Formulario

- $m_e = 9.109 \times 10^{-31}$ kg$= 0.511$ MeV$/c^2$ $\qquad$ $h = 6.626 \times 10^{-34}$ J s
  $|e| = 1.602 \times 10^{-19}$ C $\qquad$ $c = 2.998 \times 10^{8}$ m s^{-1}
  $k_{\rm B} = 1.381 \times 10^{-23}$ J/K

- Radiancia espectral: $r(\lambda, T) = \dfrac{2\pi h c^2}{\lambda^5} \, \dfrac{1}{e^{hc/\lambda k_B T} - 1}\,.$

- Dispersión Compton: $\lambda - \lambda_0 = \dfrac{h}{m_e c}\,(1 - \cos\theta)\,.$

- Autoestados y autoenergías del **pozo infinito unidimensional** de anchura a centrado en el origen:

$$\left.\begin{aligned} \phi_n(x) &= \sqrt{\frac{2}{a}}\,\cos\left(\frac{n\pi x}{a}\right), \qquad & n &= 1, 3, 5, \ldots \\ \phi_n(x) &= \sqrt{\frac{2}{a}}\,\sin\left(\frac{n\pi x}{a}\right), \qquad & n &= 2, 4, 6, \ldots \end{aligned}\right\} \qquad E_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{a}\right)^2.$$

- Autoestados y autoenergías del **oscilador armónico unidimensional**:

$$\left.\begin{aligned} \phi_n(x) &= \left(\frac{\alpha}{2^{\,n}\, n!\,\sqrt{\pi}}\right)^{1/2} H_n(\alpha x)\; e^{-\alpha^2 x^2/2} \\ E_n &= \left(n + \frac{1}{2}\right)\hbar\omega \end{aligned}\right\} \qquad n = 0, 1, 2, \ldots \qquad \alpha = \sqrt{\frac{m\omega}{\hbar}}\,,$$

con $H_n(z)$ los polinomios de Hermite en la variable z,

$$H_0(z) = 1 \qquad H_1(z) = 2z \qquad H_2(z) = 2 - 4z^2 \qquad \cdots$$

Operadores creación y aniquilación:

$$\text{creación}: \quad a^{+} = \frac{1}{\sqrt{2}}\left(\alpha x - \frac{1}{\alpha}\,\frac{d}{dx}\right) = \frac{1}{\sqrt{2}}\left(\alpha X + \frac{1}{\mathrm{i}\hbar\alpha}\,P\right),$$

$$\text{aniquilación}: \quad a = \frac{1}{\sqrt{2}}\left(\alpha x + \frac{1}{\alpha}\,\frac{d}{dx}\right) = \frac{1}{\sqrt{2}}\left(\alpha X - \frac{1}{\mathrm{i}\hbar\alpha}\,P\right).$$

Conmutatodres y hamiltoniano:

$$[a, a^{+}] = 1\,, \qquad H = \hbar\omega\left(a^{+}a + \frac{1}{2}\right).$$

Acción sobre ϕ_n:

$$a\phi_n = \sqrt{n}\,\phi_{n-1} \qquad a^{+}\phi_n = \sqrt{n+1}\,\phi_{n+1}$$

• **Momento angular** $\mathbf{L} = (L_x, L_y, L_z)$. Definición y conmutadores:

$$L_x = -\mathrm{i}\hbar\,(y\partial_z - z\partial_y) \qquad L_y = -\mathrm{i}\hbar\,(z\partial_x - x\partial_z) \qquad L_z = -\mathrm{i}\hbar\,(x\partial_y - y\partial_x)$$

$$[L_x, L_y] = \mathrm{i}\hbar L_z \qquad [L_y, L_z] = \mathrm{i}\hbar L_x \qquad [L_z, L_x] = \mathrm{i}\hbar L_y\,.$$

Cuadrado del momento angular:

$$\mathbf{L}^2 = L_x^2 + L_y^2 + L_z^2\,, \qquad [\mathbf{L}^2, L_x] = [\mathbf{L}^2, L_y] = [\mathbf{L}^2, L_z] = 0\,.$$

Operadores escalón y sus conmutadores

$$L_\pm = L_x \pm \mathrm{i}L_y$$

$$[L_z, L_\pm] = \pm\hbar L_\pm \qquad [L_+, L_-] = 2\hbar L_z\,.$$

En términos de ellos se tiene

$$\mathbf{L}^2 = L_-L_+ + L_z^2 + \hbar\,L_z = L_+L_- + L_z^2 - \hbar\,L_z \qquad [\mathbf{L}^2, L_\pm] = 0\,.$$

• **Armónicos esféricos**:

$$\mathcal{Y}_\ell^m \;\leftrightarrow\; \big|\ell, m\big\rangle\,, \qquad \ell = 0, 1, 2, \cdots\,, \qquad m = -\,\ell, -\,\ell+1, \cdots, \ell\,.$$

Son los autoestados simultáneos de $\mathbf{L}^2$ y L_z:

$$\mathbf{L}^2\big|\ell, m\big\rangle = \hbar^2\ell(\ell+1)\,\big|\ell, m\big\rangle \qquad L_z = \hbar m\,\big|\ell, m\big\rangle\,.$$

Acción de $L_\pm$ sobre ellos :

$$L_\pm\big|\ell, m\big\rangle = \hbar\,\sqrt{\ell(\ell+1) - m(m\pm1)}\;\big|\ell, m\pm1\big\rangle\,.$$

• Autofunciones y autoenergías del **átomo de hidrógeno**:

$$\psi_{n\ell m} = R_{n\ell}(r)\;\mathcal{Y}_\ell^m(\theta,\phi) \qquad E_n = -\frac{1}{n^2}\,\frac{Z^2e^2}{8\pi\epsilon_0 a_0} = -\,\frac{13.6\,Z^2}{n^2}\,\mathrm{eV}\,.$$

Partes radiales para $n = 1, 2$:

$$R_{10}(r) = \left(\frac{Z}{a_0}\right)^{3/2} 2e^{-Zr/a_0}$$

$$R_{20}(r) = \left(\frac{Z}{2a_0}\right)^{3/2}\left(2 - \frac{Zr}{a_0}\right)\,e^{-Zr/2a_0} \qquad R_{21}(r) = \left(\frac{Z}{2a_0}\right)^{3/2}\,\frac{Zr}{\sqrt{3}\,a_0}\;e^{-Zr/2a_0}$$

Partes angulares:

$$\mathcal{Y}_0^0(\theta,\phi) = \frac{1}{\sqrt{4\pi}}$$

$$\mathcal{Y}_1^0(\theta,\phi) = \sqrt{\frac{3}{4\pi}}\;\cos\theta\,, \qquad \mathcal{Y}_1^{\pm1}(\theta,\phi) = \mp\sqrt{\frac{3}{8\pi}}\;\sin\theta\;e^{\pm\mathrm{i}\phi}$$