M. Pérez-Victoria

Beyond SM: Extra dimensions

Exercises

1. For AdS_5, I have used "conformally flat" coordinates, in which the metric reads $ds^2 = \frac{z_0}{z}(dx^2 - dz^2)$. In many papers you will find instead "domain wall" coordinates, in which $ds^2 = e^{-2ky}dx^2 - dy^2$. Find the coordinate transformation between both parametrizations.

2. Show that a slice of AdS_5 between two rigid flat 3-branes is a solution to Einstein equations. Include a cosmological constant term and tension (positive or negative) for the branes, and impose necessary constraints. Calculate the distance between branes necessary to achieve, through gravitational red-shift, a Planck-electroweak hierarchy between scales at both branes. Is the distance between branes determined? Is this a solution to the hierarchy problem? Discuss.

3. For a 5D scalar field ϕ in an interval,

   (a) Calculate the KK wave functions and masses in flat space and AdS space, for different combinations of Neumann and Dirichlet boundary conditions and different values of the 5D mass (including negative values). Determine a condition on the 5D mass to allow for zero modes, and the condition to avoid tachyons in the spectrum.

   (b) Obtain the new boundary conditions when a localized mass term is added on one brane, and calculate the new KK wave functions and masses in flat space.

   (c) Same as above but with a localized kinetic term $\partial_\mu\phi\partial^\mu\phi$ instead of the mass term.

4. For a scalar field, compute the UV brane to UV brane propagator for Neumann-Neumann boundary conditions in a slice of AdS and in a slice of Minkowski. Use the method you prefer. Find the poles and residues of the propagator and compare with the results in the previous exercise.

5. Write explicitly the quadratic action for a fermion in a slice of AdS space, using both conformally flat and domain wall coordinates. Simplify it as much as possible.

6. Study the size of the mass of the first excited KK mode for a fermion with (+,+), (+,-) and (-,-) boundary conditions in AdS, as a function of its 5D mass (numerically, if needed). Is it possible to find a mass parametrically smaller than the compactification scale $L^{-}1$?

7. In a flat interval, add a gauge-fixing term to the action of an abelian gauge boson A_M, such that the mixing between A_μ and A_5 is cancelled. Compute the propagator in that gauge.

8. Read and discuss hep-ph/0412089