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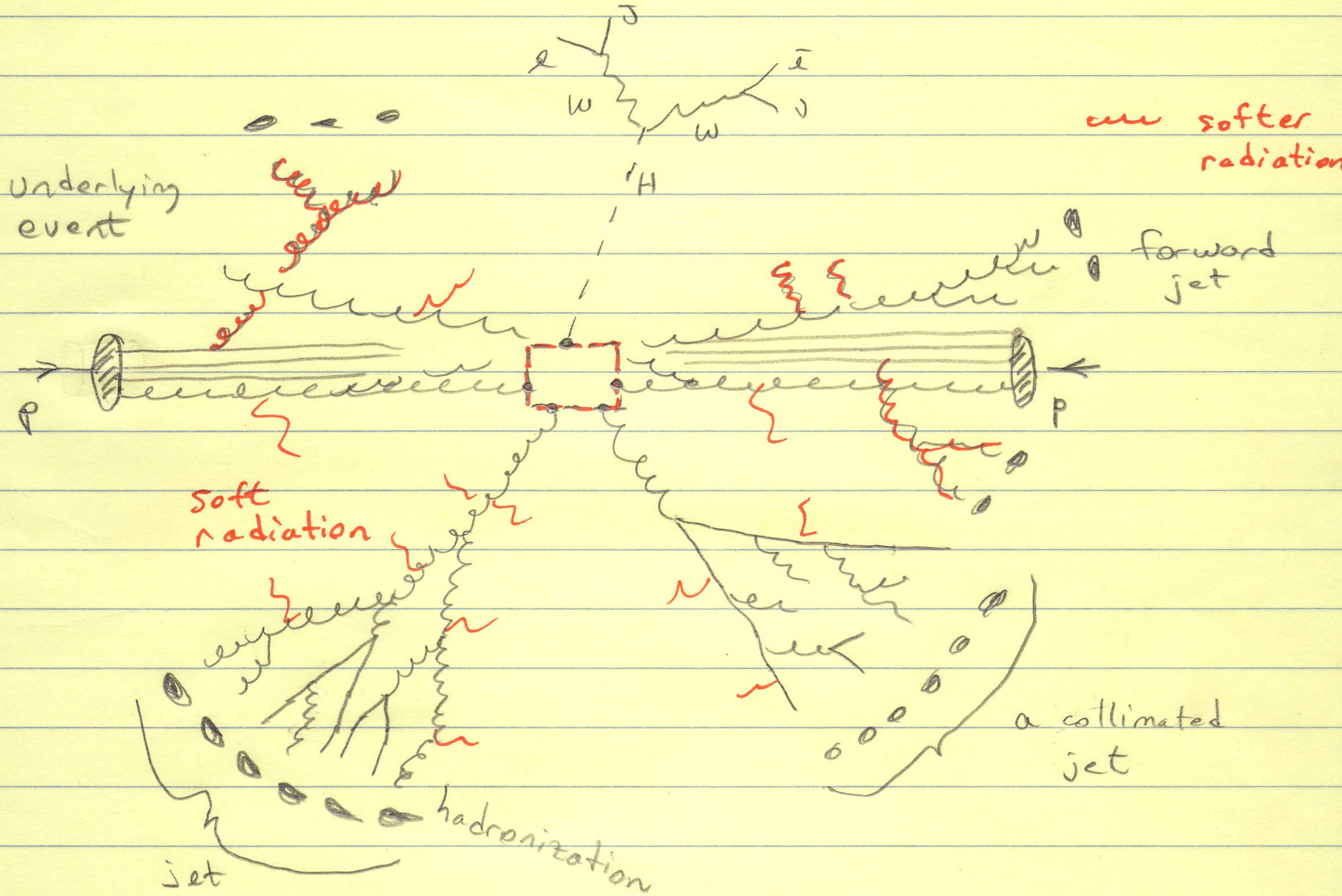
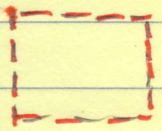
Jet Physics Lectures

Madrid
2012

TAE Summer School

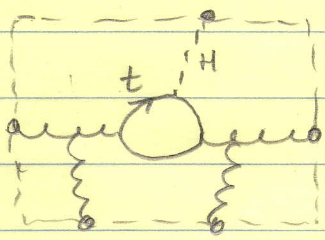
LHC collision





Short distance process

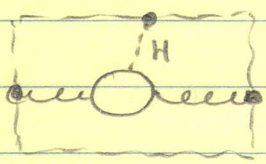
=



$gg \to H$ + two jets

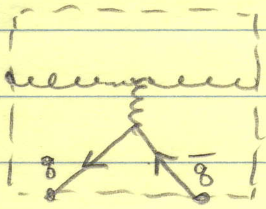
WW + two-jets

or



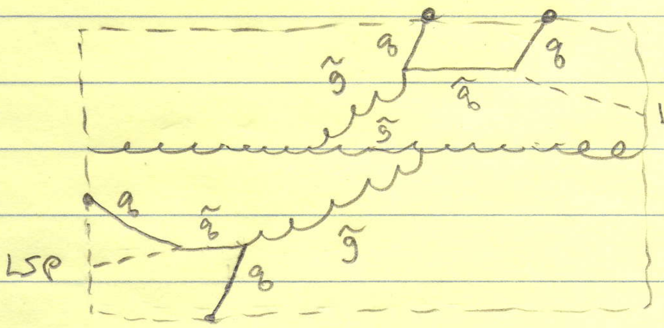
$gg \to H$

WW + 0-jets



$gg \to 2$ jets   two energetic jets

($>10^5$ more likely)

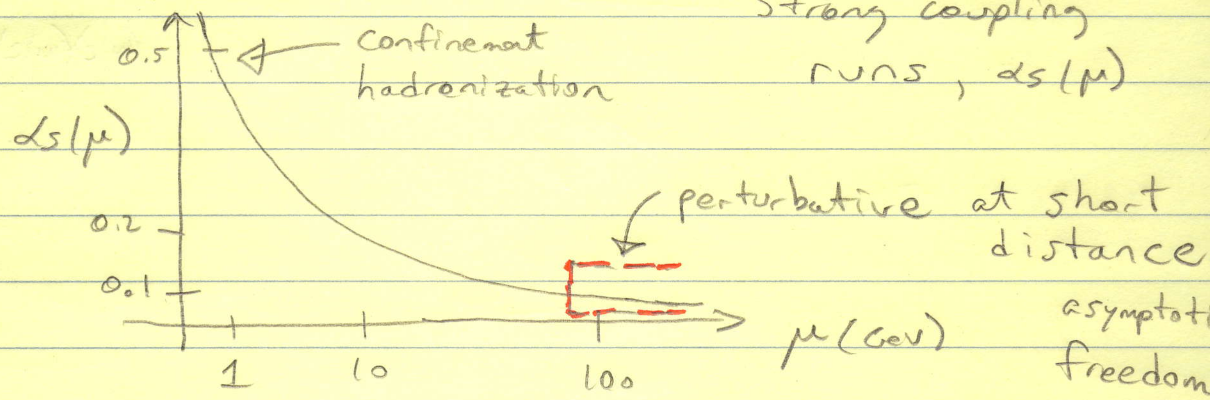


$gg \to 4$ jets + missing E

- In QCD the <u>energy scale</u> "μ" of a process is very important

$$iD^\mu = \partial^\mu + g\, T^A A^{\mu A}$$

$$\hookrightarrow \mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_i \bar{\Psi}_i (i\cancel{D} - m_i)\Psi_i$$



strong coupling runs, $\alpha_s(\mu)$

need to know the appropriate coupling to use for different aspects of the collision

- <u>Factorization</u>

A key tool to calculate cross sections for collisions is the ability to <u>independently</u> consider different parts of the process

$$d\sigma \sim \left(\begin{array}{c}\text{Prob. for} \\ \text{gluons taken} \\ \text{from protons}\end{array}\right) \boxed{\left(\begin{array}{c}\hat{\sigma}(gg \to H), \\ \hat{\sigma}(gg \to Hg), \ldots\end{array}\right)} \left(\begin{array}{c}\text{Prob. for gluons} \\ \text{to produce} \\ \text{jets}\end{array}\right)$$

etc

Explicit example: $pp \to H + X$

anything $\quad$ 0+1+2+.. jets

$$\sigma(pp \to H+X) = \int dx_1\, dx_2\, \underbrace{f_g(x_1,\mu)}_{\text{parton dist'n}}\, f_g(x_2,\mu)\, \boxed{\hat{\sigma}(gg \to H+X, x_1, x_2, \ldots}$$

short distance

parton dist'n
= Prob. of finding g in proton
with momentum fraction x_1
(probability density)

Here
- Because we sum over everything that can happen with final state quarks & gluons we are not sensitive to dynamics of jet formation

$$\sum_i (\text{Prob})(i) = 1$$

Want to be inclusive to avoid sensitivity to low energy scales & in particular the hadronization process in the final state

Practicalities Limit how much we can $\sum_i$:

Higgs search

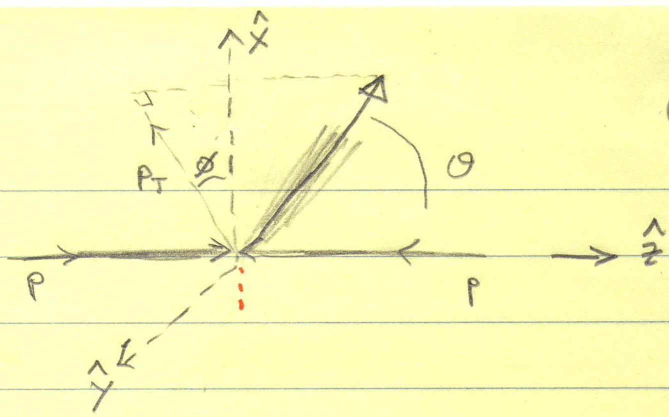
- need to make cuts on jets to control backgrounds
- enhanced signal by requireing $\geq N$ jets (eg. SUSY)

- Still want to sum over dynamics inside the jet & characterize it by a few variables

$$p_J^\mu = \sum_{i \in J} p_i^\mu \,, \quad \text{angular size of jet } R$$

(Or deeper to jet substructure  , jets inside jets)

④



Hadron Collider Variables

Know proton collision CM frame

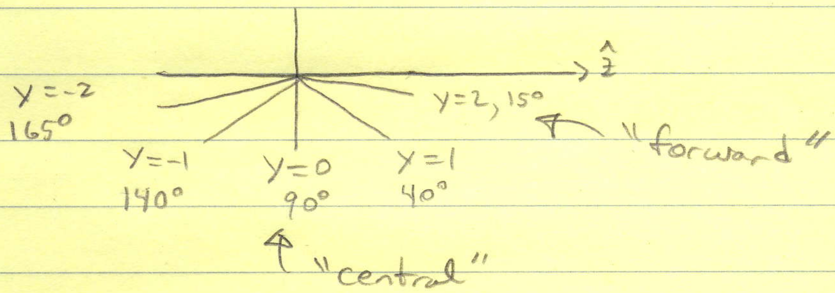
don't know gluon collision CM frame $\int dx_1 \int dx_2$

Use variables that are boost invariant along $\hat{z}$

- $\{p_x, p_y\} \Leftrightarrow \{p_T, \phi\}$

- for $\{E, p_z\}$ use $\{m, y\}$ $\qquad p^2 = m^2$

rapidity $\quad y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) \overset{m=0}{=} \ln\cot\frac{\theta}{2}$



$E = \sqrt{m^2+p_T^2}\cosh y$

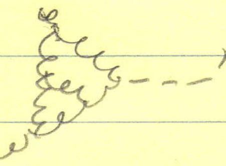
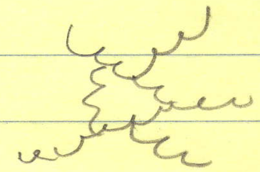
$p_z = \sqrt{m^2+p_T^2}\sinh y$

$\Delta y = y_1 - y_2$ is $\hat{z}$ boost invariant

angular size $\quad \Delta R = \sqrt{(\Delta y)^2 + (\Delta\phi)^2}$

Factorization Paradigm $\Rightarrow$ very successful!

<u>Activities of QCD collider physics community</u>

- higher order α_s^n calculations from having more loops , more legs 

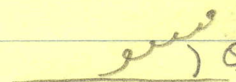
- Parton Shower Monte Carlo for initial/final state jets
  { and combining these two }

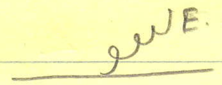
- PDF's $f_i(x,\mu)$ global fits

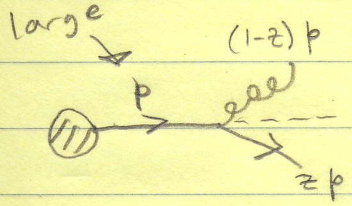
- Factorization (validity, effects of underlying event, results for more specific final states)

- Resummation $\sum_{i=0}^{\infty} c_i \alpha_s^i \ln^{2i}(\mu_1/\mu_0)$ $\mu_1 \gg \mu_0$ two scales in a measurement

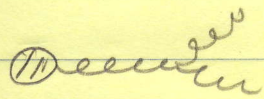
<u>Why does QCD produce Jets?</u>

enhancement from collinear  $\theta \to 0$

and soft  $E \to 0$ infrared singularities (IR)



$\propto \frac{\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\theta}$ ∞ $\theta, E \to 0$



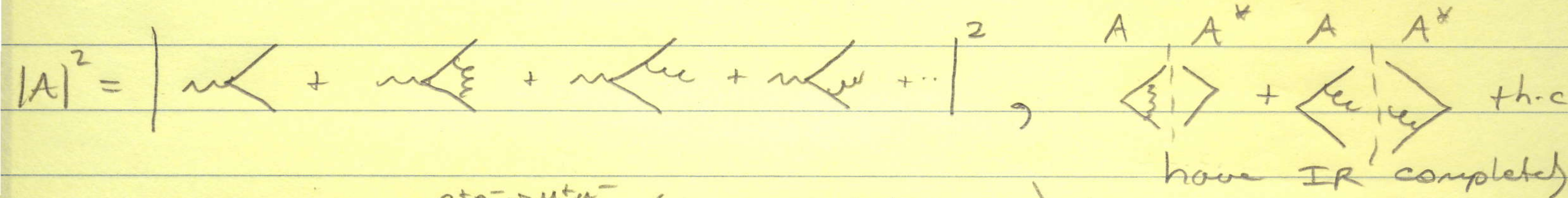
$\propto \frac{\alpha_s C_A}{\pi} \underbrace{\frac{dE}{E} \frac{d\theta}{\theta}}_{\frac{dz}{1-z} \frac{dk_T}{k_T}}$

<u>Casmir's</u>
$C_A = 3$ gluon
$C_F = 4/3$ quark

or $E = (1-z)p$
$k_T = E\sin\theta$
$z =$ energy fraction or $(E+p_z)$ fraction

In an inclusive calculation there are cancellations of IR singularities between real + virtual graphs

eg $e^+e^- \to \gamma^* \to X$ ← anything hadronic



$$\sigma = \sigma^{e^+e^-\to\mu^+\mu^-}\left(1 + \frac{\alpha_s(Q)}{\pi} + \cdots\right)$$

↑ finite short distance correction

In jet calculations we have angular cutoff R for size of jets & a cutoff on the amount of energy outside the N-jets of interest

eg. 3 jets, $R = 0.5$ with $P_T \geq 50\,\text{GeV}$ (transverse $\hat{z}$)

any remaining jets $P_T < 50\,\text{GeV} = P_T^{cut}$

$\Rightarrow$ α_s corrections become functions $g(R, P_T^{cut})$

eg. Higgs + 0 jets (used in Higgs search / discovery)

only jets with $P_T \leq 30\,\text{GeV}$

$$\sigma = \sigma_{incl}^{H+X}\left(1 - \frac{2\alpha_s C_A}{\pi}\ln^2\left(\frac{P_T^{cut}}{m_H}\right) + \cdots\right)$$ "jet veto"

LL $\quad 1 + \alpha_s L^2 + \alpha_s^2 L^4 + \cdots$ exponentiate

$$\exp\left(-2\frac{\alpha_s C_A}{\pi}\ln^2\left(\frac{P_T^{cut}}{m_H}\right) + \text{running coupling terms}\right)$$

example of Sudakov Form Factor from restricting radiation

NLL $\quad \alpha_s L + \alpha_s^2 L^3 + \cdots$

NNLL $\quad \alpha_s + \alpha_s^2 L^2 + \cdots$

$\quad + \alpha_s^2 L + \cdots$

Must Resum to all orders in α_s if $\alpha_s L^2 \sim 1$ or $\alpha_s L \sim 1$

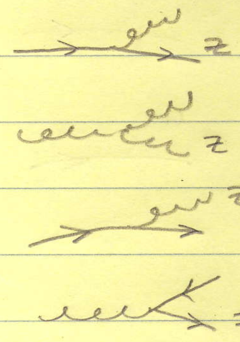
If we did not take soft limit, collinear singularity is described by "splitting functions" $P_{ij}(z)$

$$\frac{dk_T}{k_T}\, dz\, P_{qq}(z)$$

$$\text{"} \quad P_{gg}(z)$$

$$\text{"} \quad P_{gq}(z)$$

$$\text{"} \quad \underbrace{P_{qg}(z)}$$



more on these in Problem 1 of Exercises

The $P_{ij}(z)$ govern scale dependence of parton distribution

$$\mu \, d/d\mu \, \underbrace{f_i(x,\mu)} = \int \frac{dz}{z} P_{ij}(z)\, f_j\left(\frac{x}{z}, \mu\right)$$

PDF's encode universal initial state collinear singularities, common to all processes colliding high energy hadrons

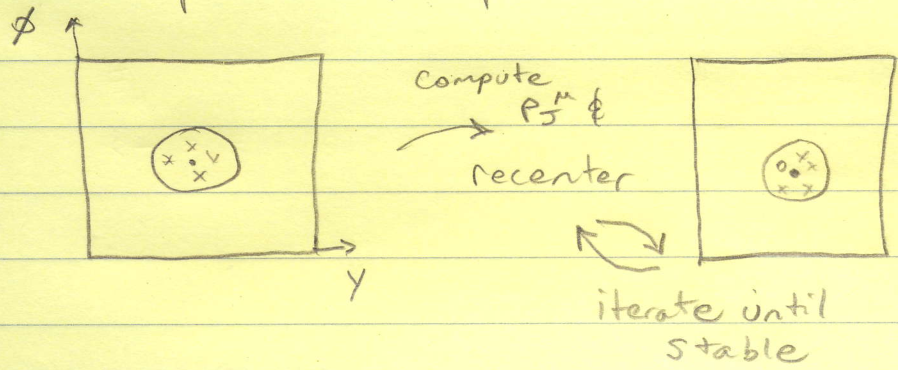
DGLAP equations

Jet Algorithms

"How precisely do we define a jet?"

Which particles do we group?

Cone Algorithms: draw cone of size R ($\Delta R \leq R$) about seed particle, particles in stable cone are "jet"

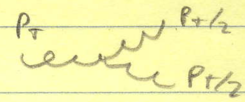


then remove particles in stable cone & continue for next jet

need seedless algorithm
eg. "SIS CONE"

How do we pick seeds?

eg. hardest p_T? not IR safe

eg. 

IR safety: invariant under $\vec{p}_i \to \vec{p}_j + \vec{p}_k$ when $\vec{p}_i \parallel \vec{p}_j$ (collinear) or one of $\vec{p}_j, \vec{p}_k$ is small

Recombination Algorithms

$$d_{ij} = \min\left(p_{Ti}^{2r}, p_{Tj}^{2r}\right) \frac{\Delta R_{ij}^2}{R^2} \qquad \text{distance}(i,j)$$

$$d_{iB} = p_{Ti}^{2r} \qquad \text{distance}(i, \text{beam})$$

Find $\min\limits_{i,j \in L} \{d_{ij}\}, \{d_{iB}\}$, L = all particles

$\{d_{ij}\}$ → join i & j into new particle in L, repeat

$\{d_{iB}\}$ → discard i

$r = 1$ k_T algorithm, clusters soft particles first (jet region not circular)

$r = 0$ Cambridge / Aachen, geometric

$r = -1$ anti-k_T, clusters harder collinear particles first (circular)

default for CMS & ATLAS

Event Shapes

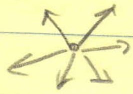
rather than using individual jets define a single variable to describe distribution of jets

eg. $e^+e^- \to$ jets "thrust" $\tau = \min\limits_{\hat{t}} \dfrac{\sum_i \left(|\vec{p}_i| - |\hat{t}\cdot\vec{p}_i|\right)}{\sum_i |\vec{p}_i|}$

 $\to \hat{t}$

[technically $\tau = 1 - T$, T = thrust]

$\tau \simeq 0$ for $\longleftarrow \bullet \longrightarrow$ $\overset{\hat{t}}{\to}$ two-jets, narrow

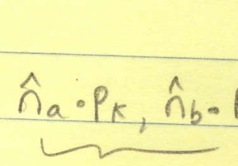
$\tau \to 1/2$ for  spherical event

eg. $pp \to \geq N$ jets, "N-jettiness"

$$\tau_N = \underbrace{\min_{\hat{n}_i}}_{\substack{\text{vary}\\ \text{axis to}\\ \text{minimize}}} \sum_K \underbrace{\min\Big(}_{\substack{\text{particle closest}\\ \text{to axis } \hat{n}_i\text{, grouped}\\ \text{with that axis}}} \hat{n}_1\cdot p_K, \hat{n}_2\cdot p_K, \ldots, \hat{n}_N\cdot p_K, \underbrace{\hat{n}_a\cdot p_K, \hat{n}_b\cdot p_K}_{\substack{\text{include two}\\ \text{beam}\\ \text{directions}}} \Big)$$

Can also consider other measures, like weighing each term by $\dfrac{E_{\text{jet}\,i}}{Q}$ to get invariant mass type measure

$\tau_N \to 0$ for N pencil-like jets & 2 narrow beams

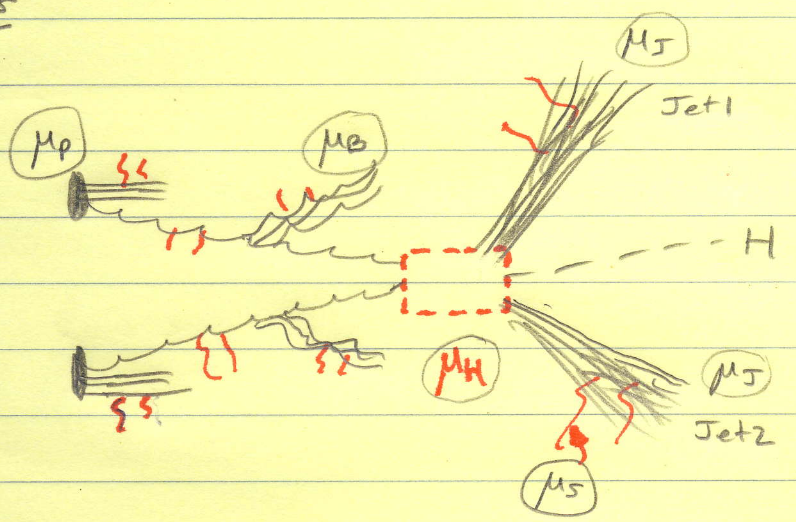


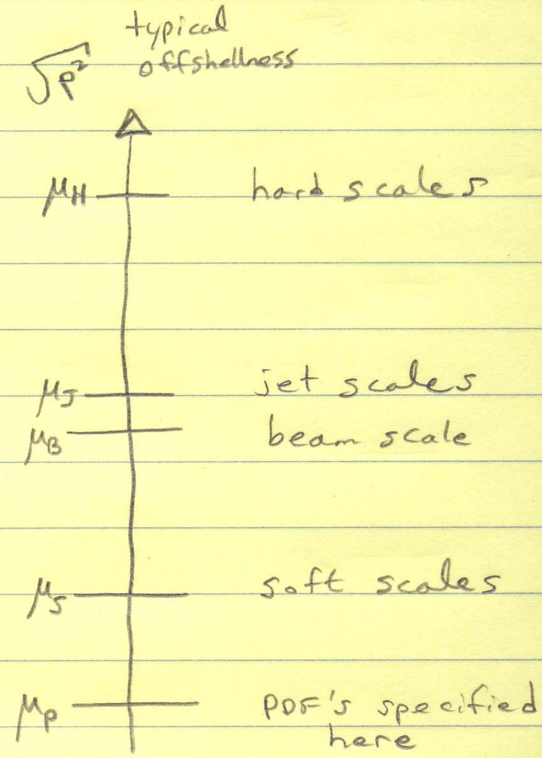
finding τ_N groups particles into N-jets and value provides quality measure

Ask: Can we use the ideas of Effective Field theory to simplify jet physics calculations?

Soft-Collinear Effective Theory

scales





hard scales

$\mu_H^2 \sim p_{J1} \cdot p_{J2}\,,\ p_H \cdot p_{J1}$ hard partonic 4-vectors

Jet scale $\mu_J^2 \simeq m_{jet}^2 = \left(\sum_{i\in J} p_i^\mu\right)^2$, $\mu_J^2 \ll \mu_H^2$ means collimated

beam scale $\mu_B^2 = m_{T,beam}^2 = \left(\sum_{i\in B} \vec{p}_{iT}\right)^2$

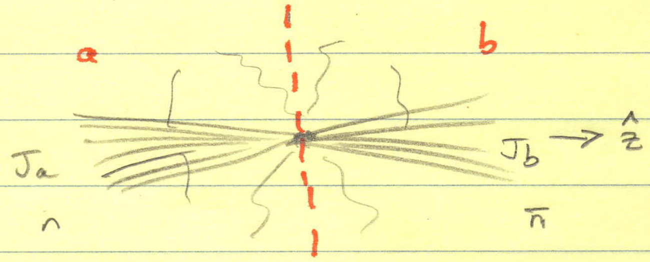
Soft scale $\mu_S \sim \mu_J^2/\mu_H\,,\ \mu_B^2/\mu_H$, energy scale for perturbative soft radiation

proton scale μ_p boundary condition $f_i(x,\mu_p)$ for PDF evolution

$$q^2 = Q^2$$

Start Simple: $e^+e^- \to \gamma^* \to J_1 J_2 X_{soft}$

$\mu_H = Q$



measure hemisphere masses

$$M_a^2 = \Big(\sum_{i \in a} p_i^\mu \Big)^2 \qquad M_b^2 = \dots$$

$$\mu_J^2 \sim M_a^2 \ll Q^2$$

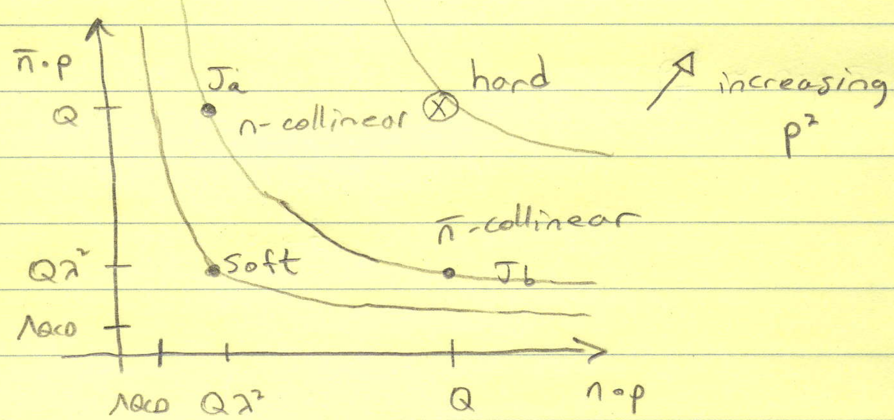
Note: thrust $\tau = \dfrac{M_a^2 + M_b^2}{Q^2}$ for $\tau \ll 1$ so $\mu_J^2 \sim Q^2 \tau$

(simpler since combines two variables)

Light-cone coords: $n = (1,0,0,-1)$, $\bar{n} = (1,0,0,1)$ $\quad n^2 = \bar{n}^2 = 0$

$$p^\mu = \frac{n^\mu}{2} \bar{n}\cdot p + \frac{\bar{n}^\mu}{2} n\cdot p + p_\perp^\mu \qquad \bar{n}\cdot p = E - p_z$$

Degrees of freedom



soft particles interact with either jet without taking them off shell

$p^2 = n\cdot p\, \bar{n}\cdot p + p_\perp^2$, $\quad p_\perp^2 \sim n\cdot p\, \bar{n}\cdot p$ to ensure $p^2 = 0$ allowed (near mass shell)

power counting parameter

$$\lambda = \sqrt{\tau}$$

Integrate out $p^2 \sim Q^2$ modes from $\mathcal{L}_{QCD}$

Gives $\quad \mathcal{L}_{SCET} = \mathcal{L}_n + \mathcal{L}_{\bar n} + \mathcal{L}_s$

$$\mathcal{L}_s = -\tfrac{1}{4}\left(G^{s\,a}_{\mu\nu}\right)^2 + \bar\psi_s\, i\not{D}_s\, \psi_s$$

soft fields QCD like

$A_s^\mu \sim \lambda^2, \quad \psi_s \sim \lambda^3$

$$\mathcal{L}_n = \bar\xi_n \frac{\not{\bar n}}{2}\Big[\underbrace{i n\cdot\partial}_{\lambda^2} + \underbrace{g n\cdot A_n}_{\lambda^2} + \underbrace{g n\cdot A_s}_{\lambda^2} + \underbrace{i\not{D}_\perp^n \frac{1}{i\bar n\cdot D_n} i\not{D}_\perp^n}_{\lambda^2}\Big]\xi_n$$

collinear quark field

only collinear momenta $\sim \lambda, \lambda^0$ & collinear fields

$i D_n^\mu = i\partial_n^\mu + g A_n^\mu$

two gluon fields are $\mathcal{O}(1)$

$\xi_n \sim \lambda$

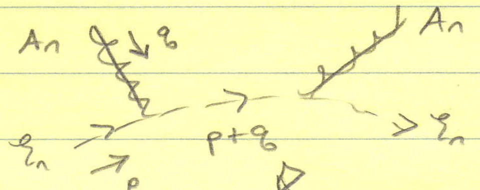
$\not{n}\,\xi_n = 0$

$\frac{\not{n}\not{\bar n}}{4}\xi_n = \xi_n$ } large spinor components

$$A_n^\mu \sim \left(\overset{n\cdot A_n}{\lambda^2}, \overset{\bar n\cdot A_n}{1}, \overset{A_n^\perp}{\lambda}\right) \quad n\text{-collinear gluon}$$

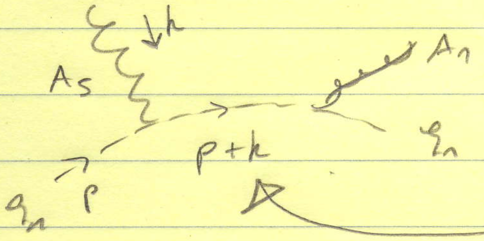
$$A_{\bar n}^\mu \sim \left(\overset{n\cdot A_{\bar n}}{1}, \overset{\bar n\cdot A_{\bar n}}{\lambda^2}, \overset{A_{\bar n}^\perp}{\lambda}\right) \quad \bar n\text{-collinear gluon}$$

Collinear Quark Propogators



$$i\frac{\not{\bar n}}{2}\,\frac{\bar n\cdot(p+q)}{(p+q)^2 + i0}$$

QCD like



$$\frac{i\not{n}}{2}\,\frac{\bar n\cdot p}{n\cdot k\,\bar n\cdot p + \underbrace{n\cdot p\,\bar n\cdot p - p_\perp^2}_{p^2=0} + i0}$$

$$= \frac{i\not{n}}{2}\,\frac{1}{n\cdot k + i0}$$

eikonal propogator (& $n\cdot A_s$ coupling)

Field redefinition

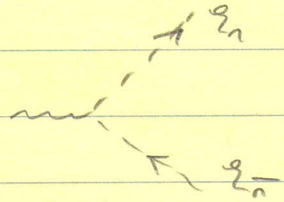
$\xi_n \to Y_n\, \xi_n$

$A_n^\mu \to Y_n\, A_n^\mu\, Y_n^\dagger$

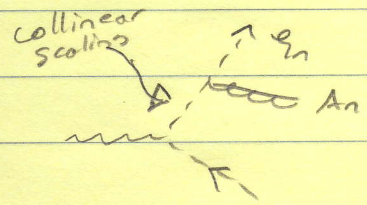
removes soft interactions from $\mathcal{L}_n$! (Exercise)

W Wilson Lines

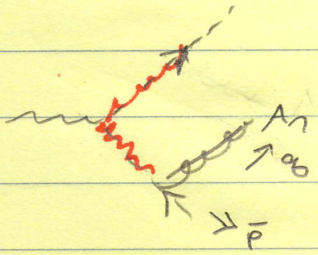
production current



$\bar\xi_n \gamma_\perp^\mu \xi_{\bar n}$



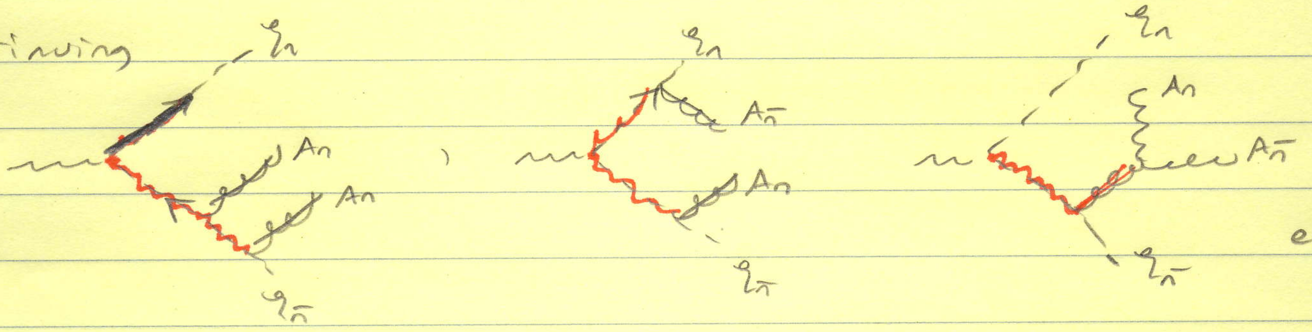
✓ Time Ordered Product in SCET



$$= \bar\xi_n \gamma_\perp^\mu \frac{g\,\bar n\cdot A_n}{\bar n\cdot q} \xi_{\bar n}$$

(Exercise)

continuing



etc

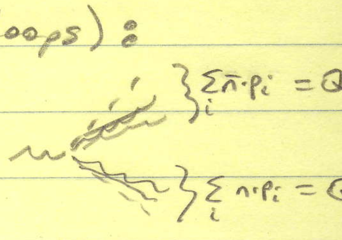
gives

$$\underbrace{(\bar\xi_n W_n)}_{\substack{\text{product is}\\ n\text{-collinear}\\ \text{gauge inv}}} \gamma_\perp^\mu \underbrace{(W_{\bar n}^\dagger \xi_{\bar n})}_{\bar n\text{-collin gauge inv.}}$$

where
$$W_n = \sum_k \sum_{\text{perms}} \frac{(-g)^k}{k!} \frac{\bar n\cdot A(q_1) \cdots \bar n\cdot A(q_n)}{\bar n\cdot q_1\ \bar n\cdot(q_1+q_2) \cdots (\bar n\cdot \Sigma q_i)}$$

position space
$$W_n(y) = P\exp\left(ig \int_{-\infty}^{y^-} ds\ \bar n\cdot A_n(\bar n s) \right)$$ Wilson Line

Add Wilson Coefficient for hard interactions (loops):
$$C(\mu, Q)\ (\bar\xi_n W_n \gamma^\mu W_{\bar n}^\dagger \xi_{\bar n})$$



All together

$$C(\mu,Q)\ \ (\bar{\xi}_n W_n)(Y_n^\dagger Y_{\bar{n}})\ \gamma^\mu\ (W_{\bar{n}}^\dagger \xi_{\bar{n}})$$
factorized fields

Plug into $\sigma = \sum_X (2\pi)^4 \delta^4(q-p_X)\, L^{e^+e^-}_{\mu\nu} \langle 0| J^{\nu\dagger}(0)|X\rangle \langle X| J^\mu(0)|0\rangle$

to derive

$$\frac{d\sigma}{d\tau} = \sigma_0\ \underbrace{H(Q,\mu)}_{\text{hard function}}\ \int ds \left[\underbrace{\int ds'\, J_n(Q^2\tau - s - s')\, J_{\bar{n}}(s')}_{\text{jet functions}}\right] \underbrace{S_\tau\!\left(\frac{s}{Q}\right)}_{\text{soft function}}$$

$|C(\mu,Q)|^2$ $\qquad \langle 0|(\bar{\xi}_n W_n)(W_n^\dagger \xi_n)|0\rangle$ $\qquad |\langle 0| Y_n^\dagger Y_{\bar{n}} |X_s\rangle$

$$Q^2 \gg Q^2\tau \gg Q^2\tau^2$$

$\int_0^{\tau_c} d\tau \frac{d\sigma}{d\tau}$ has structures

$$\begin{array}{llllll}
1 & + \alpha_s L^2 & + \alpha_s^2 L^4 & + \alpha_s^3 L^6 & + \cdots & \text{LL} \\
 & \alpha_s L & + \alpha_s^2 L^3 & + \alpha_s^3 L^5 & + \cdots & \text{NLL} \\
 & \alpha_s & + \alpha_s^2 L^2 & + \alpha_s^3 L^4 & + \cdots & \text{NNLL} \\
 & & + \alpha_s^2 L & + \alpha_s^3 L^3 & + \cdots & \text{NNLL} \\
 & & + \alpha_s^2 & + \alpha_s^3 L^2 & + \cdots & \text{N}^3\text{LL} \\
 & & & + \alpha_s^3 L & + \cdots & \text{N}^3\text{LL} \\
 & & & + \alpha_s^3 & + \cdots & \uparrow
\end{array}$$

$L = \ln \tau_c$

$\tau_c \ll$ requires resummation

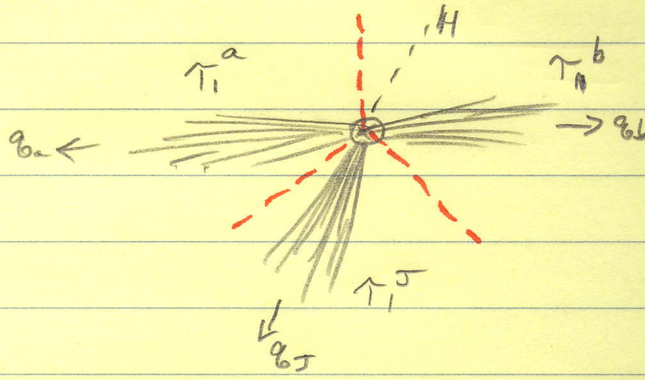
this precision achieved with SCET

Just solve Renormalization Group Eqtns for $H,\ J_n,\ S_\tau$ $\quad$ (H, Exercise at LL)

1% precision

LHC example 1-jettiness $pp \to H + 1\text{-jet}$

$$\tau_1 = \tau_1^a + \tau_1^b + \tau_1^J$$



$$\frac{d\sigma}{d\tau_1^a\, d\tau_1^b\, d\tau_1^J\, dp_T^J\, dy^J\, dY}$$

$$= \begin{pmatrix}\text{phase space}\\ \text{factor}\end{pmatrix} \sum_K \int dt_a\, B_{Ka}(t_a, x_a, \mu_B) \int dt_b\, B(t_b, x_b, \mu_B) \int ds_J\, J_{KJ}(s_J, \mu_J$$

$$* \; H^K_{H+1}(\{q_i \cdot q_j\}, x_a, x_b, \mu_H^J; \mu_H^B)\; S_1^K\left(\tau_1^a - \frac{t_a}{Q_a}, \tau_1^b - \frac{t_b}{Q_b}, \tau_1^J - \frac{s_J}{Q_J}, \{\hat{q}_i \cdot$$

$$, \mu_{SJ}, \mu_{SB}$$

where $\hat{n}_i^\mu = \frac{q_i^\mu}{Q_i} = \hat{q}_i$

K = color

here $Q_J \tau_1^J = m_J^2$ invariant mass of jet

τ_1^a, τ_1^b constrain beam radiation

$$B_i(t, x, \mu) = \sum_j \int \frac{d\xi}{\xi}\, \underbrace{\mathcal{I}_{ij}(t, x/\xi, \mu)}_{\text{initial state jet radiation}}\; \underbrace{f_j(\xi, \mu)}_{\text{standard PDF's}}$$

(measured at μ_B scale here)

References

Book: QCD and Collider Physics, by Ellis, Stirling, and Webber

Lecture Note: Elements of QCD for hadron colliders, by Gavin Salam, arXiv: 1011.5131

SCET Lecture Notes: http://www2.lns.mit.edu/~iains/talks/SCET_Lectures-Stewart-2009.pdf