm{e}^{iQ_5^a}|\psi_p\rangle)}$

  same mass but opposite parity

- VV and AA spectral functions (without pion pole):

$$
\begin{array}{ccccccc}
\langle 0|VV|0\rangle & = & \langle 0|(L+R)(L+R)|0\rangle & = & \langle 0|L^2+R^2+2LR|0\rangle & = & \langle 0|L^2+R^2|0\rangle \\
\| & & & & & & \| \\
\langle 0|AA|0\rangle & = & \langle 0|(L-R)(L-R)|0\rangle & = & \langle 0|L^2+R^2-2LR|0\rangle & = & \langle 0|L^2+R^2|0\rangle
\end{array}
$$

since L and R are orthogonal

PROPERTIES of GOLDSTONE BOSONS

- GBs are massless [no explicit symmetry breaking]

consider a broken generator $[Q, H] = 0$ but $Q|0\rangle \neq 0$

define $|\psi\rangle \equiv Q|0\rangle$

$$\longrightarrow H|\psi\rangle = HQ|0\rangle = QH|0\rangle = 0$$

$\longrightarrow$ not only G.S. $|0\rangle$ has $E = 0$

> There exist massless excitations, non-interacting as $E, p \to 0$

[NB: proper argumentation requires more precise use of the infinite volume]

- explicit symmetry breaking, perturbative [small parameter ε]

Goldstone bosons acquire a small mass $M_{\rm GB}^2 \sim \varepsilon$

In QCD, this symmetry breaking is given in terms of the light quark masses

$$\Rightarrow M_\pi^2 \sim (m_u + m_d)$$

CHIRAL EFT of QCD

Gasser, Leutwyler, Weinberg, van Kolck, Kaplan, Savage, Wise, Bernard, Kaiser, M., . . .

- Starting point: CHIRAL LAGRANGIAN (two flavors)

$$\boxed{\mathcal{L}_{\rm QCD} \longrightarrow \mathcal{L}_{\rm EFF} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \ldots}$$

- dofs: quark & gluon fields $\longrightarrow$ pions, nucleons, external sources
- Spontaneous chiral symmetry breaking of QCD $\longrightarrow$ pions are Goldstone bosons
- Systematic expansion in powers of q/Λ_χ & M_π/Λ_χ, with $\Lambda_\chi \simeq 1$ GeV
- pion and pion-nucleon sectors are perturbative in q $\longrightarrow$ chiral perturbation theory
- Parameters in $\mathcal{L}_{\pi\pi}$ and $\mathcal{L}_{\pi N}$ known from CHPT studies $\longrightarrow$ low-energy constants
- $\mathcal{L}_{NN}$ collects short-distance contact terms, to be fitted
- NN interaction requires non-perturbative resummation

$\longrightarrow$ chirally expand $V_{\rm NN}$, use in regularized LS equation

CHIRAL PERTURBATION THEORY

- Consider first the mesonic chiral effective Lagrangian

$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \ldots$$

$$\mathcal{L}^{(2)} = \frac{F_\pi^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle \qquad [U^\dagger U = UU^\dagger = 1, U \to LUR^\dagger]$$

$$U = \exp(i\Phi/F_\pi)\,, \quad \Phi = \sqrt{2}\begin{pmatrix} \pi^0/\sqrt{2}+\eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2}+\eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{pmatrix}$$

$\chi = 2B\mathcal{M} + \ldots,\ B = |\langle 0|\bar{q}q|0\rangle|/F_\pi^2 \leftarrow$ scalar quark condensate

- Two parameters:

$F_\pi \simeq 92$ MeV = pion decay constant (GB coupling to the vacuum)

$B \simeq 2$ GeV = normalized vacuum condensate

- Goldstone boson masses: $M_{\pi^+}^2 = (m_u + m_d)B\,,\ \ M_{K^+}^2 = (m_d + m_s)B\,, \ldots$

- has been extended to two loops $\mathcal{O}(q^6)$ in many cases

FROM QUARK to MESON MASSES

- symmetry breaking Lagrangian: $\mathcal{L}_{\rm SB} = \mathcal{M} \times f(U, \cancel{\partial_\mu} U, \ldots)\ , \quad \mathcal{M} = {\rm diag}(m_u, m_d)$
- LO invariants: ${\rm Tr}(\mathcal{M}U^\dagger)\ ,\ {\rm Tr}(U\mathcal{M}^\dagger)$

$$\Rightarrow \mathcal{L}_{\rm SB} = \frac{1}{2}F_\pi^2 \left\{B\, {\rm Tr}(\mathcal{M}U^\dagger + U\mathcal{M}^\dagger)\right\}$$

B is a real constant if CP is conserved

$$= (m_u + m_d)\, B \left[F_\pi^2 - \frac{1}{2}\pi^2 + \frac{\pi^4}{24F_\pi^2} + \ldots\right]$$

[expand $U = \exp(i\vec{\tau}\cdot\vec{\pi}/F_\pi)$]

First term (vacuum): $\left.\dfrac{\partial \mathcal{L}_{\rm QCD}}{\partial m_q}\right|_{m_q=0} = -\bar{q}q$

$$\Rightarrow \langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle = -BF_\pi^2\,(1 + \mathcal{O}(\mathcal{M}))$$

Second term (pion mass): $-\frac{1}{2}M_\pi^2\pi^2 \Rightarrow M_\pi^2 = (m_u + m_d)B$

combined: $\boxed{M_\pi^2 = -(m_u + m_d)\,\langle 0|\bar{u}u|0\rangle / F_\pi^2}$ Gell-Mann–Oakes–Renner rel.

repeat for SU(3) $\Rightarrow 3M_\eta^2 = 4M_K^2 - M_\pi^2$ Gell-Mann–Okubo relation

MESON MASSES → QUARK MASS RATIOS

- lowest order:

$$M^2_{\pi^+} = (m_u + m_d)B \simeq (0.140\,\mathrm{GeV})^2$$
$$M^2_{K^0} = (m_u + m_s)B \simeq (0.494\,\mathrm{GeV})^2$$
$$M^2_{K^+} = (m_d + m_s)B \simeq (0.497\,\mathrm{GeV})^2$$

$$\xrightarrow{\text{ratios}} \quad \frac{m_u}{m_d} = 0.66\,, \quad \frac{m_s}{m_d} = 20.1\,, \quad \frac{\hat{m}}{m_s} = \frac{1}{24.2} \qquad [\hat{m} = \tfrac{1}{2}(m_u + m_d)]$$

- corrections: next-to-leading order and beyond

electromagnetism — Weinberg, Gasser, Leutwyler, ...

$$\longrightarrow \quad \boxed{\frac{m_u}{m_d} = 0.553 \pm 0.043\,, \quad \frac{m_s}{m_d} = 18.9 \pm 0.8\,, \quad \frac{\hat{m}}{m_s} = \frac{1}{24.4 \pm 1.5}}$$

absolute values: sum rules or lattice QCD

no large isospin violation since $m_u - m_d$ so small vs hadronic scale

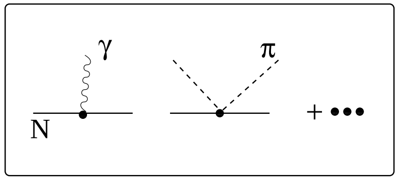
CHIRAL EFECTIVE PION-NUCLEON THEORY

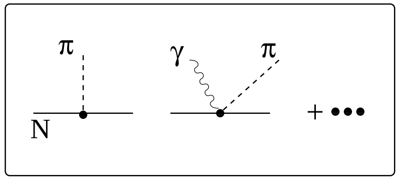
- view nucleon as matter fields [from now on, SU(2) only]
- chiral symmetry *dictates* the couplings to pions & external sources

$\downarrow$ a few steps well documented in the literature

$$\mathcal{L} = \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N} + \ldots$$

$$\mathcal{L}^{(1)}_{\pi N} = \bar{\psi}\left(i\not{D} - m_N + \tfrac{1}{2} g_A \gamma_\mu \gamma_5 u^\mu\right)\psi$$





- tree calculations: tree graphs w/ insertions from $\mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N}$
- one-loop calculations: tree graphs w/ insertions from $\mathcal{L}^{(1)}_{\pi N} + \ldots + \mathcal{L}^{(4)}_{\pi N}$

  plus loop graphs w/ (one) insertion(s) from $\mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N}$

EFFECTIVE LAGRANGIAN AT ONE LOOP

- Pion-nucleon Lagrangian: $\boxed{\mathcal{L}_{\pi N} = \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N}}$

[$^{(n)}$ = chiral dimension]

with

$$\mathcal{L}^{(1)}_{\pi N} = \bar\Psi\left(i\,\not{D} - m_N + \tfrac{1}{2} g_A \not{u}\,\gamma_5\right)\Psi$$

[$u_\mu \sim \partial_\mu \phi$]

$$\mathcal{L}^{(2)}_{\pi N} = \sum_{i=1}^{7} c_i \bar\Psi O_i^{(2)} \Psi = \bar\Psi\Bigg(c_1 \langle \chi_+ \rangle + c_2\left(-\frac{1}{8m_N^2}\langle u_\mu u_\nu \rangle D^{\mu\nu} + \text{h.c.}\right) + c_3\,\frac{1}{2}\,\langle u \cdot u \rangle$$

$$+ c_4\,\frac{i}{4}\,[u_\mu, u_\nu]\,\sigma^{\mu\nu} + c_5\,\widetilde{\chi}_+ + c_6\,\frac{1}{8m_N} F^+_{\mu\nu}\sigma^{\mu\nu} + c_7\,\frac{1}{8m_N}\left\langle F^+_{\mu\nu}\right\rangle \sigma^{\mu\nu}\Bigg)\Psi$$

- dynamical LECs $g_A \sim \partial_\mu \phi$, and $c_2, c_3, c_4 \sim \partial_\mu^2 \phi, \partial_\mu \partial_\nu \phi$
- symmetry breaking LECs $c_1 \sim m_u + m_d$, $c_5 \sim m_u - m_d$
- external probe LECs $c_6, c_7 \sim eQ\mathcal{A}_\mu$

$$\mathcal{L}^{(3)}_{\pi N} = \sum_{i=1}^{23} d_i\,\bar\Psi\,O_i^{(3)}\,\Psi\,, \qquad \mathcal{L}^{(4)}_{\pi N} = \sum_{i=1}^{118} e_i\,\bar\Psi\,O_i^{(4)}\,\Psi$$

for details, see Fettes et al., Ann. Phys. 283 (2000) 273 [hep-ph/0001308]

POWER-COUNTING in the PION-NUCLEON THEORY

- nucleon mass $m_N \sim 1$ GeV $\longrightarrow$ only three-momenta can be soft
  $\longrightarrow$ complicates the power counting (see fig.) Gasser, Sainio, Svarc, Nucl. Phys. B **307** (1988) 779

- solutions:
  (1) Heavy-baryon approach
  Jenkins, Manohar; Bernard, Kaiser, M., . . .

  $1/m_N$ expansion a la Foldy-Wouthuysen of the Lagrangian

  m_N only appears in vertices, no longer in the propagator

  (2) Infrared Regularization [or variants thereof like EOMS]
  Becher, Leutwyler; Kubis, M.; Scherer, . . .

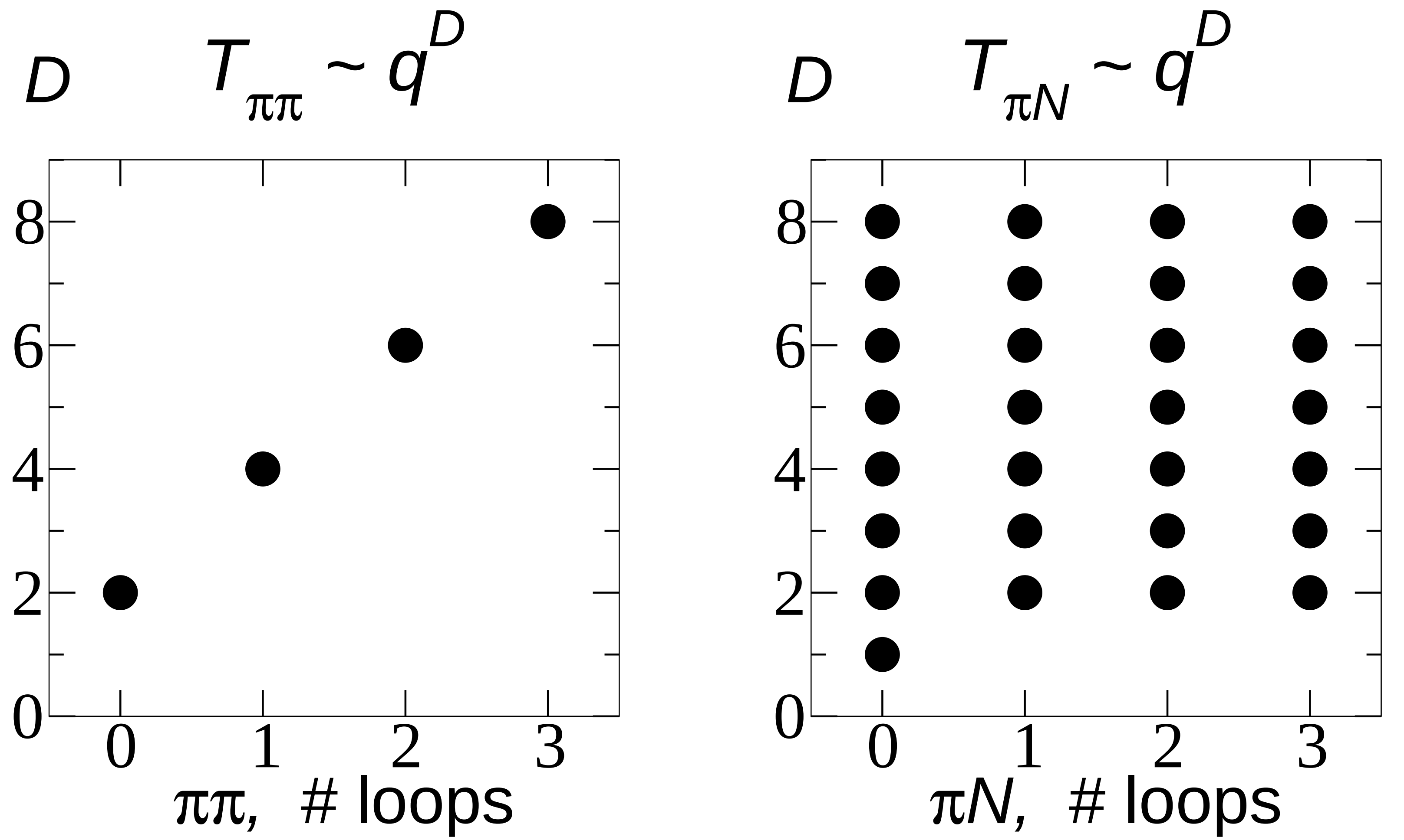
  extraction of the soft parts from the loop integrals

  easier to retain proper analytic structure

- most calculations at one loop, only two at two loop accuracy (g_A, m_N)
  Bernard, M.; Schindler, Scherer, Gegelia

FAILURE of the POWER-COUNTING

- naive extension of loop graphs from the pion to the pion-nucleon sector



HEAVY BARYON APPROACH I

- consider the nucleon as a static, heavy source $\longrightarrow$ four-velocity v_μ:

Jenkins, Manohar 1991

$$\boxed{p_\mu = m_N v_\mu + \ell_\mu}\,, \quad v^2 = 1, \quad p^2 = m_N^2, \quad v\cdot\ell \ll m_N$$

- velocity-projection: $\Psi(x) = \exp(-im_N v\cdot x)\,[H(x) + h(x)]$

with $\quad \not{v}H = H,\ \not{v}h = -h \quad$ ["large/small" components]

- H- and h-components decouple, separated by large mas gap $2m_N$:

$$\mathcal{L}_{\pi N}^{(1)} = \bar\Psi\left(i\not{D} - m_N + \tfrac{1}{2}g_A\not{u}\gamma_5\right)\Psi$$

$$u_\mu = iu^\dagger\nabla_\mu U u^\dagger,\ U = u^2$$

$$\nabla_\mu U = \partial_\mu U - ieA_\mu[Q,U],\ Q = \mathrm{diag}(1,0)$$

$$D_\mu\Psi = \partial_\mu\Psi + \tfrac{1}{2}\left(u^\dagger(\partial_\mu - ieA_\mu Q)u + u(\partial_\mu - ieA_\mu Q)u^\dagger\right)\Psi$$

$$\longrightarrow \boxed{\mathcal{L}_{\pi N}^{(1)} = \bar H\left(iv\cdot D + g_A S\cdot u\right)H + \mathcal{O}\left(\frac{1}{m_N}\right)}$$

HEAVY BARYON APPROACH II

- covariant spin-vector à la Pauli-Lubanski:

$$S_\mu = \tfrac{i}{2}\gamma_5\sigma_{\mu\nu}v^\nu, \;\; S\cdot v = 0, \;\; \{S_\mu, S_\nu\} = \tfrac{1}{2}(v_\mu v_\nu - g_{\mu\nu}), \;\; S^2 = \frac{1-d}{4}$$

- the Dirac algebra simplifies considerably (only v_μ and S_μ):

$$\bar{H}\gamma_\mu H = v_\mu \bar{H}H, \;\; \bar{H}\gamma_5 H = \mathcal{O}(\tfrac{1}{m_N}), \;\; \bar{H}\gamma_\mu\gamma_5 H = 2\bar{H}S_\mu H, \ldots$$

- propagator:

$$\boxed{S(\omega) = \frac{i}{\omega + i\eta}, \;\; \omega = v\cdot\ell, \;\; \eta \to 0^+}$$

- mass scale moved from the propagator to $1/m_N$ suppressed vertices
  $\longrightarrow$ power counting

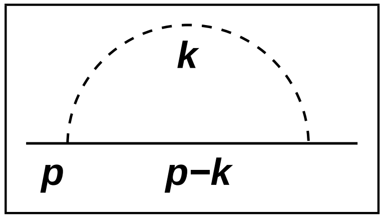
- can be systematically extended to arbitrary orders in $1/m_N$
  Bernard, Kaiser, Kambor, M., 1992

INFRARED REGULARIZATION I

- relativistic calculation of the nucleon self-energy:

Gasser, Sainio, Švarč, 1988, Becher, Leutwyler 1999

$$H(p^2) = \frac{1}{i}\int \frac{d^dk}{(2\pi)^d}\frac{1}{M_\pi^2 - k^2}\frac{1}{m_N - (p-k)^2}$$



$$\to H(s_0) = c(d)\frac{M_\pi^{d-3} + m_N^{d-3}}{M_\pi + m_N} = I + R\,, \quad s_0 = (M_\pi + m_N)^2$$

infrared singular piece I: generated by momenta of the order M_π
contains the chiral physics like chiral logs etc.

infrared regular piece R: generated by momenta of the order m_N
leads to the violation of the power counting
polynomial in external momenta and quark masses
$\to$ can be absorbed in the LECs of the eff. Lagr.

INFRARED REGULARIZATION II

Becher, Leutwyler 1999

- this symmetry-preserving splitting can be *uniquely* defined for any one-loop graph

- method to separate the infrared singular and regular parts (end-point singularity at $z=1$):

$$H = \int \frac{d^dk}{(2\pi)^d}\frac{1}{AB} = \int_0^1 dz \int \frac{d^dk}{(2\pi)^d}\frac{1}{[(1-z)A+zB]^2}$$

$$= \left\{\int_0^\infty - \int_1^\infty\right\} dz \int \frac{d^dk}{(2\pi)^d}\frac{1}{[(1-z)A+zB]^2} = I + R$$

$$A = M_\pi^2 - k^2 - i\eta\ ,\ \ B = m^2 - (p-k)^2 - i\eta\ ,\ \ \eta \to 0^+$$

- preserves the low-energy analytic structure of any one-loop graph

- extension to higher loop graphs difficult

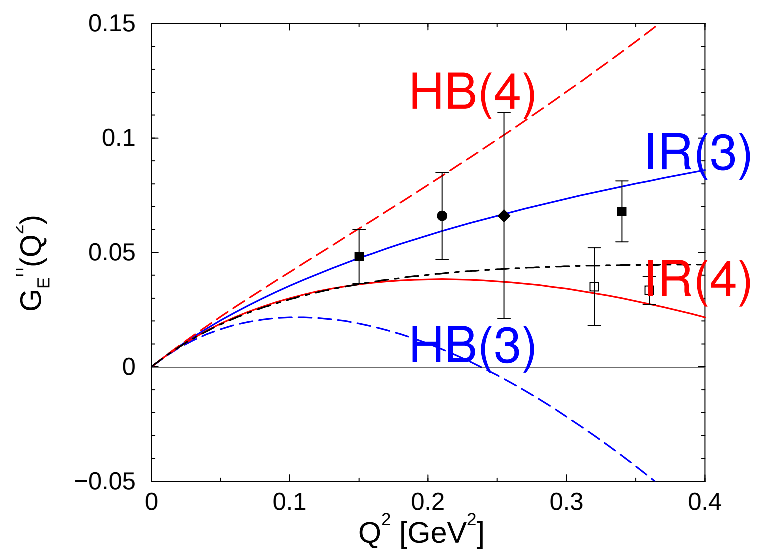
Lehmann, Prezeau, 2002

HEAVY BARYON vs INFRARED REGULARIZATION

- Heavy baryon (HB) is algebraically much simpler than infrared regularization (IR)
- HB can be extended to higher loop orders (IR requires modifications)
- Strict HB approach sometimes at odds with the analytic structure, IR not, e.g. anomalous threshold in triangle diagram (isovector em form factors)

$$t_c = 4M_\pi^2 - M_\pi^4/m_N^2 \stackrel{HB}{=} 4M_\pi^2 + \mathcal{O}(1/m_N^2)$$

- IR resums kinetic energy insertions
  $\longrightarrow$ sometimes improves convergence
  e.g. neutron electric ff $G_E^n(Q^2)$
  Kubis, M., 2001



- for a detailed discussion, see the review
  Bernard, Prog. Nucl. Part. Phys. **60** (2008) 82

POWER COUNTING in the PION-NUCLEON SYSTEM II

- consider the nucleon mass being eliminated, e.g. in the heavy baryon scheme $S(q) \sim 1/(v \cdot q)$ and vertices with $d \geq 1$

- Goldstone bosons as before, $d \geq 2$ and $D(q) \sim 1/(q^2 - M^2)$

- consider an L-loop diagram with I_B internal baryon lines, I_M internal meson lines, V_d^M mesonic vertices and V_d^{MB} meson-nucleon vertices of order d

$$Amp \propto \int (d^4q)^L \frac{1}{(q^2)^{I_M}} \frac{1}{(q)^{I_B}} \prod_d (q^d)^{(V_d^M + V_d^{MB})}$$

- let $Amp \sim q^\nu \to \nu = 4L - 2I_M + I_B + \sum_d d(V_d^M + V_d^{MB})$

- topology: $L = I_M + I_B - \sum_d (V_d^m + V_d^{MB}) + 1$
  and **one** baryon line through the diagram: $\sum_d V_d^{MB} = I_B + 1$

- eliminate I_M: $\boxed{\nu = 1 + 2L + \sum_d V_d^m (d-2) + \sum_d (d-1) V_d^{MB}}$ $\boxed{\to \nu \geq 1}$

STRUCTURE of the PION-NUCLEON INTERACTION

- Pion-nucleon scattering in chiral pertubation theory

Leading order (LO) ($\nu = 1$):

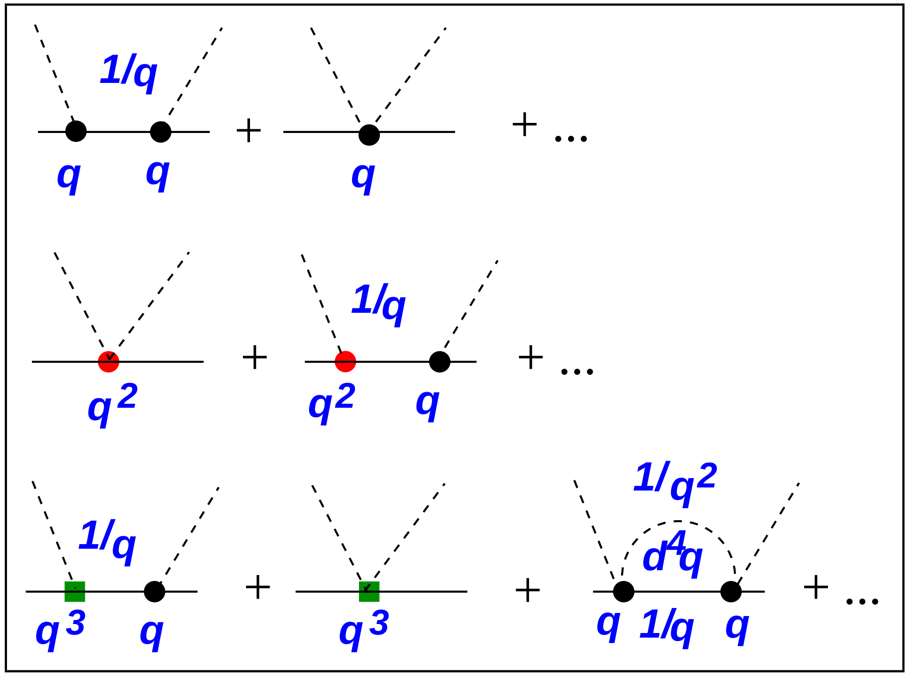
tree graphs w/ insertions with $d = 1$

Next-to-leading order (NLO) ($\nu = 2$):

tree graphs w/ insertions with $d = 1, 2$

Next-to-next-to-leading order (NNLO) ($\nu = 3$):

tree graphs w/ insertions with $d = 1, 2, 3$
and one-loop graphs w/insertion with $d = 1$



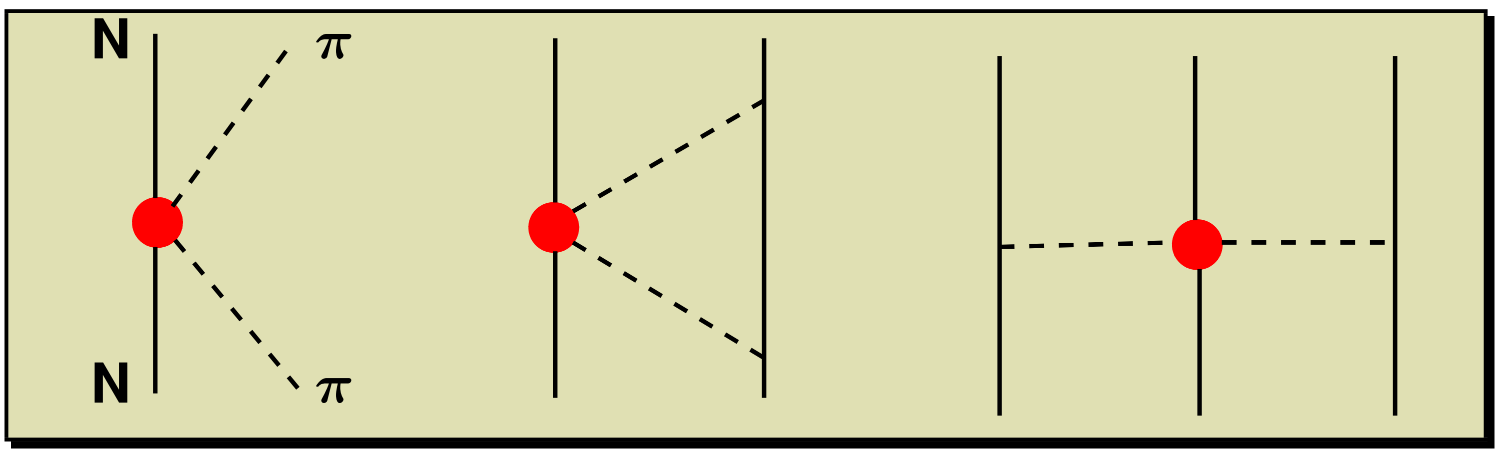
- calculations have been performed up to $\nu = 4$ (NNNLO = complete one-loop):

  heavy-baryon scheme Fettes, M., Nucl. Phys. A **676** (2000) 311

  infrared-regularization scheme Becher, Leutwyler, JHEP **06** (2001) 017

APPLICATION: DIMENSION-TWO LECS

- Low-energy constants (LECs) relate ***many*** processes
- e.g. the dimension-two LECs c_i in πN, NN, NNN, $\ldots$



● = operator from $\mathcal{L}_{\pi N}^{(2)} \propto c_i \;\; (i = 1, 2, 3, 4)$

Here:
- $\boxed{\text{determine the } c_i \text{ from the purest process } \pi N \to \pi N}$
- $\boxed{\text{later use in the calculation of nuclear forces}}$

DETERMINATION OF THE LECs

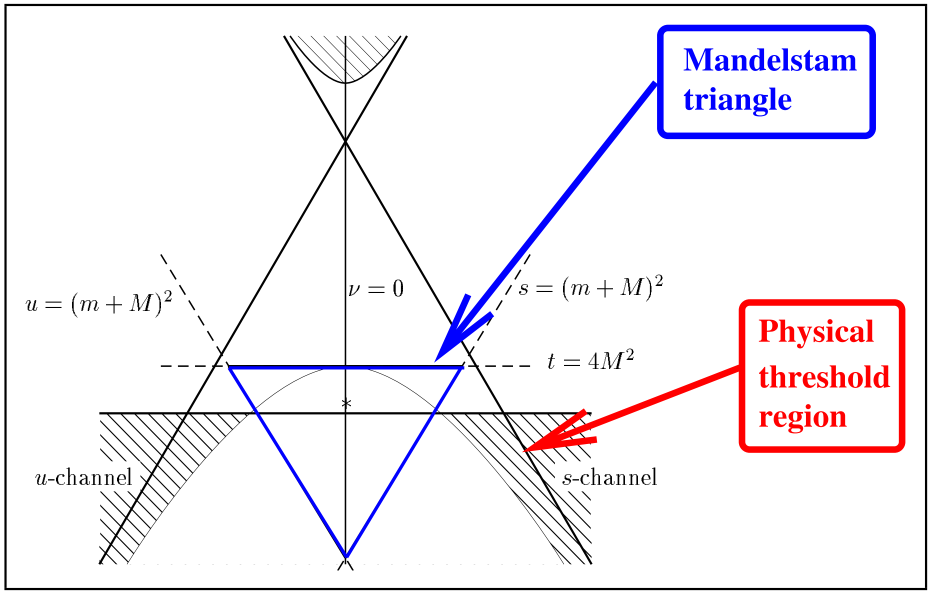
- πN scattering data can be explored in different ways:

- πN scattering inside the Mandelstam triangle:

$\longrightarrow$ best convergence, relies on dispersive analysis

$\longrightarrow$ not sensitive to all LECs, esp. c_2

Büttiker, M., Nucl. Phys. A 668 (2000) 97 [hep-ph/9908247]



- πN scattering in the threshold region:

$\longrightarrow$ large data basis, not all consistent

$\longrightarrow$ use threshold parameters and global fits

$\longrightarrow$ tree level $\mathcal{O}(p^2)$ fits tends to underestimate the LECs

$$\boxed{c_3 = -a_{01}^+ F_\pi^2 - \underbrace{\frac{g_A^2 M_\pi}{16\pi F_\pi^2}\left(g_A^2 + \frac{77}{48}\right)}_{40\%\ \text{correction}}}$$

Bernard, Kaiser, M., Nucl. Phys. A 615 (1997) 483 [hep-ph/9611253]

Fettes, M., Steininger, Nucl. Phys. A 640 (1998) 119 [hep-ph/9803266]

Fettes, M., Nucl. Phys. A 676 (2000) 311 [hep-ph/0002182]

Becher, Leutwyler, JHEP 0106 (2001) 017 [arXiv:hep-ph/0103263]

VALUES OF THE LECs

$\Rightarrow$ Resulting values in GeV^{-1}:

$$\boxed{\boldsymbol{c_1 = -0.9^{+0.2}_{-0.5}\ ,\quad c_2 = 3.3 \pm 0.2\ ,\quad c_3 = -4.7^{+1.2}_{-1.0}\ ,\quad c_4 = 3.5^{+0.5}_{-0.2}}}$$

Remarks:

- $\boldsymbol{c_{2,3,4}}$ larger than natural values: $\boldsymbol{c_i \sim g_A/\Lambda_\chi \simeq 1.1 \ldots 1.5}$
- fits with smaller sigma-term preferred $\boldsymbol{\sigma_{\pi N}(0) \simeq 45}$ MeV

Gasser, Leutwyler, Sainio, Phys. Lett. B 253 (1991) 252

- uncertainty in $\boldsymbol{c_1}$ accomodates larger $\boldsymbol{\sigma}$-term (like e.g. in GWU/VPI)
- consistent w/ determinations from peripheral NN waves:

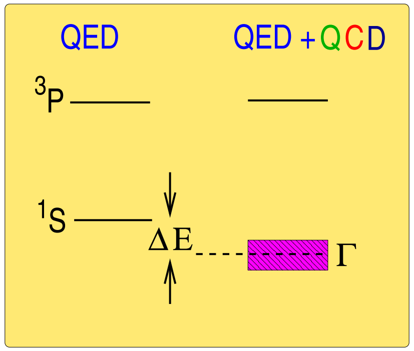
$$c_1 = 0.76(7)\ ,\quad c_3 = -4.78(10)\ ,\quad c_4 = 3.96(22)$$

Rentmeester, Timmermans, de Swart, Phys. Rev. C 67 (2003) 044001

- accuracy challenged — Entem, Machleidt, nucl-th/0303017

BOUND STATE EFT: HADRONIC ATOMS

- Hadronic atoms are bound by the static Coulomb force (QED)
- Many species: $\pi^+\pi^-$, $\pi^\pm K^\mp$, $\pi^- p$, $\boxed{\pi^- d,\ K^- p,\ K^- d,}$ $\ldots$
- Bohr radii $\gg$ typical scale of strong interactions
- Small average momenta $\Rightarrow$ non-relativistic approach
- Observable effects of QCD
  - $\star$ energy shift $\boldsymbol{\Delta E}$ from the Coulomb value
  - $\star$ decay width $\boldsymbol{\Gamma}$



$\Rightarrow$ access to scattering at zero energy! = S-wave scattering lengths

- These scattering lengths are very sensitive to the chiral & isospin symmetry breaking in QCD
  Weinberg, Gasser, Leutwyler, . . .
- can be analyzed systematically & consistently in the framework of low-energy Effective Field Theory (including virtual photons)

EFFECTIVE FIELD THEORY for HADRONIC ATOMS

- Three step procedure utilizing *nested* effective field theories

- Step 1:

  Construct non-relativistic effective Lagrangian (complex couplings)
  & solve Coulomb problem exactly, corrections in perturbation theory

- Step 2: *matching*

  relate complex couplings of $\mathcal{L}_{\rm eff}$ to QCD parameters, e.g. scattering lengths
  & express complex energy shift in terms of QCD parameters

- Step 3:

  extract scattering length(s) from the measured complex energy shift

  $\Rightarrow$ most precise way of determining hadron-hadron scattering lengths

  $\longrightarrow$ study kaonic hydrogen as one example

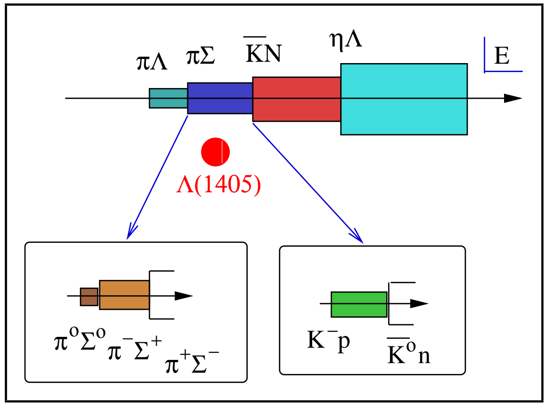
FEATURES OF KAONIC HYDROGEN

- Strong ($K^- p \to \pi^0\Lambda, \pi^\pm\Sigma^\mp, \ldots$) and weaker electromagnetic ($K^- p \to \gamma\Lambda, \gamma\Sigma^0, \ldots$) decays

  $\to$ complicated (interesting) analytical structure

- Average momentum $\langle p^2 \rangle = \alpha\,\mu \simeq 2$ MeV

  $\to$ highly non-relativistic



- Bohr radius $r_B = 1/(\alpha\,\mu) \simeq 100$ fm

- Binding energy $E_{1s} = \frac{1}{2}\,\alpha^2\,\mu + \ldots \simeq 8$ keV

- Width $\Gamma_{1s} \simeq 250$ eV $\ll E_{1s}$

- $\mathcal{M} = m_n + M_{K^0} - m_p + M_{K^-} > 0 \Rightarrow$ unitary cusp

- Isospin breaking, small parameter $\delta \sim \alpha \sim (m_d - m_u)$

$$\Delta E = \underbrace{\delta^3}_{\rm LO} + \underbrace{\delta^4}_{\rm NLO} + \ldots$$

NON-RELATIVISTIC EFFECTIVE LAGRANGIAN

$$
\begin{aligned}
\mathcal{L} \;=\;& -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \psi^\dagger \left\{ i\mathcal{D}_t - m_p + \frac{\mathcal{D}^2}{2m_p} + \frac{\mathcal{D}^4}{8m_p^3} + \cdots \right. \\
& - \mathbf{c_p^F} \frac{e\sigma\mathbf{B}}{2m_p} - \mathbf{c_p^D} \frac{e(\mathcal{D}\mathbf{E} - \mathbf{E}\mathcal{D})}{8m_p^2} - \mathbf{c_p^S} \frac{ie\sigma(\mathcal{D}\times\mathbf{E} - \mathbf{E}\times\mathcal{D})}{8m_p^2} + \cdots \left. \right\} \psi \qquad \boxed{\text{proton}} \\
& + \chi^\dagger \left\{ i\partial_t - m_n + \frac{\nabla^2}{2m_n} + \frac{\nabla^4}{8m_n^3} + \cdots \right\} \chi \qquad \boxed{\text{neutron}} \\
& + \sum_\pm (K^\pm)^\dagger \left\{ iD_t - M_{K^+} + \frac{\mathbf{D}^2}{2M_{K^+}} + \frac{\mathbf{D}^4}{8M_{K^+}^3} + \cdots \mp \mathbf{c_K^R} \frac{e(\mathbf{DE} - \mathbf{ED})}{6M_{K^+}^2} + \cdots \right\} K^\pm \\
& + (\bar{K}^0)^\dagger \left\{ i\partial_t - M_{\bar{K}^0} + \frac{\nabla^2}{2M_{\bar{K}^0}} + \frac{\nabla^4}{8M_{\bar{K}^0}^3} + \cdots \right\} \bar{K}^0 \qquad \boxed{\text{kaons}} \\
& + \tilde{\mathbf{d}}_\mathbf{1}\, \psi^\dagger\psi\, (K^-)^\dagger K^- + \tilde{\mathbf{d}}_\mathbf{2} (\psi^\dagger\chi\, (K^-)^\dagger \bar{K}^0 + h.c.) + \tilde{\mathbf{d}}_\mathbf{3} \chi^\dagger\chi\, (\bar{K}^0)^\dagger \bar{K}^0 + \cdots .
\end{aligned}
$$

$\rightarrow$ calculate electromagnetic levels and the strong shift (note: $\tilde{d}_i$ complex!)

ENERGY SHIFT in KAONIC HYDROGEN

a) recoil corrections

b) transverse photon exchange

c) finite size corrections

d) vacuum polarisation

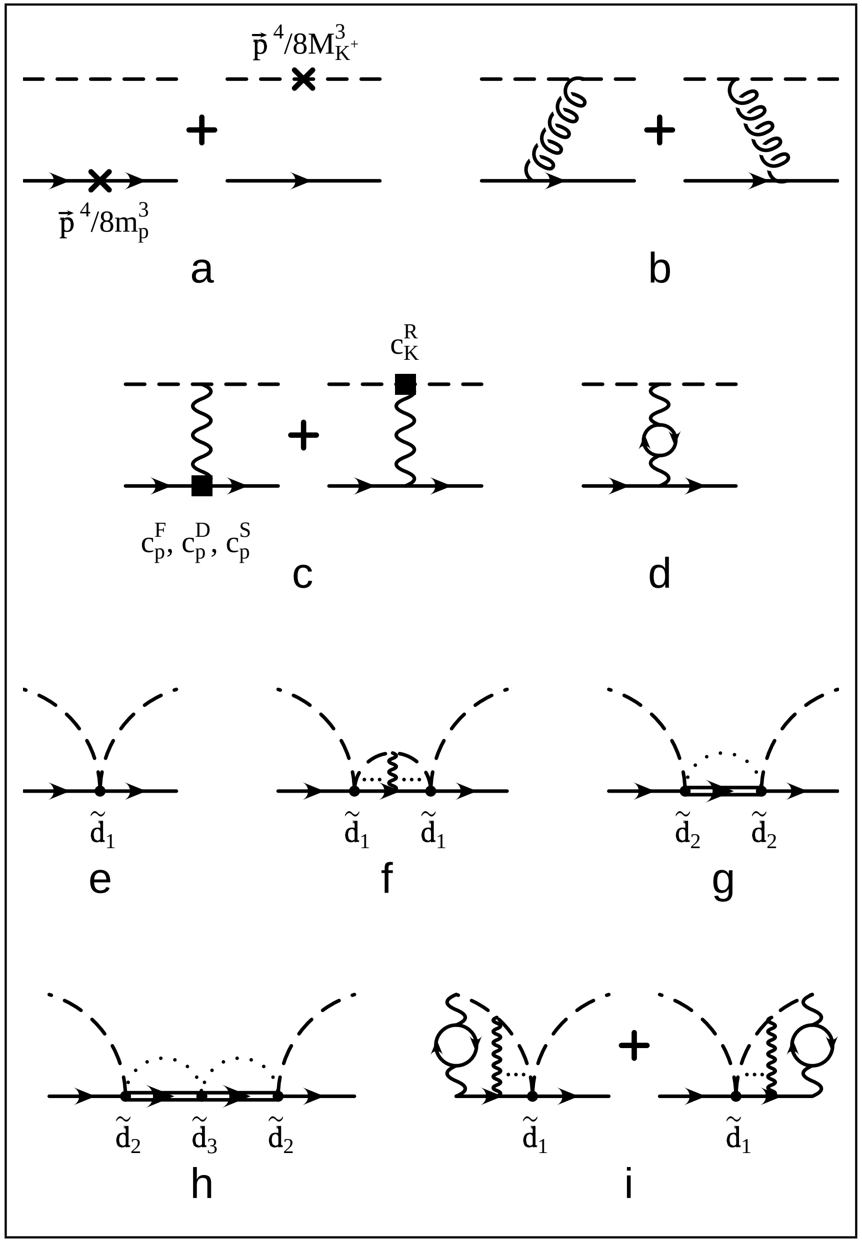
e) leading K^-p interaction

f) K^-p interaction w/ Coulomb ladders

g) leading $\bar{K}^0 n$ intermediate state

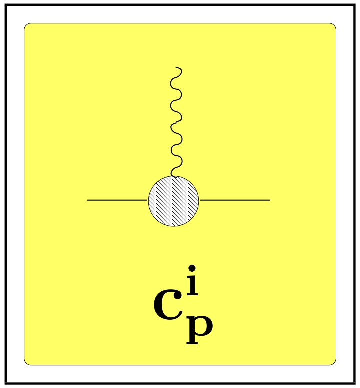
h) iterated $\bar{K}^0 n$ intermediate state

i) Coulomb ladders in the K^-p interaction



MATCHING CONDITIONS

- Electromagnetic form factors



$$\boxed{c_p^F = 1 + \mu_p,\, c_p^D = 1 + 2\mu_p + \tfrac{4}{3}\, m_p^2 \langle r_p^2 \rangle,\, c_p^S = 1 + 2\mu_p}$$

$$\boxed{c_K^R = M_{K^+}^2 \langle r_K^2 \rangle}$$

- Kaon–nucleon scattering amplitude

matching allows to express the complex strong energy shift in terms of the threshold amplitude (kaon-nucleon scattering lengths a_0 and a_1)

$$\boxed{\Delta E_n^s - \frac{i}{2}\,\Gamma_n = -\frac{\alpha^3 \mu_c^3}{2\pi M_{K^+} n^3}\, \mathcal{T}_{KN} \left\{ 1 - \frac{\alpha \mu_c^2}{4\pi M_{K^+}}\, \mathcal{T}_{KN} (s_n(\alpha) + 2\pi i) + \delta_n^{\rm vac} \right\}}$$

with $\mathcal{T}_{KN} = 4\pi \left(1 + \frac{M_{K^+}}{m_p} \right) \frac{1}{2}\,(a_0 + a_1) + O(\sqrt{\delta})$

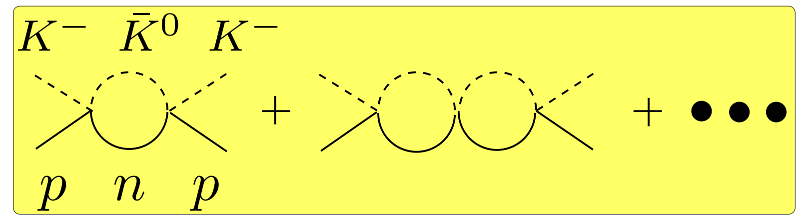
$$s_n(\alpha) = 2(\psi(n) - \psi(1) - \tfrac{1}{n} + \ln \alpha - \ln n)$$

$\Rightarrow$ correct, but not sufficiently accurate

UNITARY CUSP

- Corrections at $\mathcal{O}(\sqrt{\delta})$ can be expressed entirely in terms of a_0 and a_1

$\to$ resum the fundamental bubble
to account for the unitary cusp



$$\boxed{\mathcal{T}_{KN}^{(0)} = 4\pi\left(1 + \frac{M_{K^+}}{m_p}\right)\frac{\frac{1}{2}\,(a_0+a_1)+q_0 a_0 a_1}{1+\frac{q_0}{2}\,(a_0+a_1)}\,, \qquad q_0 = \sqrt{2\mu_0\Delta\mathcal{M}}}$$

$\star$ agrees with R.H. Dalitz and S.F.Tuan, Ann. Phys. **3** (1960) 307

$\star$ all corrections at $\mathcal{O}(\sqrt{\delta})$ included

$$\mathcal{T}_{KN} = \mathcal{T}_{KN}^{(0)} + \frac{i\alpha\mu_c^2}{2M_{K^+}}\,(\mathcal{T}_{KN}^{(0)})^2 + \underbrace{\delta\mathcal{T}_{KN}}_{\mathcal{O}(\delta)} + o(\delta)$$

$\Rightarrow$ These further $\mathcal{O}(\delta)$ corrections are expected to be small

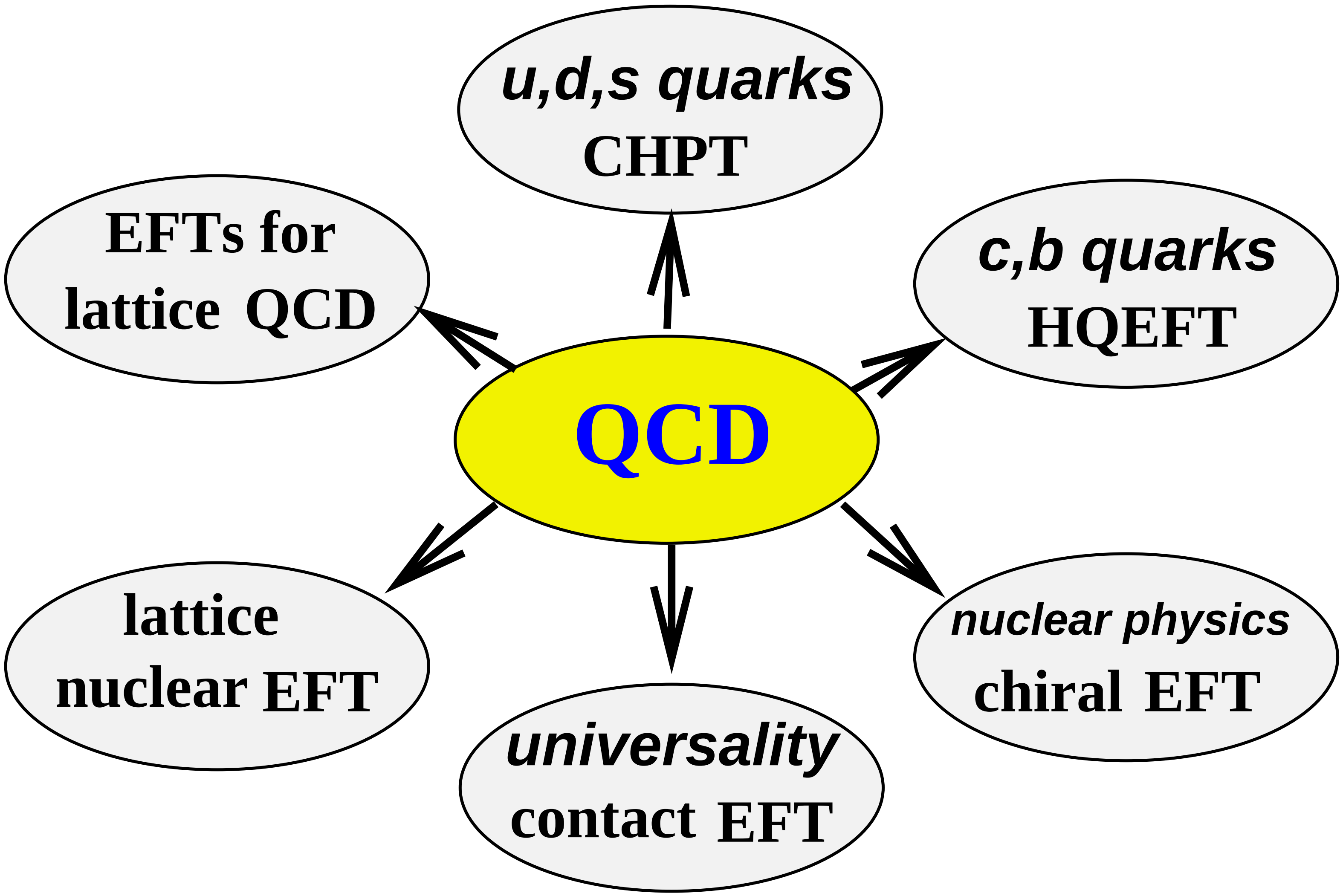
FINAL FORMULA to ANALYZE the DATA

$$\boxed{\Delta E_n^s - \frac{i}{2}\,\Gamma_n = -\frac{\alpha^3\mu_c^3}{2\pi M_{K^+} n^3}\left(\mathcal{T}_{KN}^{(0)} + \delta\mathcal{T}_{KN}\right)\left\{1 - \frac{\alpha\mu_c^2 s_n(\alpha)}{4\pi M_{K^+}}\,\mathcal{T}_{KN}^{(0)} + \delta_n^{\rm vac}\right\}}$$

- $\mathcal{O}(\sqrt{\delta})$ and $\mathcal{O}(\delta\ln\delta)$ terms:
  - $\star$ Parameter-free, expressed in terms of a_0 and a_1
  - $\star$ Numerically by far dominant
- Estimate of $\delta\mathcal{T}_{KN}$ in CHPT
  - $\star$ $\delta\mathcal{T}_{KN}/\mathcal{T}_{KN} = (-0.5 \pm 0.4)\cdot 10^{-2}$ at $O(p^2)$
  - $\star$ should be improved (loops, unitarization, influence of $\Lambda(1405)$, etc.)
- vacuum polarization calculation: $\delta_n^{\rm vac} \simeq 1\%$

D. Eiras and J. Soto, Phys. Lett. **B 491** (2000) 101 [hep-ph/0005066]

EFTs of the STRONG INTERACTIONS



- strongly intertwined

INTERMEDIATE SUMMARY

- QCD has a chiral symmetry in the light quark sector (neglecting quark masses)
- Chiral symmetry is *spontaneously* and *explicitely* broken

  appearance of almost massless Goldstone bosons (π, K, η)

  Goldstone boson interactions vanish as $E, p \to 0$
- Chiral perturbation theory is the EFT of QCD that explores chiral symmetry
- Meson sector: only even powers in small momenta, many successes
- Single nucleon sector: odd & even powers, quite a few successes
- Low-energy constants relate many processes (in particular the c_i)
- Isospin-breaking can be systematically incorporated $\longrightarrow$ spares
- NREFT can be set up for hadronic atoms $\longrightarrow$ extraction of scattering lengths

Testing chiral dynamics in hadron-hadron scattering

WHY HADRON-HADRON SCATTERING?

- Weinberg's 1966 paper "Pion scattering lengths"

Weinberg, Phys. Rev. Lett. **17** (1066) 616

– pion scattering on a target with mass m_t and isospin T_t:

$$a_T = -\frac{L}{1 + M_\pi/m_t} \left[T(T+1) - T_t(T_t+1) - 2\right]$$

– pion scattering on a pion ["the more complicated case"]:

$$a_0 = \frac{7}{4}L\,, \quad a_2 = -\frac{1}{2}L \qquad \boxed{L = \frac{g_V^2 M_\pi}{2\pi F_\pi^2} \simeq 0.1\, M_\pi^{-1}}$$

- amazing predictions - witness to the power of chiral symmetry
- what have we learned since then?

Example 1

ELASTIC PION-PION SCATTERING

- Purest process in two-flavor chiral dynamics (really light quarks)

- scattering amplitude at threshold: two numbers (a_0, a_2)

- History of the prediction for a_0:

LO (tree):	$a_0 = 0.16$	Weinberg 1966
NLO (1-loop):	$a_0 = 0.20 \pm 0.01$	Gasser, Leutwyler 1983
NNLO (2-loop):	$a_0 = 0.217 \pm 0.009$	Bijnens et al. 1996

- even better: match 2-loop representation to Roy equation solution

Roy + 2-loop:	$a_0 = 0.220 \pm 0.005$	Colangelo et al. 2000

$\Rightarrow$ this is an *amazing* prediction!

- same precision for a_2, but corrections very small ...

HOW ABOUT EXPERIMENT?

- Kaon decays (K_{e4} and $K^0 \to 3\pi^0$): most precise
- Lifetime of pionium: experimentally more difficult

Kaon decays:

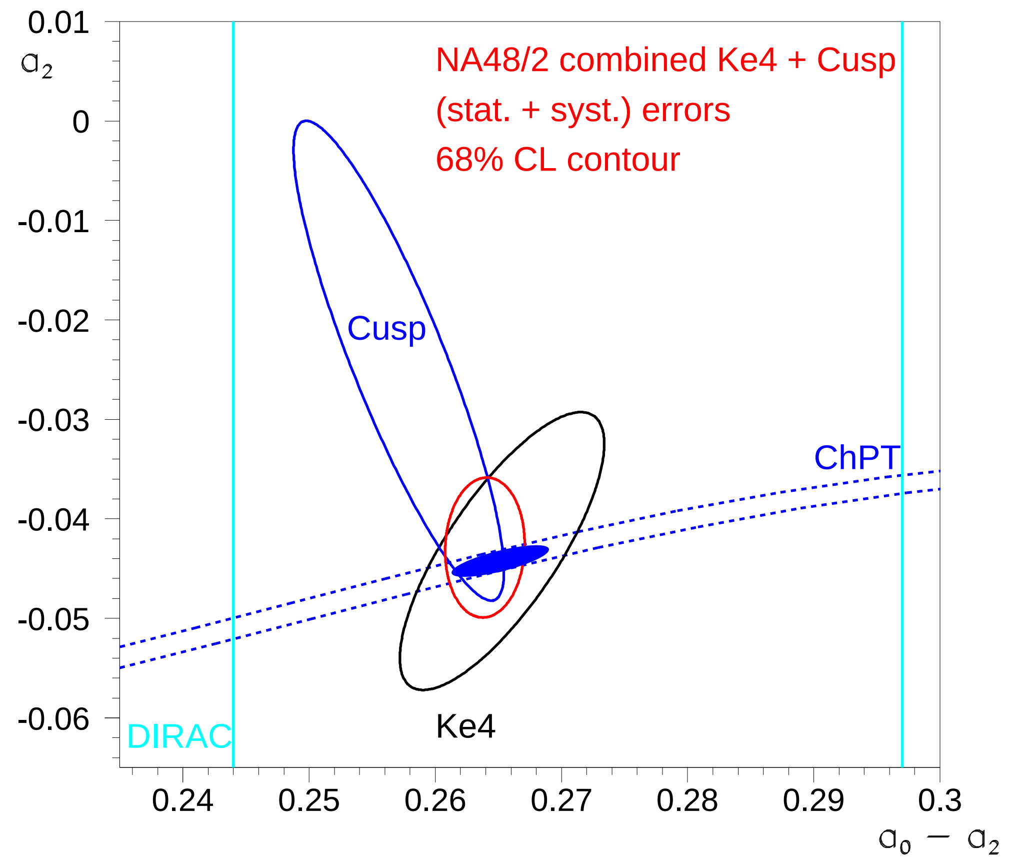
$$a_0^0 = 0.2210 \pm 0.0047_{\rm stat} \pm 0.0040_{\rm sys}$$
$$a_0^2 = -0.0429 \pm 0.0044_{\rm stat} \pm 0.0028_{\rm sys}$$

J. R. Batley et al. [NA48/2 Coll.] EPJ C 79 (2010) 635

Pionium lifetime:

$$|a_0^0 - a_0^2| = 0.264^{+0.033}_{-0.020}$$

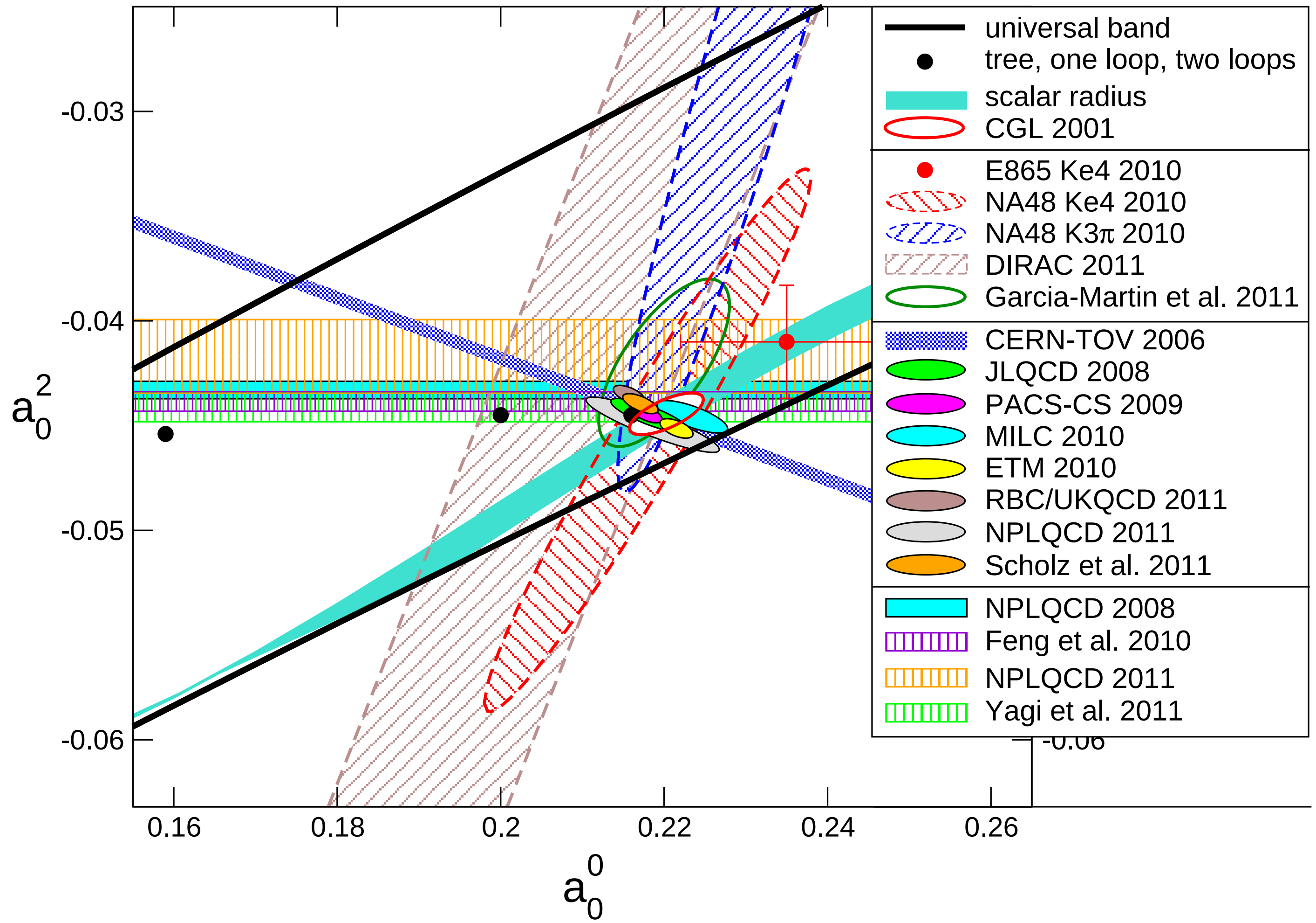
B. Adeva et al. [DIRAC Coll.] PL B 619 (2005) 50



- and how about the lattice?

$\Rightarrow$ direct and indirect determinations of the scattering lengths

Fig. courtesy Heiri Leutwyler 2012



- one of the finest tests of the Standard Model

(but: direct lattice a_0 missing)

Example 2

STRANGE QUARK MYSTERIES

- Is the strange quark really light? $\boxed{m_s \sim \Lambda_{\rm QCD}}$

$\longrightarrow$ expansion parameter: $\xi_s = \frac{M_K^2}{(4\pi F_\pi)^2} \simeq 0.18$ $\left[\text{SU(2): } \xi = \frac{M_\pi^2}{(4\pi F_\pi)^2} \simeq 0.014\right]$

- many predictions of SU(3) CHPT work quite well, but:

$\hookrightarrow$ indications of bad convergence in some recent lattice calculations:

$\star$ masses and decay constants — Allton et al. 2008

$\star$ $K_{\ell 3}$-decays — Boyle et al. 2008

$\hookrightarrow$ suppression of the three-flavor condensate?

$\star$ sum rule: $\Sigma(3) = \Sigma(2)[1 - 0.54 \pm 0.27]$ — Moussallam 2000

$\star$ lattice: $\Sigma(3) = \Sigma(2)[1 - 0.23 \pm 0.39]$ — Fukuya et al. 2011

ELASTIC PION-KAON SCATTERING

- Purest process in three-flavor chiral dynamics

- scattering amplitude at threshold: two numbers $(a_0^{1/2}, a_0^{3/2})$

- History of the chiral predictions:

	CA [1]	1-loop [2]	2-loop [3]
$a_0^{1/2}$	0.14	0.18 ± 0.03	0.220 [0.17 ... 0.225]
$a_0^{3/2}$	-0.07	-0.05 ± 0.02	$-0.047[-0.075 \ldots -0.04]$

[1] Weinberg 1966, Griffith 1969

[2] Bernard, Kaiser, UGM 1990

[3] Bijnens, Dhonte, Talavera 2004

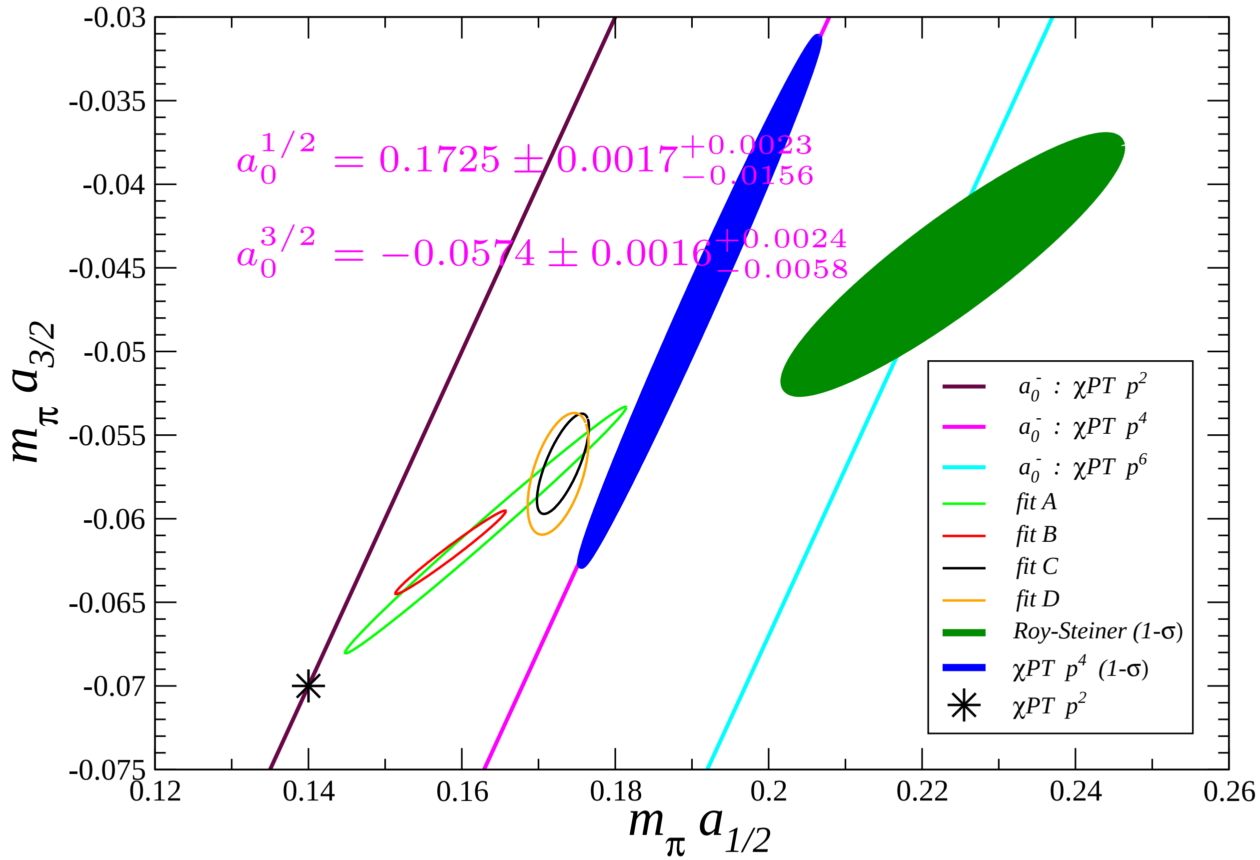
- match 1-loop representation to Roy-Steiner equation solution

$$a_0^{1/2} = 0.224 \pm 0.022\ , \quad a_0^{3/2} = -0.0448 \pm 0.0077$$

Büttiker et al. 2003

THE GRAND PICTURE

Fig. courtesy Silas Beane



- tension between lattice and Roy-Steiner (loophole: inconsistencies)
- need improved lattice results (direct calculations)

$\Rightarrow$ work required

Example 3

PION-NUCLEON SCATTERING

- simplest scattering process involving nucleons
- intriguing LO prediction for isoscalar/isovector scattering length:

$$a^+_{\rm CA} = 0, \quad a^-_{\rm CA} = \frac{1}{1+M_\pi/m_p}\frac{M_\pi^2}{8\pi F_\pi^2} = 79.5\cdot 10^{-3}/M_\pi,$$

- chiral corrections:
  - chiral expansion for a^- converges fast — Bernard, Kaiser, UGM 1995
  - large cancellations in a^+, even sign not known from scattering data

	$\mathcal{O}(q)$	$\mathcal{O}(q^2)$	$\mathcal{O}(q^3)$	$\mathcal{O}(q^4)$
fit to KA85	0.0	0.46	-1.00	-0.96
fit to EM98	0.0	0.24	0.49	0.45
fit to SP98	0.0	1.01	0.14	0.27

Fettes, UGM 2000

A WONDERFUL ALTERNATIVE: HADRONIC ATOMS

- Hadronic atoms: bound by the static Coulomb force (QED)

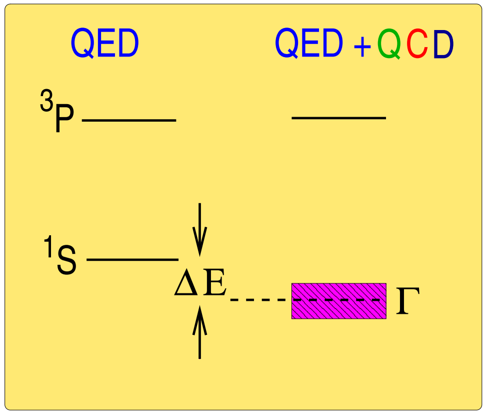
- Many species: $\pi^+\pi^-$, $\pi^\pm K^\mp$, $\pi^- p$, $\pi^- d$, $K^- p$, $K^- d$, ...

- Observable effects of QCD: strong interactions as **small** perturbations

  $\star$ energy shift $\boldsymbol{\Delta E}$

  $\star$ deacy width $\boldsymbol{\Gamma}$

  $\Rightarrow$ access to scattering at zero energy!
  = S-wave scattering lengths



- can be analyzed in suitable NREFTs

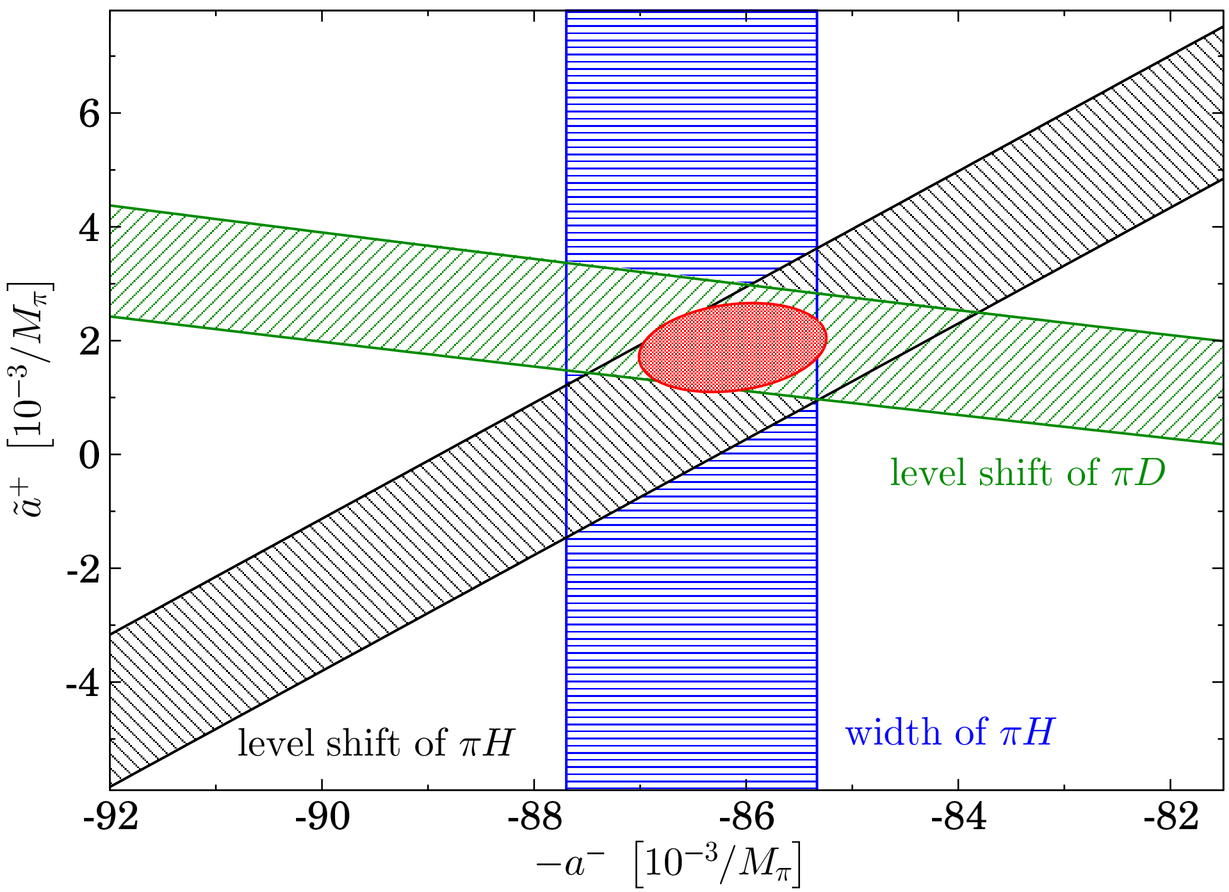
  Pionic hydrogen — Gasser, Rusetsky, . . . 2002

  Pionic deuterium — Baru, Hoferichter, Kubis . . . 2011

PION–NUCLEON SCATTERING LENGTHS

- superbe experiments performed at PSI

Gotta et al.



$$a^+ = (7.6 \pm 3.1) \cdot 10^{-3}/M_\pi$$

$$a^- = (86.1 \pm 0.9) \cdot 10^{-3}/M_\pi$$

GMO sum rule:

$$\frac{g^2_{\pi N}}{4\pi} = 13.69(12)(15)$$

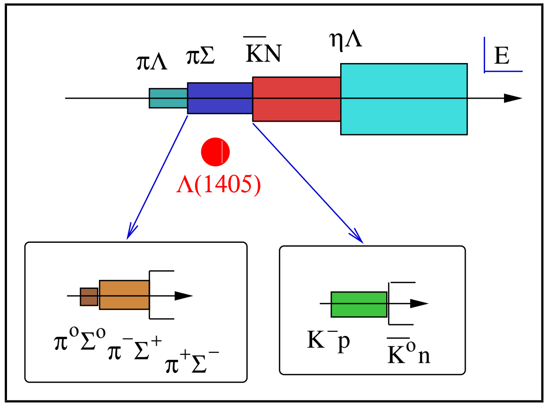
$\Rightarrow$ very precise value for a^- & first time definite sign for a^+

Example 4

ANTIKAON-NUCLEON SCATTERING

- $K^-p \rightarrow K^-p$: fundamental scattering process with strange quarks

- coupled channel dynamics



- dynamic generation of the $\Lambda(1405)$

  Dalitz, Tuan 1960

- major playground of **unitarized CHPT**

- chiral Lagrangian + unitarization leads to generation of certain resonances

  like e.g. the $\Lambda(1405)$, $S_{11}(1535)$, $S_{11}(1650)$, ...

  Kaiser, Siegel, Weise, Oset, Ramos, Oller, UGM, Lutz, ...

- loopholes: convergence a posteriori, crossing symmetry,
  on-shell approximation, unphysical poles, ...

A PUZZLE RESOLVED

- DEAR data inconsistent with scattering data

  UGM, Raha, Rusetsky 2004

$\Rightarrow$ vaste number of papers ...

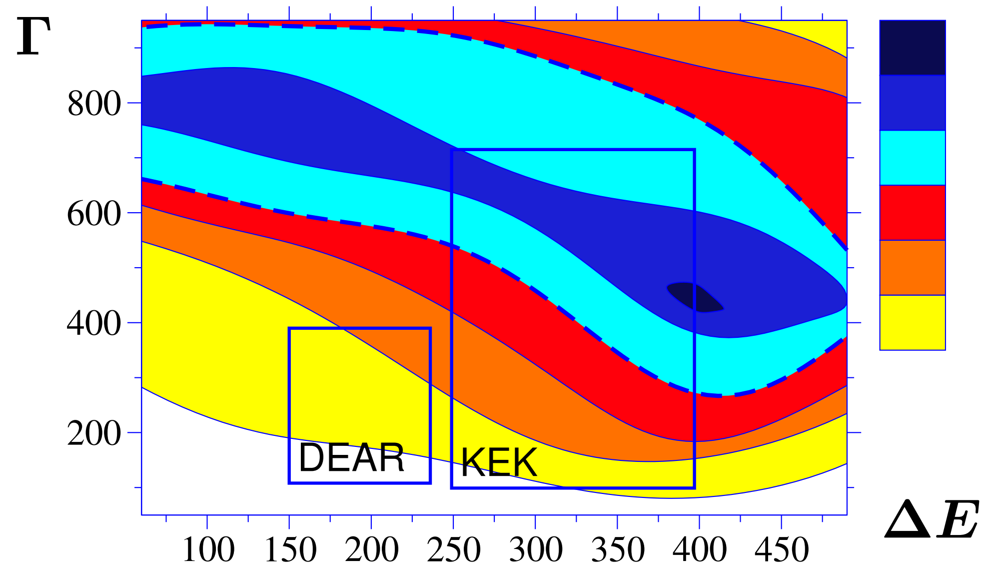
- SIDDHARTA to the rescue

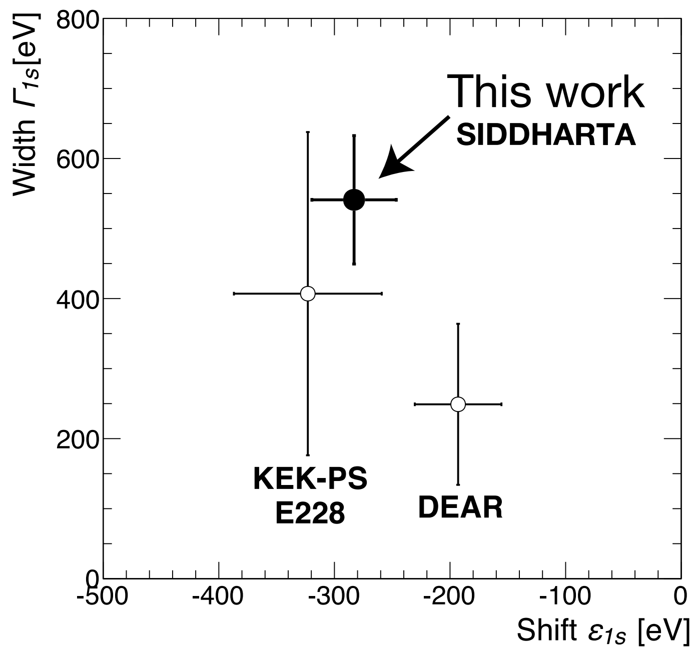
  Bazzi et al. 2011

$\Rightarrow$ more precise, consistent with KpX

$$\epsilon_{1s} = -283 \pm 36(\text{stat}) \pm 6(\text{syst}) \text{ eV}$$

$$\Gamma_{1s} = 541 \pm 89(\text{stat}) \pm 22(\text{syst}) \text{ eV}$$





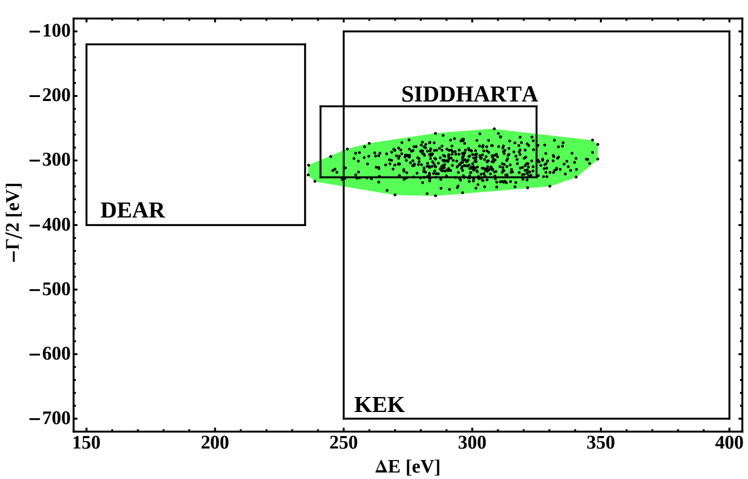
CONSISTENT ANALYSIS

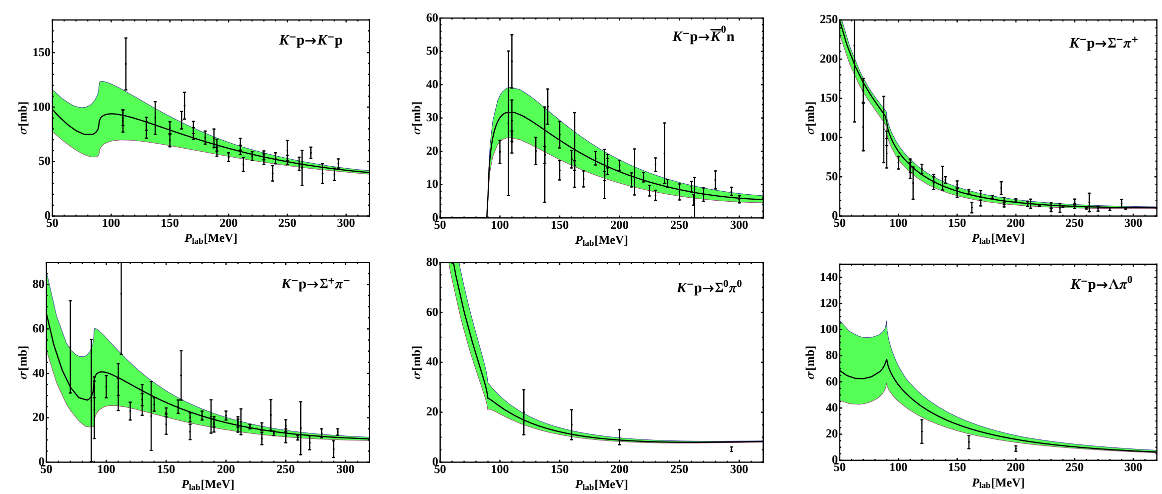
- kaonic hydrogen + scattering data can now be analyzed consistently
- use the chiral Lagrangian at NLO, two groups (different schemes)

Ikeda, Hyodo, Weise 2011; UGM, Mai 2012

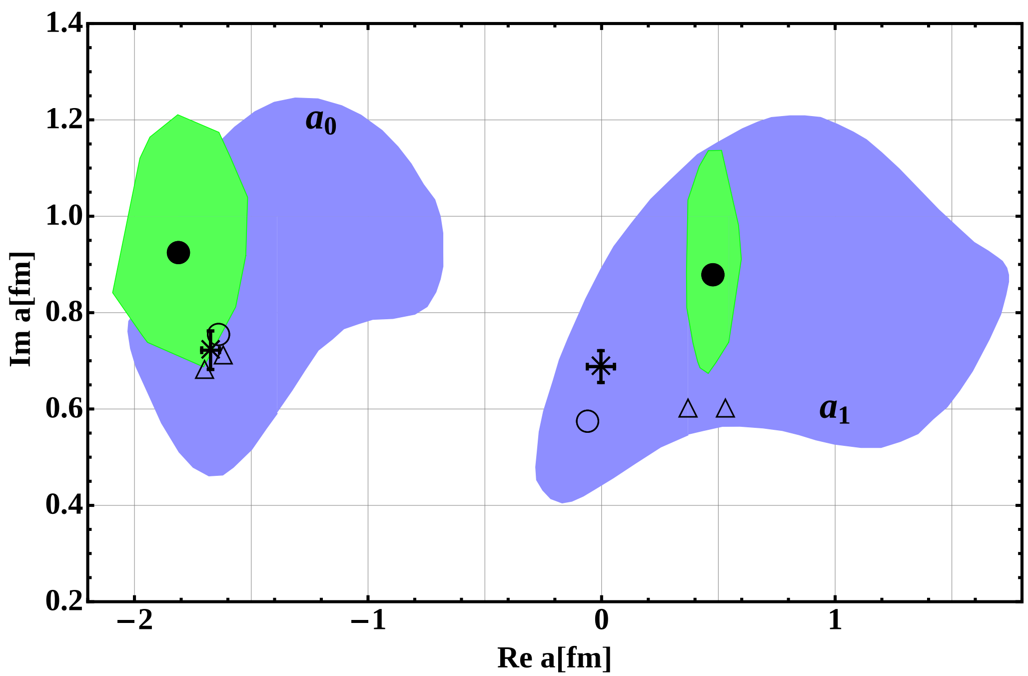
- 14 LECs and 3 subtraction constants to fit

$\Rightarrow$ simultaneous description of the SIDDHARTA and the scattering data





KAON–NUCLEON SCATTERING LENGTHS



$$a_0 = -1.81^{+0.30}_{-0.28} + i\,0.92^{+0.29}_{-0.23}\ \mathrm{fm}$$

$$a_1 = +0.48^{+0.12}_{-0.11} + i\,0.87^{+0.26}_{-0.20}\ \mathrm{fm}$$

$$a_{K^-p} = -0.68^{+0.18}_{-0.17} + i\,0.90^{+0.13}_{-0.13}\ \mathrm{fm}$$

SIDDHARTA only:

$$a_{K^-p} = -0.65^{+0.15}_{-0.15} + i\,0.81^{+0.18}_{-0.18}\ \mathrm{fm}$$

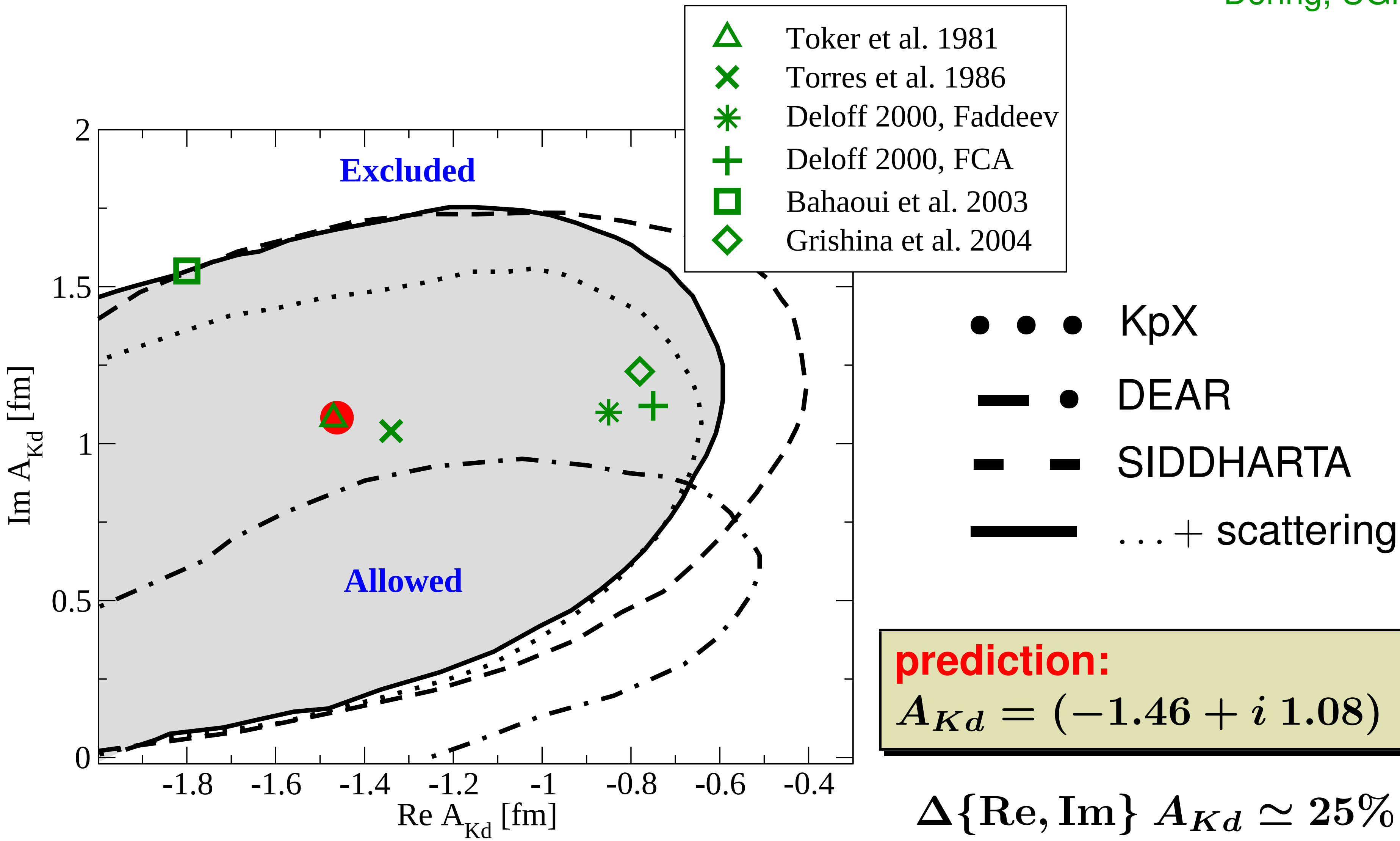
- clear improvement compared to scattering data only

$\Rightarrow$ fundamental parameters to within about 15% accuracy

KAON–DEUTERON SCATTERING LENGTH

- analyze K^-d, imposing consistency with the $\bar{K}N$ scattering lengths

Döring, UGM 2011



prediction:
$A_{Kd} = (-1.46 + i\,1.08)$ fm

$\Delta\{\mathrm{Re},\mathrm{Im}\}\,A_{Kd} \simeq 25\%$

Example 5

EFFECTIVE LAGRANGIAN for $\phi D \to \phi D$

- Goldstone boson octet (π, K, η) scatters off D-meson triplet (D^0, D^+, D_s^+)
- multi-scale/multi-faceted problem:
  - light particles, chiral symmetry $\longrightarrow$ chiral expansion in (p, m_q)
  - heavy particles, heavy quark symmetry $\longrightarrow$ expansion in $1/m_c$
  - isospin-violation $\longrightarrow$ strong = quark mass difference $m_d \neq m_u$

    $\longrightarrow$ electromagnetic = quark charge difference $q_u \neq q_d$
- 16 channels with different total strangeness and isospin
  - some are perturbative
  - some are non-perturbative, require resummation $\longrightarrow$ possible molecules

RESULTS for $\phi D \to \phi D$

- $T(\phi D \to \phi D)$ depends on two LECs at NLO, called h_3 and h_5:

  h_3 the mass of the $D^*_{s0}(2317)$ as a DK molecule

  h_5 from naturalness, $h_5/M_D^2 \in [-1,+1]$

  $\Rightarrow \boxed{\Gamma(D^*_{s0}(2317)^+ \to D_s^+\pi^0) = (180 \pm 110)\,\mathrm{keV}}$ testable prediction

  note: much smaller in quark models (a few keV)

- expectation for the scattering length for $DK(I=0)$ in the molecular picture:

$$a_{DK}^{I=0} = -g_{\rm eff}^2 \Delta_{DK} = -\frac{1}{2\sqrt{\mu_{DK}\varepsilon}} \simeq 1\,\text{fm}$$

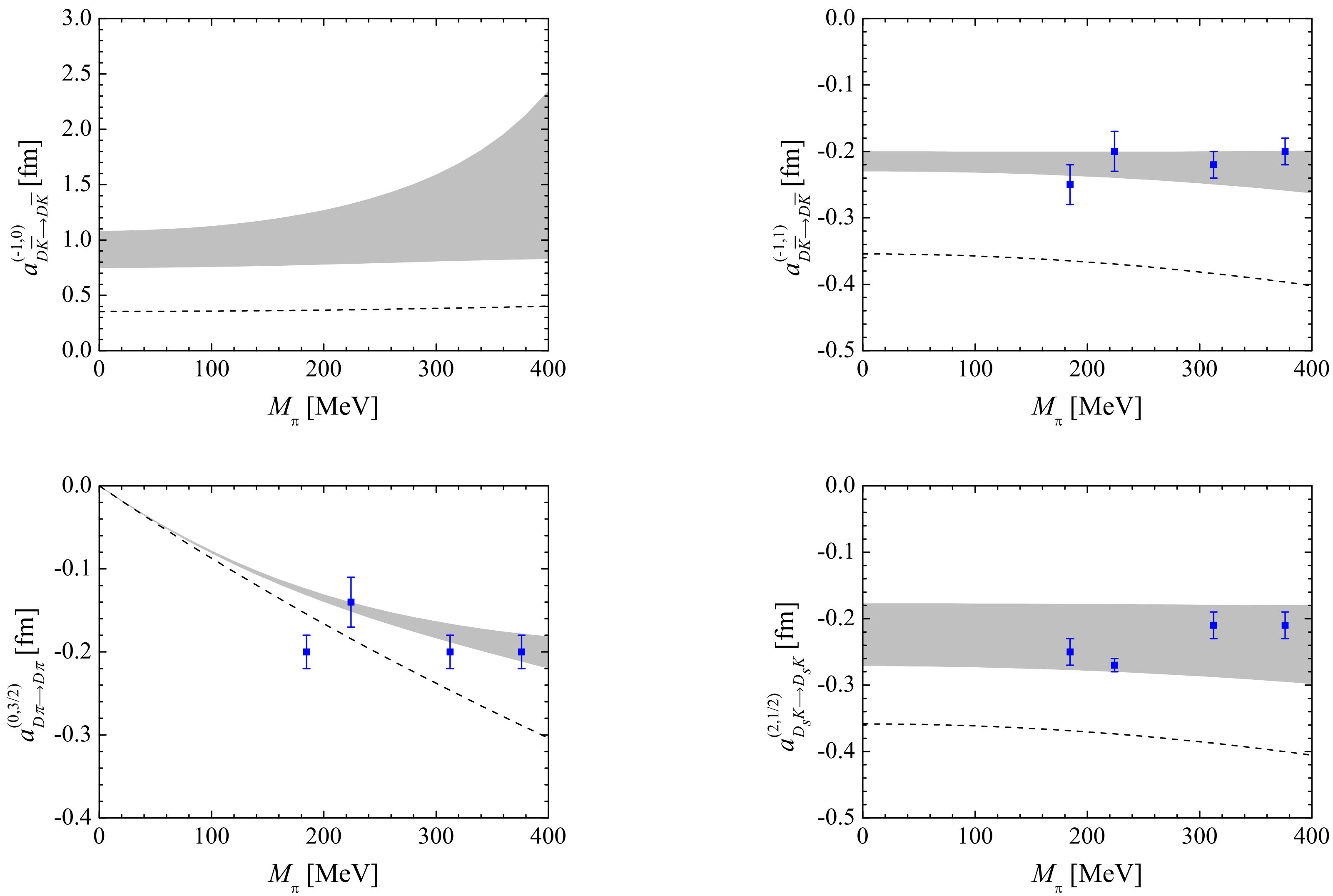
- no data, but first lattice investigations at varying quark masses

Liu, Lin, Orginos, PoS LATTICE **2008**, 112 and more to come!

QUARK MASS DEPENDENCE

- *predictions:* channels with no poles

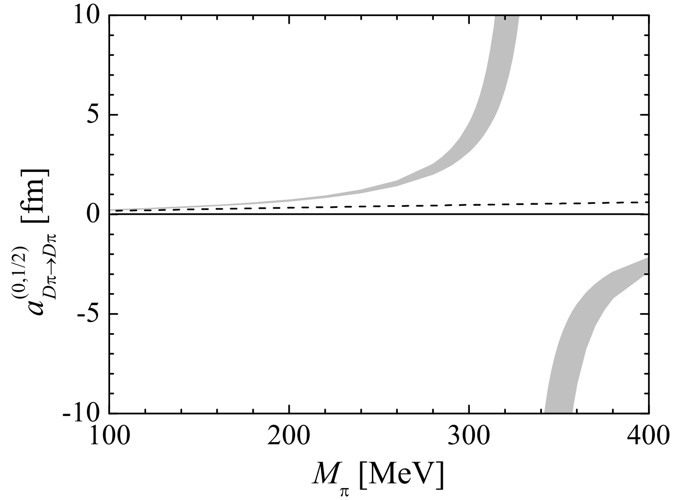
Guo, Hanhart, UGM 2009



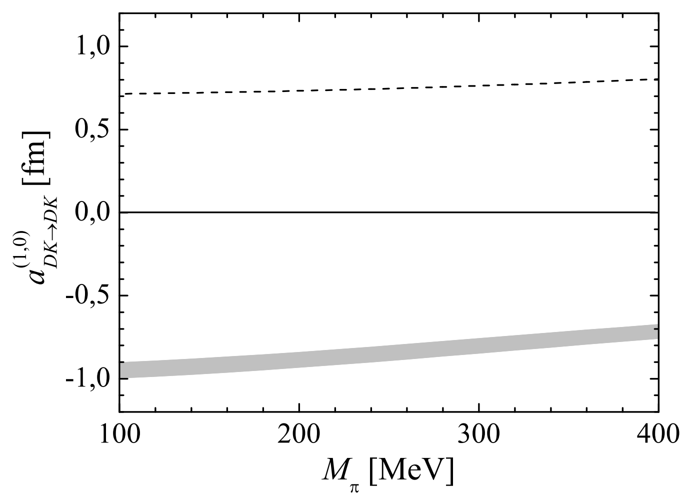
QUARK MASS DEPENDENCE cont'd

- *predictions:* channels with poles $\longrightarrow$ resonances or molecular states

$$D\pi \to D\pi\ (0,1/2)$$



$$DK \to DK\ (1,0)$$



a pair of poles above thr.

$$a^{(0,1/2)}_{D\pi} = 0.35(1)\ \text{fm}$$

a bound states below thr. $D^*_{s0}(2317)$

$$a^{(1,0)}_{DK} = -0.93(5)\ \text{fm}$$

$\Rightarrow$ lattice test of the molecular nature

NATURE of the $D_{s1}(2460)$

- Nature of the $D_{s1}(2460)$: $M_{D_{s1}(2460)} - M_{D^*_{s0}(2317)} \simeq M_{D^*} - M_D$

$\Rightarrow$ most likely a $D^\star K$ molecule (if the $D^\star_{s0}(2317)$ is DK)

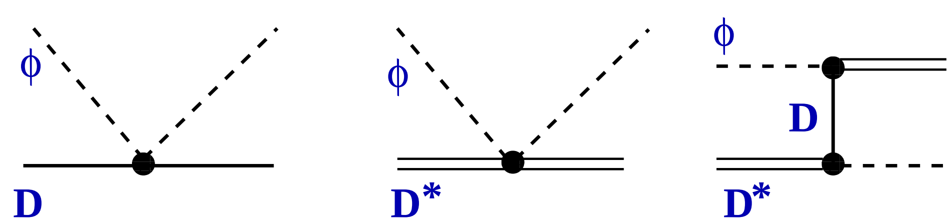
$\Rightarrow$ study Goldstone boson scattering off D- and $D^\star$-mesons

- Use heavy meson chiral perturbation theory Wise, Donoghue et al., Casalbuoni et al., ...

$$H_v = \frac{1+\not{v}}{2}\left[\not{V}^*_v + iP_v\gamma_5\right]$$

$$P = (D^0, D^+, D_s^+)\,, \quad V^*_\mu = (D^{*0}_\mu, D^{*+}_\mu, D^{*+}_{s,\mu})$$

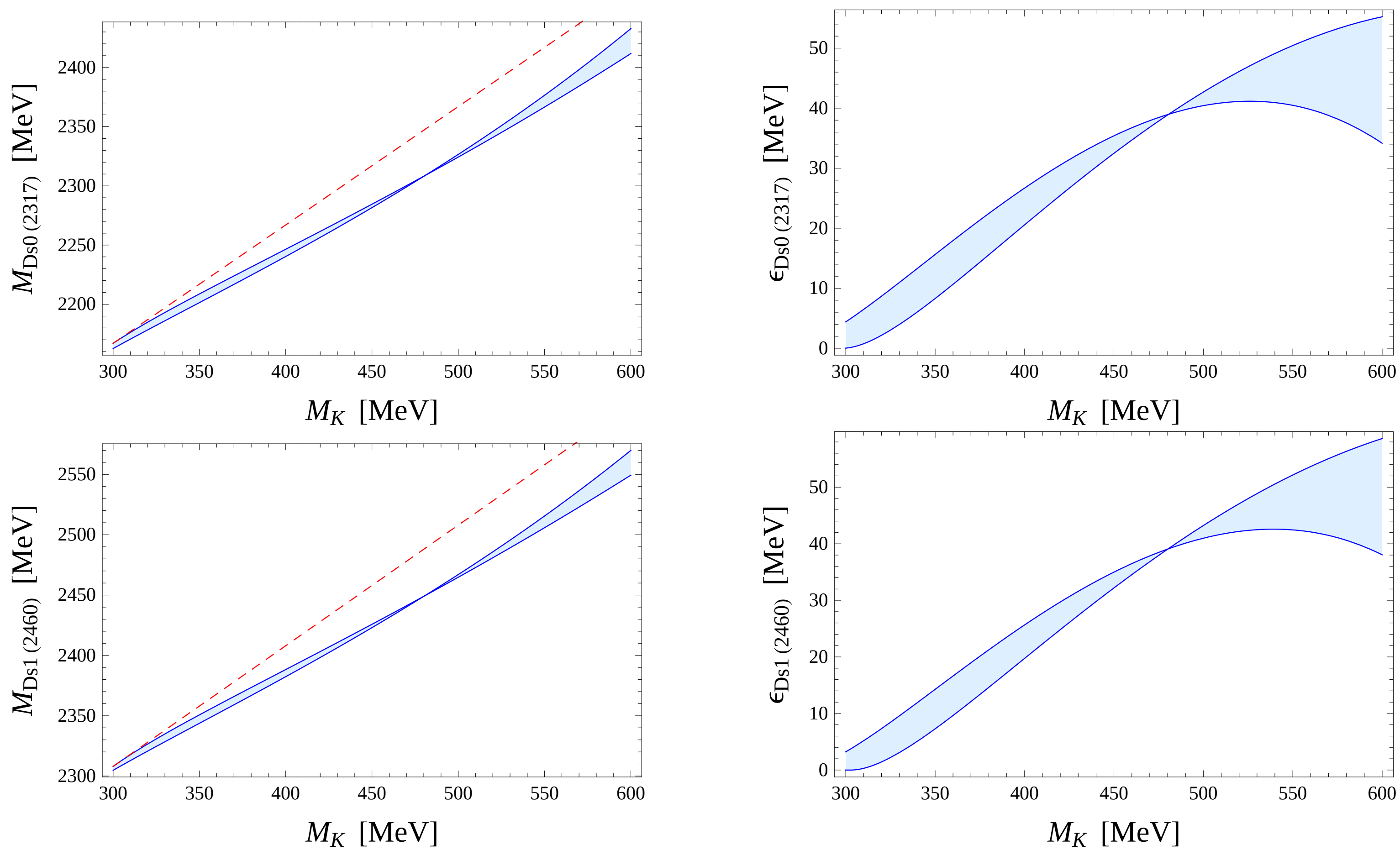
- T-matrix:



- Unitarization (as before) $\longrightarrow$ find poles in the complex plane

KAON MASS DEPENDENCE

- Mass and binding energy: $M_{\rm mol} = M_K + M_H - \epsilon$ — Guo, Hanhart, UGM 2009



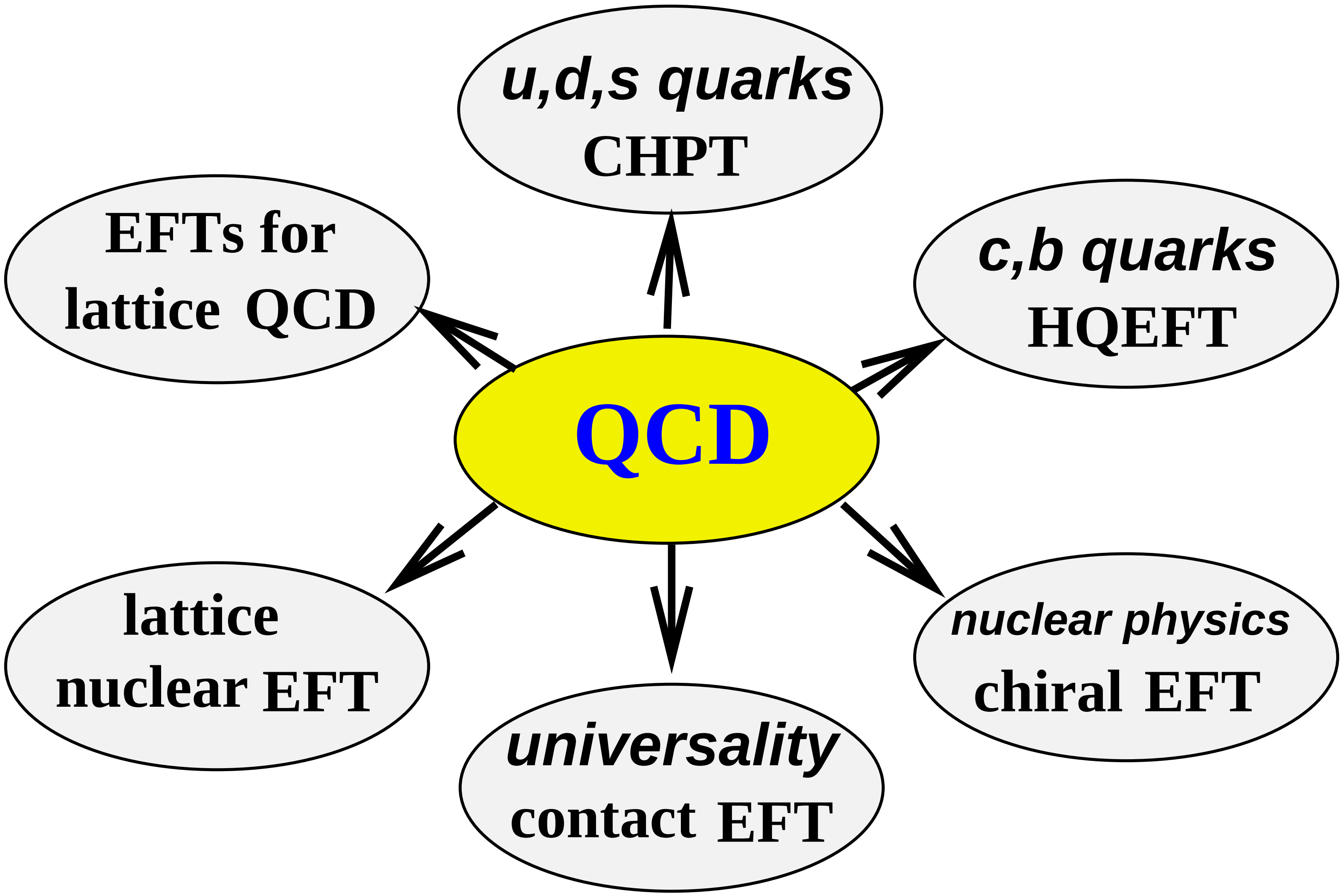
$\Rightarrow$ typical for a molecule $\rightarrow$ test in LQCD

INTERMEDIATE SUMMARY

- Hadron-hadron scattering: important role of chiral symmetry (CHPT)
  $\longrightarrow$ combine with dispersion relations, unitarization, lattice
- Pion-pion scattering
  $\longrightarrow$ a fine test of the Standard Model
- Pion-kaon scattering
  $\longrightarrow$ tension between lattice and Roy-Steiner solution
- Pion-nucleon scattering
  $\longrightarrow$ superbe accuracy from EFTs for pionic hydrogen/deuterium
- Antikaon-nucleon scattering
  $\longrightarrow$ consistent determination of the scattering lengths possible
- Goldstone-boson scattering off $D, D^\star$–mesons
  $\longrightarrow$ lattice test of molecular states possible

$\Rightarrow$ exciting times ahead of us

EFTs of the STRONG INTERACTIONS



- strongly intertwined

FINAL SUMMARY

- Basic ideas underlying EFT:
  Separate different scales, identify proper degrees of freedom
- Implement the consequences of symmetries
- EFT allows you to compute using dimensional analysis
  (even if you don't know the theory)
- EFT is very useful way of thinking about problems
- All quantum field theories are EFTs

SPARES ...

Isospin symmetry and isospin violation

ISOSPIN SYMMETRY

- For $m_u = m_d$, QCD is invariant under *SU(2) isospin* transformations:

$$\boxed{q \to q' = U q\,, \quad q = \begin{pmatrix} u \\ d \end{pmatrix}, \quad U = \begin{pmatrix} a^* & b^* \\ -b & a \end{pmatrix}, \quad |a|^2 + |b|^2 = 1}$$

  – NB: Charge symmetry = 180° rotation in iso-space

- Rewriting of the QCD quark mass term:

$$\boxed{\mathcal{H}^{\rm SB}_{\rm QCD} = m_u\,\bar{u}u + m_d\,\bar{d}d = \underbrace{\frac{1}{2}(m_u + m_d)(\bar{u}u + \bar{d}d)}_{I=0} + \underbrace{\frac{1}{2}(m_u - m_d)(\bar{u}u - \bar{d}d)}_{I=1}}$$

Strong isospin violation (IV)

- Competing effect: QED $\longrightarrow$ can be treated in CHPT
- Standard situation: small IV on top of large iso-symmetric background, requires precise machinery to perform accurate calculations

Gasser, Urech, Steininger, Fettes, M., Knecht, Kubis, . . .

ISOSPIN VIOLATION - PIONS & KAONS

Weinberg, Gasser, Leutwyler, Urech, Neufeld, Knecht, M., Müller, Steininger, . . .

- SU(2) effective Lagrangian w/ virtual photons to leading order:

Q = quark charge matrix

$$\boxed{\mathcal{L}^{(2)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{2}(\partial_\mu A^\mu)^2 + \frac{F_\pi^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle + C\langle QUQU^\dagger\rangle}$$

⋆ pion mass difference of em origin, $M_{\pi^+}^2 - M_{\pi^0}^2 = 2Ce^2/F_\pi^2$

⋆ no strong isospin breaking at LO, absence of D-symbol

⋆ strong and em corrections at NLO worked out

M., Müller, Steininger, Phys. Lett. B 406 (1997) 154

- Three-flavor chiral perturbation theory:

⋆ for $m_u = m_d \Rightarrow M_{K^+}^2 - M_{K^0}^2 = M_{\pi^+}^2 - M_{\pi^0}^2 = \frac{2Ce^2}{F_\pi^2}$ – Dashen's theorem

⋆ for $m_u \neq m_d \Rightarrow$ leading order strong kaon mass difference:

$$\boxed{(M_{K^0}^2 - M_{K^+}^2)^{\rm strong} = (m_u - m_d)B_0 + \mathcal{O}(m_q^2)}$$

$B_0 = |\langle 0|\bar q q|0\rangle|/F_\pi^2$

⋆ strong and em corrections at NLO incl. leptons worked out

Urech, Nucl. Phys. B 433 (1995) 234

Knecht, Neufeld, Rupertsberger, Talavera, Eur. Phys. J. C 12 (2000) 469

ISOSPIN VIOLATION - NUCLEONS

Weinberg, . . ., Fettes, M., Müller, Steininger

- Effective Lagrangian for isospin violation (to leading order):

$$\boxed{\mathcal{L}_{\pi N}^{(2,\mathrm{IV})} = \bar{N}\{c_5 \underbrace{(\chi_+ - \tfrac{1}{2}\langle\chi_+\rangle)}_{\sim m_u - m_d} + f_1 \underbrace{\langle\hat{Q}_+^2 - Q_-^2\rangle}_{\sim q_u - q_d} + f_2 \underbrace{\hat{Q}_+\langle Q_+\rangle}_{\sim q_u - q_d}\} N + \mathcal{O}(q^3)}$$

  – Three LECs parameterize the leading strong (c_5) & em (f_1, f_2) IV effects
  – These LECs link various observables/processes:

$$m_n - m_p = 4\, c_5\, B_0 (m_u - m_d) + 2\, e^2\, f_2\, F_\pi^2 + \ldots$$ fairly well known

Gasser, Leutwyler, . . .

$$a(\pi^0 p) - a(\pi^0 n) = \mathbf{const}\ (-4\, c_5\, B_0\, (m_u - m_d)) + \ldots$$

extremely hard to measure

Weinberg, M., Steininger

  – IV in πN scattering analyzed in CHPT $\longrightarrow$ intriguing results

Fettes & M., Nucl. Phys. A 693 (2001) 693; Hoferichter, Kubis, M., Phys. Lett. B 678 (2009) 65

- also access to IV in $np \longrightarrow d\pi^0$ and $dd \longrightarrow \alpha\pi^0$ (spin-isospin filter)

$\longrightarrow$ need to develop a high-precision EFT for few-nucleon systems