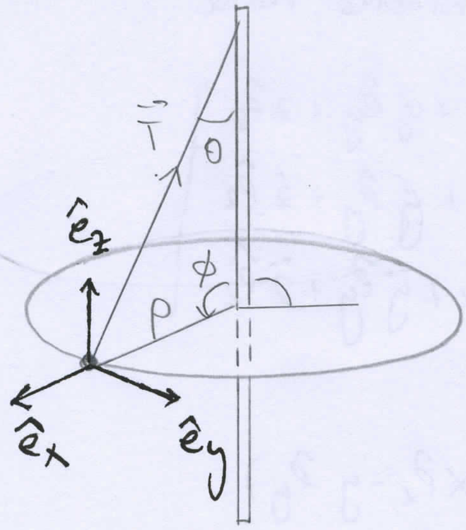
Tiovivo en equilibrio



$$\hat{e}_x = \hat{e}_\rho = \cos\phi\, \hat{e}_x' + \sin\phi\, \hat{e}_y'$$

$$\hat{e}_y = \hat{e}_\phi = (-\sin\phi\, \hat{e}_x' + \cos\phi\, \hat{e}_y')$$

$$\hat{e}_z = \hat{e}_z'$$

$$\vec{g}_0 = -g_0\, \hat{e}_z$$

$$\vec{T} = T(-\sin\theta\, \hat{e}_x + \cos\theta\, \hat{e}_z)$$

1ª forma. Trabajar en sistema de referencia inercial

$$\vec{x} = \rho\, \hat{e}_\rho + z\, \hat{e}_z$$

$$\dot{\vec{x}} = \dot{\rho}\, \hat{e}_\rho + \rho\dot{\phi}\, \hat{e}_\phi + \dot{z}\, \hat{e}_z$$

$$\ddot{\vec{x}} = (\ddot{\rho} - \rho\dot{\phi}^2)\, \hat{e}_\rho + (\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi})\, \hat{e}_\phi + \ddot{z}\, \hat{e}_z$$

$\ddot{\vec{x}} = \vec{g}_0 + \dfrac{\vec{T}}{m}$ se desdobla en tres ecuaciones:

$$\left.\begin{aligned} &\ddot{\rho} - \rho\dot{\phi}^2 = -\frac{T}{m}\sin\theta \\ &\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi} = 0 \\ &\ddot{z} = -g_0 + \frac{T}{m}\cos\theta \end{aligned}\right|$$

Equilibrio:

$$\left.\begin{aligned} &\rho = R \Rightarrow \dot{\rho} = \ddot{\rho} = 0 \\ &\dot{\phi} = \omega \\ &z = z_0 \Rightarrow \dot{z} = \ddot{z} = 0 \end{aligned}\right| \Rightarrow \left.\begin{aligned} &R\omega^2 = \frac{T}{m}\sin\theta \\ &g_0 = \frac{T}{m}\cos\theta \end{aligned}\right| \Rightarrow$$

$$\Rightarrow \quad T = \frac{m g_0}{\cos\theta}, \qquad \tan\theta = \frac{R\omega^2}{g_0}$$

2ª forma: Usar $\quad \ddot{\vec{x}} = \vec{g}_0 + \frac{\vec{T}}{m} - \vec{\omega}\wedge(\vec{\omega}\wedge\vec{x}) - 2\vec{\omega}\wedge\dot{\vec{x}} - \dot{\vec{\omega}}\wedge\vec{x}$

com $\vec{x}$ no sistema móvel

$$\left.\begin{aligned} \vec{g}_0 &= -g_0\hat{e}_z \\ \vec{T} &= T(-\sin\theta\,\hat{e}_x + \cos\theta\,\hat{e}_z) \end{aligned}\right| \quad \left.\begin{aligned} \vec{x} &= x\hat{e}_x + y\hat{e}_y + z\hat{e}_z \\ \dot{\vec{x}} &= \dot{x}\hat{e}_x + \dot{y}\hat{e}_y + \dot{z}\hat{e}_z \\ \ddot{\vec{x}} &= \ddot{x}\hat{e}_x + \ddot{y}\hat{e}_y + \ddot{z}\hat{e}_z \end{aligned}\right|$$

$$\vec{\omega}\wedge\vec{x} = \omega\hat{e}_z\wedge\vec{x} = \omega(x\hat{e}_y - y\hat{e}_x)$$

$$\vec{\omega}\wedge(\vec{\omega}\wedge\vec{x}) = \omega\hat{e}_z\wedge(\vec{\omega}\wedge\vec{x}) = \omega^2(-x\hat{e}_x - y\hat{e}_y)$$

$$\vec{\omega}\wedge\dot{\vec{x}} = \omega\hat{e}_z\wedge\dot{\vec{x}} = \omega(\dot{x}\hat{e}_y - \dot{y}\hat{e}_x)$$

Em componentes

$$\left.\begin{aligned} \ddot{x} &= -\frac{T}{m}\sin\theta - \omega^2 x + 2\omega\dot{y} \\ \ddot{y} &= -\omega^2 y - 2\omega\dot{x} \\ \ddot{z} &= -g_0 + \frac{T}{m}\cos\theta \end{aligned}\right|$$

Em equilíbrio $\quad x = -R \Rightarrow \dot{x} = \ddot{x} = 0$

$$z = z_0 \Rightarrow \dot{z} = \ddot{z} = 0$$

$$y = 0 \Rightarrow \dot{y} = \ddot{y} = 0$$

$$\Rightarrow \left.\begin{aligned} -\frac{T}{m}\sin\theta + \omega^2 R &= 0 \\ -g_0 + \frac{T}{m}\cos\theta &= 0 \end{aligned}\right\} \text{ al igual que antes}$$