Física computacional

Problema. Sea el problema de contorno no lineal con condiciones de contorno mixtas

$$\left.\begin{array}{ll} y''(x) = f\big(x,\, y(x),\, y'(x)\big) & a \le x \le b \\ y'(a) = \alpha & \\ y(b) = \beta & \end{array}\right\} . \tag{17}$$

Para resolverlo se utiliza el método de disparo de forma similar al caso de condiciones de contorno Dirichlet. Para ello se substituye el problema dado por un problema de valores iniciales que se resuelve iterativamente hasta que la solución satisface las condiciones de contorno en (17). Si el método iterativo que se usa es el de Newton, formular la sucesión de problemas iniciales que son necesarios resolver y escribir en términos de sus soluciones la ecuación que define la iteración de Newton.

Proceeding as for the shooting method for Dirichlet boundary conditions, we replace problem (17) with the sequence of initial value problems

$$\left.\begin{array}{ll} y_k''(x) = F\big(x,\, y_k(x),\, y_k'(x)\big) & x \ge a \\ y_k(a) = s_k & \\ y_k'(a) = \alpha & \end{array}\right\} , \qquad (\mathrm{IVP})_k$$

where the parameter s_k is iteratively chosen according to Newton's method until condition

$$y_k(b) - \beta = 0$$

is satisfied. Newton's iteration gives $s_0,\ s_1, \ldots$ as

$$s_{k+1} = s_k - \frac{y_k(b) - \beta}{\frac{dy_k}{ds_k}(b)}$$

and involves the derivative $dy_k(x)/ds_k$ at $x = b$, which is not provided by the solution to $(\mathrm{IVP})_k$. Hence we need an auxiliary problem for

$$z_s(x) := \frac{dy_s(x)}{ds} . \tag{18}$$

This is formulated analogously as for the case of Dirichlet boundary conditions. Noting that

$$\frac{dz_s}{dx} = \frac{dy_s'}{ds}$$

$$\frac{d^2 z_s}{dx^2} = \frac{\partial f}{\partial y}\Big|_{y_s} z_s + \frac{\partial f}{\partial y'}\Big|_{y_s} z_s'$$

and

$$z_s(a) = \frac{dy_s(x)}{ds}\Big|_{x=a} = 1$$

$$z'_s(a) = \frac{dy'_s(x)}{ds}\Big|_{x=a} = 0\,.$$

we write the sequence the sequence initial value problems for $z_k = \dfrac{dy_k}{ds_k}$

$$\left.\begin{array}{ll} z''_k(x) = \dfrac{\partial f}{\partial y_k}\, z_k + \dfrac{\partial f}{\partial y'_k}\, z'_k & a \le x \\ z_k(a) = 1 & \\ z'_k(a) = 0 & \end{array}\right\} \quad (\text{Aux-IVP})_k\,.$$

In terms of its solution, Newton's iteration reads

$$s_{k+1} = s_k - \frac{y_k(b) - \beta}{z_k(b)}\,. \tag{19}$$

All in all, we have two sequences of initial value problems $(\text{IVP})_k$ and $(\text{Aux-IVP})_k$ that must be solved simultaneously for every k, the value of s_k in $(\text{IVP})_k$ being given by eq. (19).