Introduction to SUPERSYMMETRY

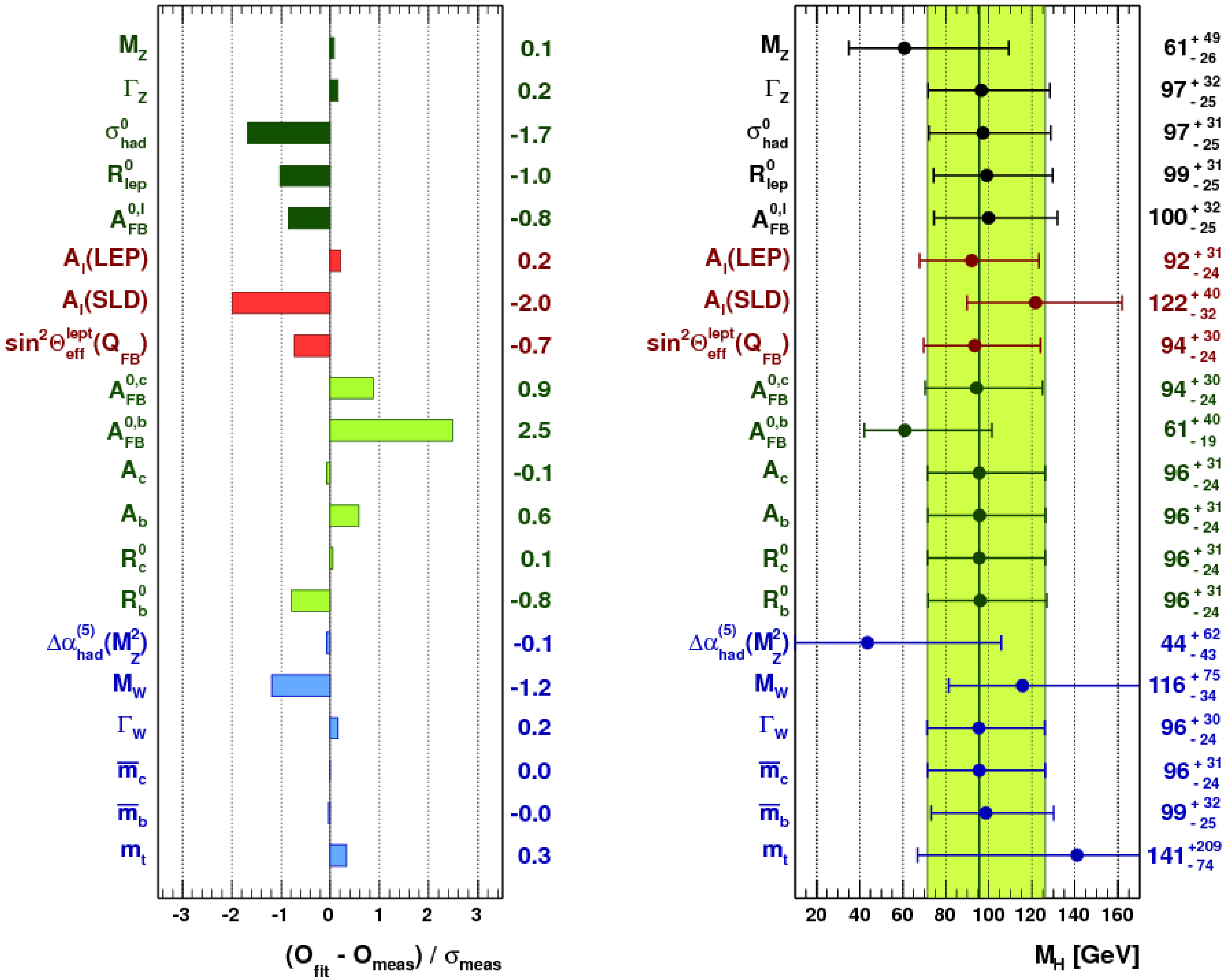
1.- Motivation and Wess-Zumino model

Oscar VIVES

TAE 2012

U. Complutense, *16–27/07/2012*

Electroweak Global Fit



Standard Model "Problems"

Standard Model "Problems"

Theoretical "Problems"

- Assignment of matter Quantum Numbers
- Unification of Gauge Couplings
- Origin of Spontaneous Symmetry Breaking
- Accomodate Quantum Gravity
- Large number of Flavour Parameters

STANDARD MODEL "PROBLEMS"

Theoretical "Problems"

- Assignment of matter Quantum Numbers
- Unification of Gauge Couplings
- Origin of Spontaneous Symmetry Breaking
- Accomodate Quantum Gravity
- Large number of Flavour Parameters

Observational "Problems"

- Need of non-baryonic Dark Matter
- CP violation source for Baryogenesis
- Mechanism of Inflation

Motivations for Supersymmetry

- Only possible extension of symmetry beyond Lie Symmetries (Coleman–Mandula Theorem).
- Correct Unification of Gauge couplings at M_{GUT}, GUT assignment of Quantum numbers (anomaly cancellation).
- Solution of the Hierarchy Problem, strong motivation for low-energy SUSY.
- "Natural" Mechanism of Electroweak Symmetry Breaking, Radiative Symmetry Breaking.
- SUSY is a necessary ingredient in String Theory. Local Supersymmetry $\Leftrightarrow$ Supergravity.

Coleman–Mandula Theorem

In the 60's attempts to combine internal and Lorentz symmetries ...

The only conserved quantities that transform as tensors under Lorentz transformations in a theory with non-zero scattering amplitudes in 4D are the generators of the Poincare group and Lorentz invariant quantum numbers (scalar charges).

2×2 spinless particle scattering, bosonic conserved charge, $\Sigma_{\mu\nu}$,

$$\langle 1|\Sigma_{\mu\nu}|1\rangle = \alpha p^1_\mu p^1_\nu + \beta g_{\mu\nu}$$

So, in the scattering process,

$$p^1_\mu p^1_\nu + p^2_\mu p^2_\nu = p^3_\mu p^3_\nu + p^4_\mu p^4_\nu \quad \& \quad p^1_\mu + p^2_\mu = p^3_\mu + p^4_\mu$$

Not possible in a theory with non-zero scattering.

However: Coleman–Mandula theorem does not forbid conserved spinor charges, Q_α (transforming like fermions under Lorentz)

$$Q_\alpha|\text{Boson}\rangle = |\text{Fermion}\rangle \qquad Q_\alpha|\text{Fermion}\rangle = |\text{Boson}\rangle$$

$$[Q_\alpha, H] = 0 \qquad [\{Q_\alpha, \bar{Q}_{\dot\beta}\}, H] = 0$$

Supersymmetry Algebra

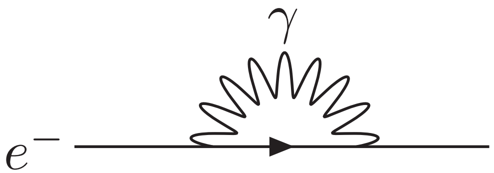
$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu \qquad \{Q_\alpha, Q_\beta\} = 0 \qquad \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0$$

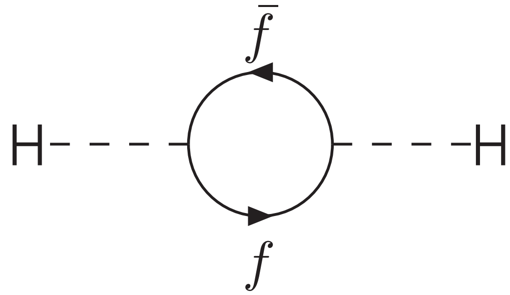
Supersymmetry relates particles of different spin with equal quantum numbers and identical masses.

$$\begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix} \sim \begin{pmatrix} q \text{ (quark)} \\ \tilde{q} \text{ (squark)} \end{pmatrix} \qquad \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \sim \begin{pmatrix} g \text{ (gluon)} \\ \tilde{g} \text{ (gluino)} \end{pmatrix}$$

Chiral supermultiplet   Gauge supermultiplet

Hierarchy Problem





$$\delta m_e = 2\frac{\alpha_{em}}{\pi} m_e \log \frac{\Lambda}{m_e}$$

with $m_e \to 0$, chiral symmetry

$$\delta m_H^2(f) = -2N_f \frac{|\lambda_f|^2}{16\pi^2}[\Lambda^2 - \ 2m_f^2 \ln \frac{\Lambda}{m_f}]$$

No symmetry protects m_H^2 . . .

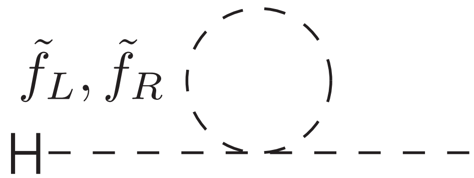
Quadratic div.!!

$\Lambda = 10^{19}$ GeV

$$\delta m_e \ = \ 0.24 \, m_e$$

$$\delta m_H^2 \ \simeq \ 10^{30} \text{ GeV}$$

Supersymmetry $\Rightarrow$ $+$



$$\delta m_H^2(\tilde{f}) = -2N_{\tilde{f}} \frac{\lambda_{\tilde{f}}}{16\pi^2}[\Lambda^2 - \ 2m_{\tilde{f}}^2 \ln \frac{\Lambda}{m_{\tilde{f}}} \ + \ldots]$$

$\delta m_H^2(f) + \delta m_H^2(\tilde{f}) = 0$ if $N_f = N_{\tilde{f}}, |\lambda_f|^2 = -\lambda_{\tilde{f}}$ and $m_f = m_{\tilde{f}}$

$\Rightarrow$ Supersymmetry + Chiral symmetry solve hierarchy problem.

But ... no scalars degenerate with the SM fermions, SUSY broken !!

$$\delta m_H^2(\tilde{f}) = -2N_{\tilde{f}} \frac{\lambda_{\tilde{f}}}{16\pi^2}[\Lambda^2 - \ 2m_{\tilde{f}}^2 \ln \frac{\Lambda}{m_{\tilde{f}}} \ + \ldots]$$

$\delta m_H^2(f) + \delta m_H^2(\tilde{f}) = 0$ if $N_f = N_{\tilde{f}}, |\lambda_f|^2 = -\lambda_{\tilde{f}}$ and $m_f = m_{\tilde{f}}$

$\Rightarrow$ Supersymmetry + Chiral symmetry solve hierarchy problem.

But ... no scalars degenerate with the SM fermions, SUSY broken !!

Soft Supersymmetry breaking

- Preserve cancellation of Quadratic divergencies requires dimensionless couplings still supersymmetric: $|\lambda_f|^2 = -\lambda_{\tilde{f}}$
- SUSY only broken in couplings with possitive mass dimension:

Soft Breaking $m_{\tilde{f}}^2 = m_f^2 + \delta^2$

$$\delta m_H^2(f) + \delta m_H^2(\tilde{f}) \simeq 2N_f \frac{|\lambda_f|^2}{16\pi^2} \delta^2 \ln \frac{\Lambda}{m_{\tilde{f}}} + \ldots$$

to solve hierarchy problem $\delta \lesssim 1$ TeV

GUT and coupling unification

Grand Unification

- Simple gauge group unifying $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$
- All matter multiplets in 1 generation unified in a single (two) representation of the gauge group:

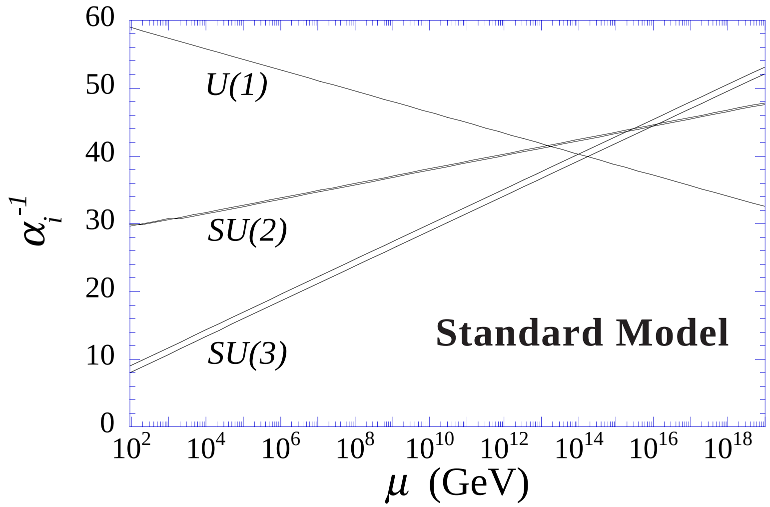
$$SU(5) \quad \bar{5} = \begin{pmatrix} d^c \\ d^c \\ d^c \\ \nu_e \\ e^- \end{pmatrix}_L \qquad 10 = \begin{pmatrix} 0 & u^c & u^c & u & d \\ & 0 & u^c & u & d \\ & & 0 & u & d \\ & & & 0 & e^c \\ & & & & 0 \end{pmatrix}_L$$

$$S0(10) \quad 16 = 10 + \bar{5} + 1 \quad \left(\bar{5} = (\bar{3}, 1) + (1, 2), \quad 10 = (3, 2) + (\bar{3}, 1) + (1, 1)\right)$$

$\Rightarrow$Explains the assignment of quantum numbers in the SM

RGE evolution of gauge couplings Running the couplings to high energies they come close at $M_{GUT} \simeq 10^{16}$ GeV

In SM

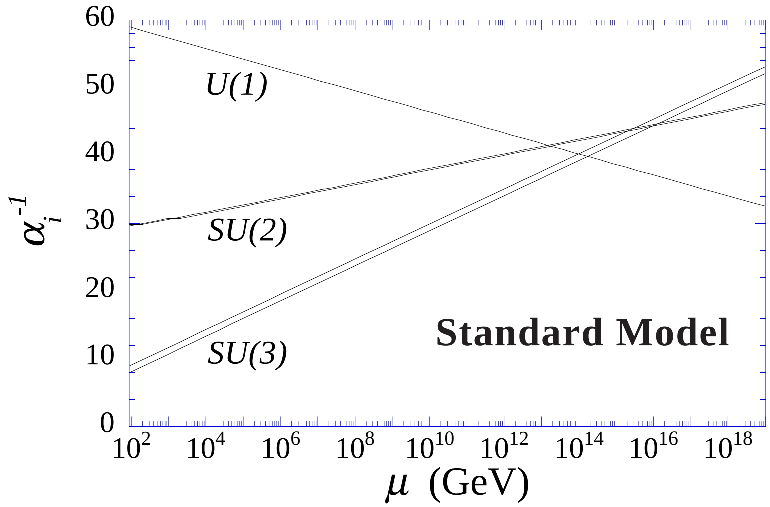


Using $SU(3)$ and $SU(2)_L$ to predict $\sin\theta_W$ with correct unification :

$$\sin\theta_W^{th} = 0.214 \pm 0.004$$
$$\sin\theta_W^{exp} = 0.23149 \pm 0.00017$$

RGE evolution of gauge couplings Running the couplings to high energies they come close at $M_{GUT} \simeq 10^{16}$ GeV

In SM

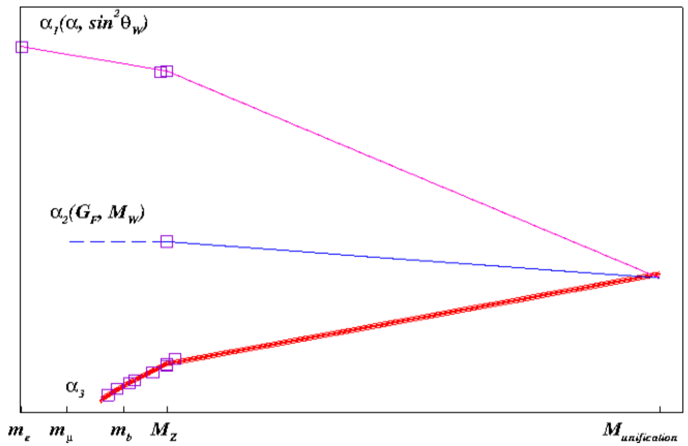


Using $SU(3)$ and $SU(2)_L$ to predict $\sin\theta_W$ with correct unification :

$$\sin\theta_W^{th} = 0.214 \pm 0.004$$

$$\sin\theta_W^{exp} = 0.23149 \pm 0.00017$$

In MSSM



Much better agreement:

$$\sin\theta_W^{th} \simeq 0.232$$

$$\sin\theta_W^{exp} = 0.23149 \pm 0.00017$$

iif ($M_{SUSY} \simeq 1$ TeV).

$\Rightarrow$ Strongly suggests Supersymmetric Grand Unification !!

Radiative Symmetry Breaking

- In MSSM many scalars but (typically) only Higgs gets a vev
- All soft masses possitive $\mathcal{O}(M_W)$ at M_{GUT}
- $\mu H_1 H_2 \in W$, $\mu \sim \mathcal{O}(M_W)$ (SUSY μ problem)
- Approx. RGE evolution from M_{GUT} to M_W:

$$
\begin{aligned}
16\pi^2 \frac{d}{dt} m_{H_2}^2 &= 6Y_t^2(m_{H_2}^2 + m_{Q_3}^2 + m_{U_3}^2) - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2, \\
16\pi^2 \frac{d}{dt} m_{H_1}^2 &= -6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2, \\
16\pi^2 \frac{d}{dt} m_{Q_3}^2 &= 2Y_t^2(m_{H_2}^2 + m_{Q_3}^2 + m_{U_3}^2) - \frac{32}{3} g_3^2 M_3^2 - \\
& \quad 6g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2,
\end{aligned}
$$

$\Rightarrow$ $m_{H_2}^2$ pushed down by Y_t and not $SU(3)$ coupling

EW symmetry breaking occurs naturally as a radiative effect

Gravity and strings ...

$$\text{Global bosonic sym.} \xrightarrow{Local} \text{Gauge Theory}$$

$$\text{Global SUSY} \xrightarrow{Local} \text{Supergravity}$$

• SUSY transformation parameters ξ^α deppend on space-time position. Anticonmutator of 2 SUSY transformations is a translation

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$$

$$\Downarrow$$

Local SUSY implies local coordinate tranformations: Gravity

• Superstring can unify gravity with gauge interactions. Supersymmetry necessary ingredient of consistent String Theory

Wess-Zumino model

Weyl spinors Susy described in terms of 2-comp. Weyl spinors:

$$\mathcal{L}_{\mathrm{Dirac}} = i\bar{\Psi}\gamma^\mu \partial_\mu \Psi - M\bar{\Psi}\Psi$$

Weyl repres.

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix} \qquad \gamma_5 = \begin{pmatrix} -I_2 & 0 \\ 0 & I_2 \end{pmatrix}$$

$$(\sigma_\mu)_{\alpha\dot{\alpha}} = (I_2, \sigma_i)_{\alpha\dot{\alpha}} \qquad (\bar{\sigma}_\mu)^{\dot{\alpha}\alpha} = (I_2, -\sigma_i)^{\dot{\alpha}\alpha}$$

$$\psi_D = \begin{pmatrix} \xi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} = \psi_L + \psi_R = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}, \quad \bar{\psi}_D = \psi_D^\dagger \gamma^0 = \left(\chi^\alpha, \ \bar{\xi}_{\dot{\alpha}}\right), \quad (\xi_\alpha)^\dagger = \bar{\xi}_{\dot{\alpha}}$$

$$\Longrightarrow \qquad \mathcal{L} = i\bar{\xi}\bar{\sigma}^\mu \partial_\mu \xi + i\chi\sigma^\mu \partial_\mu \bar{\chi} - M(\xi\chi + \bar{\xi}\bar{\chi})$$

Dirac $\rightarrow$ Weyl:

$$\bar{\Psi}_1 \gamma^\mu \, L \, \Psi_2 = \bar{\xi}_1 \bar{\sigma}_\mu \xi_2 \qquad \bar{\Psi}_1 \gamma^\mu \, R \, \Psi_2 = \chi_1 \sigma_\mu \bar{\chi}_2$$

$$\bar{\Psi}_1 \, L \, \Psi_2 = \chi_1 \xi_2 \qquad \bar{\Psi}_1 \, R \, \Psi_2 = \bar{\xi}_1 \bar{\chi}_2$$

Non-interacting Wess–Zumino

Weyl fermion + complex boson, no interactions

$$S = \int d^4xL \; = \; \int d^4x \, (i\bar\psi\bar\sigma^\mu\partial_\mu\psi \; + \; \partial^\mu\phi^*\partial_\mu\phi)$$

SUSY transformation: scalar $\leftrightarrow$ fermion, with ξ Weyl spinor ($[\xi] = -1/2$) transf. parameter (Ex. 1: Check L inv. under SUSY)

$$\delta\phi = \sqrt{2}\xi^\alpha\psi_\alpha \equiv \sqrt{2}\xi\psi \qquad \delta\phi^* = \sqrt{2}\bar\xi_{\dot\alpha}\bar\psi^{\dot\alpha} \equiv \sqrt{2}\bar\xi\bar\psi$$

$$\delta L_{scal} = +\sqrt{2}\xi\;\partial^\mu\psi\;\partial_\mu\phi^* + \sqrt{2}\bar\xi\;\partial^\mu\bar\psi\;\partial_\mu\phi$$

for a fermion, comparing δL_{scal} and L_{ferm} must be:

$$\delta\psi_\alpha = i\sqrt{2}(\sigma^\mu\bar\xi)_\alpha\partial_\mu\phi \qquad \delta\bar\psi_{\dot\alpha} = -i\sqrt{2}(\xi\sigma^\mu)_{\dot\alpha}\partial_\mu\phi^*$$

$$\delta L_{fer} = \sqrt{2}(\xi\;\sigma^\mu\bar\sigma^\nu\partial_\nu\psi)\partial_\mu\phi^* - \sqrt{2}(\bar\psi\;\bar\sigma^\nu\sigma^\mu\bar\xi)\partial_\nu\partial_\mu\phi$$

using the Pauli matrix identities ($+\partial^\mu\partial^\nu = \partial^\nu\partial^\mu$)

$$[\sigma^\nu\bar\sigma^\mu + \sigma^\mu\bar\sigma^\nu]_\alpha^\beta = -2\eta_{\mu\nu}\delta_\alpha^\beta, \;\; [\bar\sigma^\mu\sigma^\nu + \bar\sigma^\nu\sigma^\mu]_{\dot\alpha}^{\dot\beta} = -2\eta_{\mu\nu}\delta_{\dot\alpha}^{\dot\beta}$$

So, we arrive at

$$\delta L_{fermion} = -\sqrt{2}\xi\, \partial^\mu \psi\, \partial_\mu \phi^* - \sqrt{2}\bar{\xi}\, \partial^\mu \bar{\psi}\, \partial_\mu \phi +$$
$$\sqrt{2}\, \partial^\mu \left[\bar{\xi}\, \bar{\psi}\, \partial_\mu \phi + \xi \psi \partial_\mu \phi^* + (\xi\, \sigma^\nu \bar{\sigma}^\mu \psi)\, \partial_\mu \phi^*\right]$$
$$\Rightarrow \delta S = \int d^4x (\delta L_{scalar} + \delta L_{fermion}) = 0$$

Theory is invariant only if the algebra closes

$$(\delta_{\xi_2}\delta_{\xi_1} - \delta_{\xi_1}\delta_{\xi_2})\phi = 2i\left(\xi_1 \sigma^\mu \bar{\xi}_2 - \xi_2 \sigma^\mu \bar{\xi}_1\right) \partial_\mu \phi$$

for the fermion field, using Fierz identity, $\chi_\alpha(\xi\eta) = -\xi_\alpha(\eta\chi) - \eta_\alpha(\chi\xi)$ and $\bar{\xi}\bar{\sigma}^\mu \chi = -\chi \sigma^\mu \bar{\xi}$:

$$(\delta_{\xi_2}\delta_{\xi_1} - \delta_{\xi_1}\delta_{\xi_2})\psi_\alpha = 2i\left(\xi_1 \sigma^\mu \bar{\xi}_2 - \xi_2 \sigma^\mu \bar{\xi}_1\right) \partial_\mu \psi_\alpha - i\xi_{1\alpha}\, \bar{\xi}_2 \bar{\sigma}^\mu \partial_\mu \psi \; + \; i\xi_{2\alpha}\, \bar{\xi}_1 \bar{\sigma}^\mu \partial_\mu \psi$$

only on-shell, $\bar{\sigma}^\mu \partial_\mu \psi = 0$, it closes. Off-shell we must introduce auxiliary field, F with $L_{aux} = F^* F$ and dimensions $mass^2$. It does not propagate, the eqs. motion are $F = F^* = 0$

we modify the transformation properties

$$\delta\phi = \sqrt{2}\xi\psi \qquad\qquad \delta\phi^* = \sqrt{2}\bar{\xi}\bar{\psi}$$

$$\delta\psi_\alpha = i\sqrt{2}\,(\sigma^\mu\bar{\xi})_\alpha\partial_\mu\phi \;+\; \sqrt{2}\,\xi_\alpha F$$

$$\delta\bar{\psi}_{\dot\alpha} = -i\sqrt{2}\,(\xi\sigma^\mu)_{\dot\alpha}\partial_\mu\phi^* \;+\; \sqrt{2}\bar{\xi}_{\dot\alpha}F^*$$

$$\delta F = i\sqrt{2}\,\bar{\xi}\bar{\sigma}^\mu\partial_\mu\psi \qquad \delta F^* = -i\sqrt{2}\,\partial_\mu\bar{\psi}\bar{\sigma}^\mu\xi$$

Now theory is Supersymmetry invariant even off-shell:

$$L_{WZ} = i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi \;+\; \partial^\mu\phi^*\partial_\mu\phi + F^*F$$

$$(\delta_{\xi_2}\delta_{\xi_1} - \delta_{\xi_1}\delta_{\xi_2})X = 2i\left(\xi_1\sigma^\mu\bar{\xi}_2 - \xi_2\sigma^\mu\bar{\xi}_1\right)\partial_\mu X$$

we modify the transformation properties

$$\delta\phi = \sqrt{2}\xi\psi \qquad \delta\phi^* = \sqrt{2}\bar{\xi}\bar{\psi}$$

$$\delta\psi_\alpha = i\sqrt{2}\,(\sigma^\mu\bar{\xi})_\alpha\partial_\mu\phi \;+\; \sqrt{2}\,\xi_\alpha F$$

$$\delta\bar{\psi}_{\dot{\alpha}} = -i\sqrt{2}\,(\xi\sigma^\mu)_{\dot{\alpha}}\partial_\mu\phi^* \;+\; \sqrt{2}\bar{\xi}_{\dot{\alpha}}F^*$$

$$\delta F = i\sqrt{2}\,\bar{\xi}\bar{\sigma}^\mu\partial_\mu\psi \qquad \delta F^* = -i\sqrt{2}\,\partial_\mu\bar{\psi}\bar{\sigma}^\mu\xi$$

Now theory is Supersymmetry invariant even off-shell:

$$L_{WZ} = i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi \;+\; \partial^\mu\phi^*\partial_\mu\phi + F^*F$$

$(\delta_{\xi_2}\delta_{\xi_1} - \delta_{\xi_1}\delta_{\xi_2})X = 2i\left(\xi_1\sigma^\mu\bar{\xi}_2 - \xi_2\sigma^\mu\bar{\xi}_1\right)\partial_\mu X$ for $X = \phi, \psi, F$

Supersymmetry Algebra

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}}P_\mu \qquad [P_\mu, Q_\alpha] = 0$$

$$\{Q_\alpha, Q_\beta\} = 0 \qquad \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0$$

Chiral Supermultiplet$= \psi, \phi, F$ with four fermionic d.o.f. (2 complex components of ψ) and four bosonic d.o.f. (2 complex scalars, ϕ and F)

Matter Interactions

General set of renormalizable matter SUSY interactions is given by,

$$L_{int} = -\frac{1}{2} W^{ij} \psi_i \psi_j + W^i F_i + c.c.$$

with W^{ij}, W^i functions of bosonic fields $[W^{ij}] = M$ and $[W^i] = M^2$

1.- $[L] = 4$, so W^{ij} and W^i cannot be functions of ψ and F.

2.- L_{int} must be SUSY invariant by itself

3.- L_{int} cannot include $f(\phi, \phi^*)$, it transforms in $\xi\psi\phi\phi^*$... and cannot be cancelled (all other terms at least 1 derivative or F)

$$\delta L_{int} = -\sqrt{2}\left(\frac{\delta W^{ij}}{\delta \phi_k}(\xi \psi_k) + \frac{\delta W^{ij}}{\delta \phi_k^*}(\bar{\xi}\bar{\psi}_k)\right)(\psi_i \psi_j) - i\left(\sqrt{2}\, W^{ij}\, \partial_\mu \phi_j\, \psi_i \sigma^\mu \bar{\xi}\right.$$

$$\left. + \sqrt{2}\, W^i\, \partial_\mu \psi_i \sigma^\mu \bar{\xi}\right) + \sqrt{2}\left(\frac{\delta W^i}{\delta \phi_j}\,(\xi \psi_j)\, F_i + \sqrt{2}\,\frac{\delta W^i}{\delta \phi_j^*}\,(\bar{\xi}\bar{\psi}_j)\, F_i - \sqrt{2}\, W^{ij}\,(\xi \psi_i)\, F_j + c.c.\right)$$

1^{st} term . No other term with 4 spinors, however Fierz identity

$$(\xi\psi_i)(\psi_j\psi_k) + (\xi\psi_j)(\psi_k\psi_i) + (\xi\psi_k)(\psi_i\psi_j) = 0$$

so it cancels iif $\delta W^{ij}/\delta\phi_k$ is totally symmetric in i, j, k. No similar identity for $(\bar{\xi}\bar{\psi}_k)(\psi_i\psi_j)$ therefore $\delta W^{ij}/\delta\phi_k^* = 0$

2^{nd} term. Total derivative iif $W^{ij}\partial_\mu\phi_j = \partial_\mu W^i = (\delta W^i/\delta\phi_j)\partial_\mu\phi_j$

3^{rd} term. Vanishes under the above conditions.

1^{st} term . No other term with 4 spinors, however Fierz identity

$$(\xi\psi_i)(\psi_j\psi_k) + (\xi\psi_j)(\psi_k\psi_i) + (\xi\psi_k)(\psi_i\psi_j) = 0$$

so it cancels iif $\delta W^{ij}/\delta\phi_k$ is totally symmetric in i, j, k. No similar identity for $(\bar{\xi}\bar{\psi}_k)(\psi_i\psi_j)$ therefore $\delta W^{ij}/\delta\phi_k^* = 0$
2^{nd} term. Total derivative iif $W^{ij}\partial_\mu\phi_j = \partial_\mu W^i = (\delta W^i/\delta\phi_j)\partial_\mu\phi_j$
3^{rd} term. Vanishes under the above conditions.

$\Rightarrow$ **Superpotential W**

Analitic function of the (complex) scalar fields (not of ϕ^*), at most cubic in ϕ and dimensions of mass^3

$$W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}\lambda^{ijk}\phi_i\phi_j\phi_k, \qquad W^i = \frac{\delta W}{\delta\phi_i} \qquad W^{ij} = \frac{\delta W}{\delta\phi_i\delta\phi_j}$$

F_i, F^{i*} eliminated using equations of motion: $F_i = -W_i^*$ $F^{i*} = -W^i$, i.e. functions of scalar fields, no derivatives. Then Lagrangian,

$$L_{chiral} = L_{WZ} + L_{int} = i\bar{\psi}\bar{\sigma}^\mu \partial_\mu \psi + \partial^\mu \phi^* \partial_\mu \phi + \\ - \frac{1}{2} W^{ij} \psi_i \psi_j - \frac{1}{2} W^{ij*} \bar{\psi}_i \bar{\psi}_j - W^i W_i^*$$

The scalar potential $V(\phi, \phi^*)$ is given by

$$V(\phi, \phi^*) = W^i W_i^* = F^i F_i^* = M_{ji}^* M^{ik} \phi^{j*} \phi_k + \frac{1}{2} M^{ik} \lambda_{jnk}^* \phi_i \phi^{j*} \phi^{n*} \\ + \frac{1}{2} M_{ik}^* \lambda^{jnk} \phi^{i*} \phi_j \phi_n + \frac{1}{4} \lambda^{ijn} \lambda_{kln}^* \phi_i \phi_j \phi^{k*} \phi^{l*}$$

Superpotential: general SUSY invariant matter interactions

Example: top Yukawa

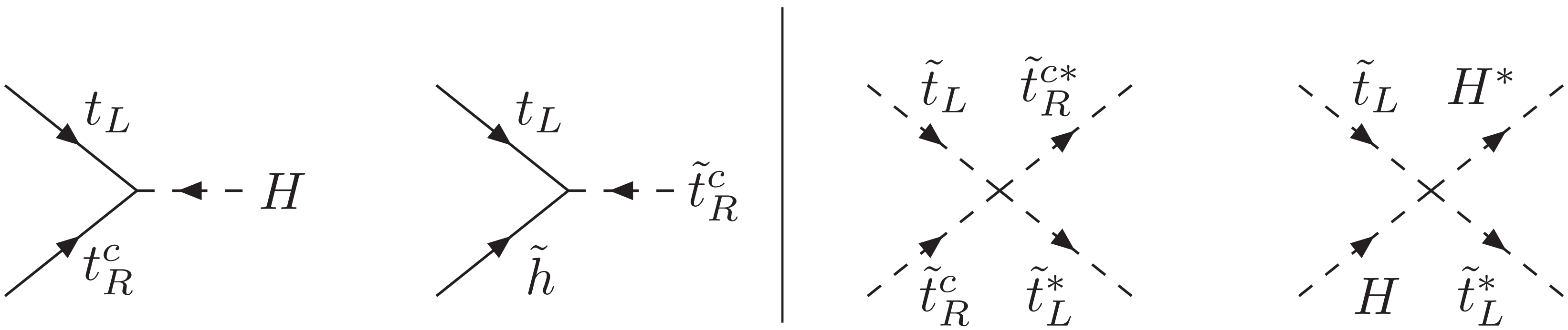
$W = h_t\, Q_L H t^c_R$ Gives rise to the Lagrangian,

$$L_{int} = -\frac{1}{2} W^{ij}\, \psi_i \psi_j - \frac{1}{2} W^{ij*}\, \bar{\psi}_i \bar{\psi}_j - W^i W^*_i$$

$$= -\frac{1}{2}\, h_t \left[H\, Q_L t^c_R + \tilde{Q}_L\, \tilde{h} t^c_R + \tilde{t}^c_R\, Q_l \tilde{h} \right] + c.c.$$

$$- h_t^2 \left(|H \tilde{t}^c_R|^2 + |H \tilde{Q}_L|^2 + |\tilde{Q}_L \tilde{t}^c_R|^2 \right)$$

and the Feynman rules



$-\, i\, h_t\, L$ $\qquad$ $-\, i\, h_t^2$

Vector Superfields

Gauge multilplet: massless vector boson A_μ^a + Weyl gaugino λ^a + auxiliary field D^a

$$L_{gauge} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + i\bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2} D^a D^a$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c \quad D_\mu \lambda^a = \partial_\mu \lambda^a - g f^{abc} A_\mu^b \lambda^c$$

L_{gauge} is already Supersymmetric with the transformations (Wess-Zumino gauge)

$$\begin{aligned} \delta A_\mu^a &= i\bar{\xi}\bar{\sigma}_\mu \lambda^a - i\bar{\lambda}^a \bar{\sigma}_\mu \xi \\ \delta\lambda_\alpha^a &= -\frac{1}{2} (\sigma^\mu \bar{\sigma}^\nu \xi)_\alpha F_{\mu\nu}^a - i\xi_\alpha D^a \\ \delta D^a &= -\bar{\xi}\bar{\sigma}^\mu D_\mu \lambda^a + D_\mu \bar{\lambda}^a \bar{\sigma}^\mu \xi \end{aligned}$$

Gauge Interactions

Replacing $\partial_\mu \to D_\mu = \partial_\mu + igT^a A^a_\mu$ obtain Gauge invariant Lagrangian, but not SUSY invariant, we need gaugino and D interactions,

$$\phi^* T^a \psi \, \lambda^a \qquad \bar{\lambda}^a \, \bar{\psi} T^a \phi \qquad \phi^* T^a \phi \, D^a$$

Then the full Lagrangian is Supersymmetric replacing also in the SUSY transformations derivatives by covariant derivatives,

$$L = L_{gauge} \; + \; L_{chiral} + \; i\sqrt{2}\, g \; \left[\phi^* T^a \psi \, \lambda^a \; + \; \bar{\lambda}^a \, \bar{\psi} T^a \phi\right] + \; g \, \phi^* T^a \phi \, D^a$$

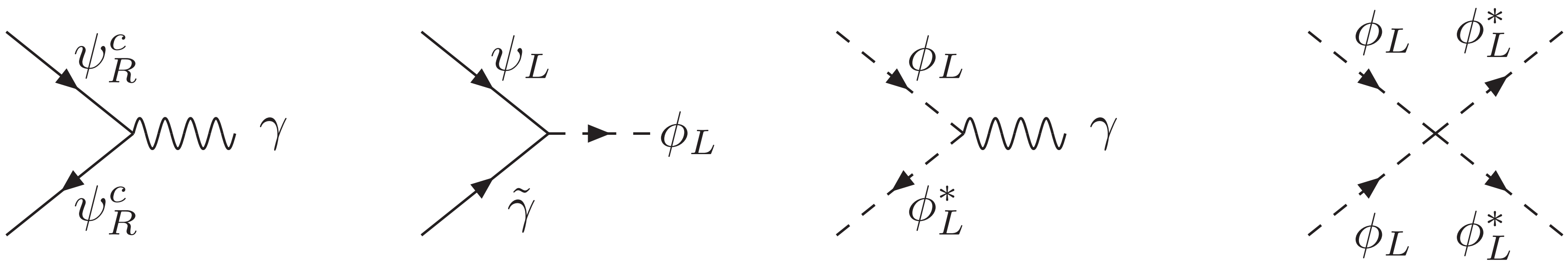
where W must also be Gauge invariant by itself. Moreover D^a can be eliminated using eqs of motion,

$$D^a \;=\; - \, g \, \phi^* T^a \phi$$

The gauge invariant SUSY Lagrangian,

$$L = i\bar{\psi}_L \bar{\sigma}^\mu D_\mu \psi_L + D^\mu \phi_L^* D_\mu \phi_L + (L \to R^c) + \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} - i\,\bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda^a$$
$$+\, i\,\sqrt{2}\, g\, \left[\phi_i^* T^a \psi_i\, \lambda^a + \bar{\lambda}^a\, \bar{\psi}_i T^a \phi_i\right] - \frac{1}{2} W^{ij}\, \psi_i \psi_j - \frac{1}{2} W^{ij*}\, \bar{\psi}_i \bar{\psi}_j$$
$$-\, W^i W_i^* - \frac{1}{2}\, g^2\, \left(\sum\nolimits_i \phi_i^* T^a \phi_i\right)^2$$

therefore the gauge interactions are ($U(1)$),



$-\,i\,g\,\gamma_\mu$ $\quad$ $i\,g\sqrt{2}\,L$ $\quad$ $-\,i\,g\,(p+p')_\mu$ $\quad$ $-\,2\,i\,g^2$

SUSY Lagrangian

- All interactions determined by gauge quantum numbers and Superpotential
- Write all kinetic terms with covariant derivatives plus gaugino interactions and D–terms
- W must be a gauge invariant analitic function of the scalar fields
- Matter interactions determined by $W^i W_i^*$ and $W^{ij}\psi_i\psi_j$
- $\Longrightarrow$ Lagrangian is Supersymmetric