Spherical coordinates

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

$$\left.\begin{aligned}
\hat{e}_r &= \hat{e}_1 \sin\theta\cos\phi + \hat{e}_2 \sin\theta\sin\phi + \hat{e}_3\cos\theta \\
\hat{e}_\theta &= \hat{e}_1 \cos\theta\cos\phi + \hat{e}_2\cos\theta\sin\phi - \hat{e}_3\sin\theta \\
\hat{e}_\phi &= -\hat{e}_1\sin\phi + \hat{e}_2\cos\phi
\end{aligned}\right\} \Rightarrow$$

$$\begin{aligned}
\Rightarrow \quad \dot{\hat{e}}_r &= \hat{e}_1(\dot\theta\cos\theta\cos\phi - \dot\phi\sin\theta\sin\phi) \\
&\quad + \hat{e}_2(\dot\theta\cos\theta\sin\phi + \dot\phi\sin\theta\cos\phi) \\
&\quad - \hat{e}_3\dot\theta\sin\theta \\
&= \dot\theta(\hat{e}_1\cos\theta\cos\phi + \hat{e}_2\cos\theta\sin\phi - \hat{e}_3\sin\theta) \\
&\quad + \dot\phi\sin\theta(-\hat{e}_1\sin\phi + \hat{e}_2\cos\phi) \\
&= \dot\theta\,\hat{e}_\theta + \dot\phi\sin\theta\,\hat{e}_\phi
\end{aligned}$$

$$\dot{\hat{e}}_\theta = \text{similarly} = -\dot\theta\,\hat{e}_r + \dot\phi\cos\theta\,\hat{e}_\phi$$

$$\dot{\hat{e}}_\phi = \text{similarly} = -\dot\phi(\sin\theta\,\hat{e}_r + \cos\theta\,\hat{e}_\theta)$$

Hence

$$\vec{x} = r\hat{e}_r \quad \Rightarrow \quad \dot{\vec{x}} = \dot r\hat{e}_r + r\dot{\hat{e}}_r = \dot r\hat{e}_r + r(\dot\theta\,\hat{e}_\theta + \dot\phi\sin\theta\,\hat{e}_\phi)$$

$$\dot{\vec{x}} = \dot r\hat{e}_r + r\dot\theta\,\hat{e}_\theta + r\dot\phi\sin\theta\,\hat{e}_\phi$$

Analogously

$$\ddot{\vec{x}} = (\ddot r - r\dot\theta^2 - r\dot\phi^2\sin^2\theta)\hat{e}_r + (r\ddot\theta + 2\dot r\dot\theta - r\dot\phi^2\sin\theta\cos\theta)\hat{e}_\theta$$

$$+ (2\dot r\dot\phi\sin\theta + r\ddot\phi\sin\theta + 2r\dot\theta\dot\phi\cos\phi)\hat{e}_\phi$$