

MM2

Soluciones

Julio 2019

UNIVERSIDAD COMPLUTENSE DE MADRID
FACULTAD DE CIENCIAS FÍSICAS

2 de Julio de 2019(11)

Examen Extraordinario de MM2, grupo C

Nombre y Apellidos:

Firma y DNI:

Consejo muy útil: Siempre que sea **razonable** es conveniente comprobar los resultados.

Importante: el profesor no operará nada de nada de lo que usted deje sin operar. Más aún, los resultados que no estén debidamente operados no se puntuarán.

1. [2.5 puntos] Los dos primeros polinomios de Legendre son $P_0 = 1$ y $P_1 = x$.

a) Comprobar, haciendo las integrales correspondientes, que P_0 y P_1 son ortogonales en $[-1, 1]$ pero no en $[0, 1]$.

b) Calcular el valor de la constante d para que el polinomio $P_2 = \frac{1}{2}(dx^2 - 1)$ sea ortogonal a P_0 en $[-1, 1]$.

c) Muestre para qué valores de d son P_1 y P_2 ortogonales en $[-1, 1]$.

d) Tomaremos de ahora en adelante para d el valor obtenido en el apartado (b). Calcular las constantes a, b, c de la **serie de Fourier** para que

$$\cos x \sim a P_0 + b P_1 + c P_2$$

en $[-1, 1]$. El símbolo \sim es una manera de escribir: un coseno no es nunca un polinomio, es simplemente que se ha cortado la serie de Fourier después de P_2 . Una vez que conozca estas constantes habrá determinado lo que los libros llaman “el polinomio de segundo orden que mejor approxima a $\cos x$ en $[-1, 1]$ en el sentido de *mínimos cuadrados*”. Con esto los libros quieren decir que el valor de la integral

$$I \equiv \int_{-1}^1 dx \left(\cos x - a P_0 - b P_1 - c P_2 \right)^2$$

donde a, b, c son las que usted ha determinado, es más pequeño que el valor que se obtiene con otras cualesquiera constantes. El profesor ha calculado la integral anterior, y vale

$$\frac{121}{2} \sin(2) - 48 \cos^2(1) - 41 \simeq 0.00001839,$$

la calculadora en modo radianes.

e) Pruebe el último párrafo como sigue: elija para a, b, c las constantes reales que usted quiera (fáciles, por favor, pero no triviales, no coja, 0, 0, 0), que no sean las obtenidas en d); calcule la integral I anterior con las constantes que usted ha elegido, apunte el resultado y compruebe que es mayor que 0.00001839. Es un teorema, así que es siempre cierto.

2. [2.5 puntos] Resolver por separación de variables el problema

$$\begin{cases} u_{tt} - u_{xx} = 0, & 0 < x < 2\pi, \quad t > 0 \\ u(x, 0) = \begin{cases} 2 \sin x & 0 \leq x \leq \pi \\ 0 & \pi \leq x \leq 2\pi \end{cases}, & u_t(x, 0) = 0, \\ u(0, t) = u(2\pi, t) = 0. \end{cases}$$

Comprobar que $u(x, \pi) = -\sin x$.

Corrección de este examen en la última página.

3. [2.5 puntos] a) Calcular por **separación de variables** la única solución **acotada** del problema del plano que cumple

$$\begin{cases} \Delta u = 0, & r < 2, \\ u_r(2, \theta) + u(2, \theta) = \cos \theta, & \theta \in [0, 2\pi]. \end{cases}$$

Dibujar el recinto de integración.

- b) Si se cambia $+u(2, \theta)$ por $-u(2, \theta)$, el problema tiene infinitas soluciones. Calcular estas soluciones.

4. [1+1.5 puntos] a) Resolver por el método de las características

$$\begin{cases} 2u_y + u_x = yu \\ u(x, 0) = e^{-x^2}. \end{cases}$$

- b) Sea la ecuación diferencial

$$xy'' + xy' + (x^2 - 1)y = 0$$

considerada en un entorno de $x = 0$. Escribir los cuatro primeros términos **no nulos** de una solución que se anula en $x = 0$. No se pide la regla de recurrencia de los coeficientes, pero sí las soluciones del polinomio indicial.

Escriba a continuación. No deje hojas en blanco y haga los problemas en el orden que usted prefiera.

④

$$\begin{cases} 2u_y + u_x = yu \\ u(x, 0) = e^{-x^2} \end{cases}$$

$$(u_x + u_y - 1) \cdot (1, 2, yu) = 0$$

$$dx = \underbrace{\frac{dy}{2}}_{2} = \underbrace{\frac{du}{yu}}_{\text{equations (2) of the characteristics}}$$

$$\boxed{2x - 1 = C_1} \quad \text{1st characteristic curve}$$

$$\frac{dy}{2} = \frac{du}{yu} \quad \text{or} \quad \frac{dy}{2} = \frac{du}{u} \quad \text{or}$$

$$\frac{y^2}{4} + C_1 = \log u \quad (\text{integrating}),$$

which is also

$$\boxed{u = C_2 e^{y^2/4}} \quad \text{2nd characteristic.}$$

General solution is

$$\boxed{u = f(2x - 1) e^{y^2/4}} \quad \text{2 function } \in C^1(\mathbb{R})$$

with function

(3)

Particular solution:

$$u(x, 0) = e^{-x^2}$$

$$e^{-x^2} = f(2x) \text{ or } f(x) = e^{-\frac{x^2}{4}},$$

which affords

$$u = e^{-\frac{1}{4}(2x-\gamma)^2 + \gamma^2/4}$$

$$= e^{-\frac{1}{4}(4x^2 + \gamma^2 - 4x\gamma) + \gamma^2/4}$$

$$= e^{-x^2 + x\gamma}$$

Particular solution: $u = e^{-x^2 + x\gamma}$

Following: $ux = u(-2x + \gamma)$
 $u\gamma = u(x)$

$$2u\gamma + ux = u[2x - 2x + \gamma] = \gamma u. \quad \text{OK.}$$

Also $u(x, 0) = e^{-x^2}$? Perfect!!

(4b) $x\gamma'' + x\gamma' + (x^2 - 1)\gamma = 0 \quad [\star]$

Multiply by x : $x^2\gamma'' + x^2\gamma' + x(x^2 - 1)\gamma = 0$

obtain Euler's equation around $x=0$:

$$x^2\gamma'' + 0\gamma' + 0\gamma = 0$$

or $\gamma'' = 0$, $\gamma_{\text{Euler}} = C_1 + C_2 x$

$$\begin{cases} 1 \equiv r_1 \\ 0 \equiv r_2 \end{cases}$$

The solution that vanishes at $x=0$ corresponds to the solution that vanishes at $x \rightarrow 0$: this solution is stated in Frobenius theorem

$$\gamma = \sum_0^\infty a_n x^{n+1}, \quad a_0 \neq 0 \quad (\star\star)$$

This solution of (\star) has no logarithms, as stated in Frobenius. Inserted $(\star\star)$ in (\star) , we obtain

$$0 = \sum_{n=0}^\infty n(n+1)a_n x^n + \sum_{n=0}^\infty n a_n x^{n+1} + \sum_{n=0}^\infty a_n x^{n+3}$$

$$\begin{aligned}
 &= x^0 [0 \cdot 1 a_0] \\
 &+ x [1 \cdot 2 a_1 + 0 \cdot a_0] \\
 &+ x^2 [2 \cdot 3 a_2 + 1 \cdot a_0] \\
 &+ x^3 [3 \cdot 4 a_3 + 2 \cdot a_2 + a_0] \\
 &+ x^4 [4 \cdot 5 a_4 + 3 a_3 + a_1] \\
 &+ x^5 [5 \cdot 6 a_5 + 4 a_4 + a_2] \\
 &+ x^6 [6 \cdot 7 a_6 + 5 a_5 + a_3] \\
 &+ x^7 [7 \cdot 8 a_7 + 6 a_6 + a_4]
 \end{aligned}$$

⋮

From where

$$a_0 = \text{free}$$

$$a_1 = 0$$

$$a_2 = 0$$

$$a_3 = -\frac{a_0}{12}$$

$$a_4 = -\frac{3a_3}{4 \cdot 5} = \frac{3 \cdot a_0}{4 \cdot 5 \cdot 12} = \frac{a_0}{80} \quad \text{term three}$$

$$a_5 = -\frac{4a_4}{5 \cdot 6} = -\frac{4a_0}{5 \cdot 6 \cdot 80} = -\frac{a_0}{600} \quad \text{term four (enough!!),}$$

or similarly, $a_0 = 1$ taken,

$$\gamma = x \left(1 - \frac{x^3}{12} + \frac{x^4}{80} - \frac{x^5}{600} + \dots \right).$$

End of the step.

① P_0 is orthogonal to P_1 in $[-1, 1]$ if

$$\int_{-1}^1 dx P_0 P_1 = 0$$

what is true since $\int_{-1}^1 dx \xrightarrow{\text{odd factor}} 0$. however

$$\int_0^1 dx P_0 P_1 = \frac{1}{2} \neq 0,$$

and P_0 and P_1 are not orthogonal in $[0, 1]$.

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6) $d=3$ only if $d=3$,
 $\int_{-1}^1 dx P_0 P_2 = 0$,

$$\Gamma \int_{-1}^1 dx \frac{1}{2}(dx^2 - 1) = \int_0^1 dx (dx^2 - 1) = \frac{d}{3} - 1.$$

c) For all values of d , $P_1 \perp P_2$ in $[-1, 1]$.
 because no matter d we always have

$$\int_{-1}^1 dx \underbrace{\frac{x}{2}(dx^2 - 1)}_{\substack{\text{odd factor} \\ \text{integrated over } [-1, 1]}} = 0.$$

d) conclusion: the polynomials

$$P_0 = 1, \quad P_1 = x, \quad P_2 = \frac{1}{2}(3x^2 - 1)$$

are orthogonal in $[-1, 1]$ by construction.
 meaning: $\int_{-1}^1 dx P_0 P_1 = \int_{-1}^1 dx P_1 P_2 = \int_{-1}^1 dx P_0 P_2 = 0$.

\rightarrow definition of a : multiply

$$or x = a P_0 + b P_1 + c P_2$$

by the function that accompanies a , that is by

P_0 :

$$P_0 w(x) = a P_0^2 + b P_0 P_1 + c P_0 P_2,$$

integrate over the interval where $\{P_0, P_1, P_2\}$ are orthogonal (it is $[-1, 1]$):

$$\int_{-1}^1 P_0 w(x) dx = a \int_{-1}^1 P_0^2 dx + b \int_{-1}^1 P_0 P_1 dx + c \int_{-1}^1 P_0 P_2 dx,$$

then

$$a = \frac{\int_{-1}^1 P_0 w(x) dx}{\int_{-1}^1 P_0^2 dx}.$$

For b, c one proceeds multiplying by P_1, P_2

in the same manner
 correspondingly.

the result is

$$b = \frac{\int_{-1}^1 P_1 \omega x dx}{\int_{-1}^1 dx \cdot P_1^2}, \quad c = \frac{\int_{-1}^1 P_2 \omega x dx}{\int_{-1}^1 dx \cdot P_2^2}.$$

This is ALWAYS the manner in which we use Fourier series. This is "El milagro de Fourier"

thus, $a = \sin 1$ | $b = 0$ | $c = 15 \cos 1 - 10 \sin 1$

$\int_{-1}^1 dx \cdot P_0 \omega x = \int_{-1}^1 dx \cdot \omega x = 2 \int_0^1 \omega x dx = 2[\sin x]_0^1$
 $= 2 \sin 1 \quad a = \frac{2 \sin 1}{2} = \sin 1.$

"a" $\int_{-1}^1 dx \cdot P_0^2 = 2 \int_0^1 dx = 2$

L "b" $\int_{-1}^1 dx \cdot \underbrace{\omega x}_{\text{odd function}} = 0 \quad \text{by symmetry}$

L "c" $\int_{-1}^1 P_2 \omega x dx = 2 \int_0^1 P_2 \omega x dx = \int_0^1 (3x^2 - 1) \omega x dx$
 $\text{every factor} = 6 \omega \sin 1 - 4 \sin 1$

$\int_0^1 x^2 \omega x dx = 2 \cos 1 - \sin 1$

$\int_0^1 \omega x dx = \sin 1$

$\int_{-1}^1 P_2^2 = 2 \int_0^1 P_2^2 = \frac{1}{2} \int_0^1 (3x^2 - 1)^2 dx = \frac{2}{5}$

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e) Take $(a, b, c) = (\sin 1, 0, 0)$, for instance:

$$\begin{aligned} \int_{-1}^1 (\cos x - \sin 1)^2 dx &= 2 \int_0^1 dx (\cos^2 x + \sin^2 1 - 2 \sin 1 \cos x) \\ &= 2 \left[\int_0^1 \left(\frac{1}{2} + \frac{1}{2} \cos 2x \right) dx + \underbrace{\sin^2 1 - 2 \sin 1 \int_0^1 \cos x dx}_{\sin 1} \right] \\ &= 2 \left[\frac{1}{2} + \frac{1}{4} \sin 2 + \sin^2 1 - 2 \sin^2 1 \right] \\ &\approx 1 + \frac{1}{2} \sin 2 - 2 \sin^2 1 \approx 0.0386, \end{aligned}$$

greater than 0.00001839 , as expected

$$\textcircled{2} \quad \begin{cases} \cancel{U_{tt}} - U_{xx} = 0, & 0 < x < 2\pi, t > 0 \\ U(x, 0) = f(x), & U_t(x, 0) = 0 \end{cases} \quad \begin{matrix} \text{Wave} \\ \text{equation} \end{matrix}$$

$$U(0, t) = U(2\pi, t) = 0$$

$$\text{with } f(x) = \begin{cases} 2 \sin x & x \in [0, \pi] \\ 0 & x \in [\pi, 2\pi] \end{cases}$$

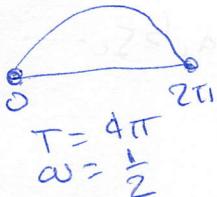
$$U = XT, \quad \frac{T''}{T} - \frac{x''}{x} = 0, \quad \frac{T''}{T} = \frac{x''}{x} = -\lambda$$

$$\begin{cases} x'' + \lambda x = 0 \\ x(0) = x(2\pi) = 0 \end{cases}; \quad \begin{cases} T'' + \frac{\lambda^2}{4} T = 0 \\ T'(0) = 0 \end{cases}$$

Prob. de contorno:
el importante

el acompañante.

$$T_n = \omega \frac{m}{2} t$$



$$T = 4\pi$$

$$\omega = \frac{1}{2}$$

$$X_n = \left\{ \sin \frac{n}{2} x \right\}, n = 1, 2, 3, \dots$$

$$\lambda_n = \frac{n^2}{4}$$

$$\text{or no forcing relations: } \int_0^{2\pi} dx \sin \frac{m}{2} x \sin \frac{n}{2} x = \pi \delta_{mn},$$

$$n, m = 1, 2, 3, \dots$$

Then

$$U = \sum_1^\infty b_m \omega \frac{m}{2} t \sin \frac{m}{2} x,$$

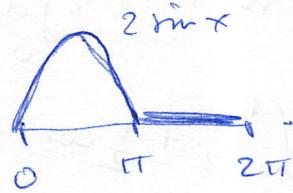
$$u = u(x, t)$$

with

$$u(x,0) = \sum_{n=1}^{\infty} b_n \sin \frac{nx}{2} = f(x) = \begin{cases} 2 \sin x & x \in [-\pi, \pi] \\ 0 & x \in [-\pi, \pi] \end{cases}$$

since

$f(x)$ is



The constants b_n are given by

$$\begin{aligned} b_n &= \frac{1}{\pi} \int_0^{2\pi} dx f(x) \sin \frac{nx}{2} \\ n = 1, 2, 3, \dots &= \frac{1}{\pi} \int_0^{\pi} dx \sin x \sin \frac{nx}{2} \\ &= \frac{2}{\pi} \int_0^{\pi} dx \sin x \sin \frac{nx}{2} \end{aligned}$$

Observation: $n=2$ is special since ' $\sin x$ ' and ' $\sin \frac{nx}{2}$ ' coincide and we have as the integrand a $\sin^2 x$. We calculate this we start with b_2 first. similar example

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$$\begin{aligned} b_2 &= \frac{2}{\pi} \int_0^{\pi} dx \sin^2 x = \frac{2}{\pi} \int_0^{\pi} \frac{1}{2} (1 - \cos 2x) \\ &\quad \text{otherwise} \\ &= \frac{2}{\pi} \cdot \frac{\pi}{2} = 1. \quad \boxed{b_2 = 1} \end{aligned}$$

Now we calculate $b_1, b_3, b_4, b_5, \dots$ the remaining ($n \neq 2$) b_n .

$$b_n = \frac{2}{\pi} \int_0^{\pi} dx \sin x \sin \frac{nx}{2}$$

$n \neq 2$

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$$\sin x \sin \frac{nx}{2} = \frac{1}{2} [\cos(x - \frac{nx}{2}) - \cos(x + \frac{nx}{2})],$$

$$\sin a \sin b = \frac{1}{2} [\cos(a-b) - \cos(a+b)]$$

$$b_n = \frac{2}{\pi} \cdot \frac{1}{2} \left[\frac{1}{1-\frac{n}{2}} \sin\left(1-\frac{n}{2}\right)x - \frac{1}{1+\frac{n}{2}} \sin\left(1+\frac{n}{2}\right)x \right]_0^\pi$$

$$= \frac{1}{\pi} \left[\frac{1}{1-\frac{n}{2}} \sin\left(1-\frac{n}{2}\right)\pi - \frac{1}{1+\frac{n}{2}} \sin\left(1+\frac{n}{2}\right)\pi \right]$$

$$= \frac{\sin n\pi/2}{\pi} \left[\frac{1}{1-\frac{n}{2}} + \frac{1}{1+\frac{n}{2}} \right]$$

$$\sin\left(1-\frac{n}{2}\right)\pi = \sin\pi \cancel{\cos\frac{n}{2}\pi} - \cos\pi \sin\frac{n\pi}{2} = \frac{\sin n\pi}{2}$$

$$\left\{ \begin{array}{l} \sin\left(1-\frac{n}{2}\right)\pi = \sin\pi \cancel{\cos\frac{n}{2}\pi} - \cos\pi \sin\frac{n\pi}{2} = \frac{\sin n\pi}{2} \\ \sin\left(1+\frac{n}{2}\right)\pi = \dots \text{change } n \text{ by } -n \text{ above} = -\frac{\sin n\pi}{2} \end{array} \right.$$

$$\sin\left(1+\frac{n}{2}\right)\pi = \frac{\sin n\pi/2}{\pi} \cdot \frac{8}{4-n^2}, \quad n=1, 2, 3, 4, 5, \dots$$

Fijense que se divide el 2
porque $4-n^2$ es divisor en $n=2$.
Por cuante se ponga, tb vale.

$$\text{Por cuante} \quad \frac{\sin n\pi/2}{\pi} \cdot \frac{8}{4-n^2} = \frac{0}{0},$$

con $n=2$, se tiene

se resuelve indefinida! (L'Hôpital,
(for instance) → se le \uparrow !!! Lo que te
le diré!!! Magnífico.

Note:

$$\sin\frac{n\pi}{2} = \begin{cases} 1 & n=1, 5, 9, 13, \dots \\ 0 & n=2, 4, 6, 8, 10, \dots \\ -1 & n=3, 7, 11, 15, \dots \end{cases}$$

$$\text{and } b_1 = \frac{8}{\pi} \cdot \frac{1}{4-1^2}, \quad b_5 = \frac{8}{\pi} \cdot \frac{1}{4-5^2}, \dots$$

$$b_3 = -\frac{8}{\pi} \cdot \frac{1}{4-3^2}, \quad b_7 = -\frac{8}{\pi} \cdot \frac{1}{4-7^2}, \dots$$

$b_2 = 1$.
even diff do zero.

$$b_4 = b_6 = b_8 = \dots = 0.$$

The solution is

$$u = \omega t \sin x + \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^2 - 4} \cos\left(\frac{2n-1}{2}\right) t \sin\left(\frac{2n-1}{2}x\right)$$

Of course,

$$u(x, \pi) = -\sin x,$$

as requested.

Following P2: $u = -r \cos \theta + a_2 r^2 \cos 2\theta + b_2 r^2 \sin 2\theta$
 $u_r = -\cos \theta + 2a_2 r \cos 2\theta + 2b_2 r \sin 2\theta$
 $u_r - u = (-1+r) \cos \theta + a_2 (2r-r^2) \cos 2\theta + b_2 (2r-r^2) \sin 2\theta$

At $r=R$, $u_r - u = \cos \theta + 0 + 0$

From our lectures: The most general solution of $\Delta u=0$ on the plane, written with polar coordinates, but has periodic boundary conditions on the angle $\theta \in [0, 2\pi]$ is

$$u = c_0 + d_0 \log r + \sum_{n=1}^{\infty} \left(a_n r^n + \frac{c_n}{r^n} \right) \cos n\theta$$

$$+ \sum_{n=1}^{\infty} \left(b_n r^n + \frac{d_n}{r^n} \right) \sin n\theta$$

with $c_0, d_0, a_n, c_n, b_n, d_n$ constants to be determined with the boundary conditions

$$u_r(2, \theta) + u(2, \theta) = \omega \sin \theta \quad (\text{problem P0})$$

or $u_r(2, \theta) - u(2, \theta) = \omega \cos \theta \quad (\text{problem P2}).$

Obviously,

$$u_r = \frac{d_0}{r} + \sum_{n=1}^{\infty} \left(a_n r^{n-1} - \frac{c_n}{r^{n+1}} \right) \cos n\theta$$

$$+ \sum_{n=1}^{\infty} n \left(b_n r^{n-1} - \frac{d_n}{r^{n+1}} \right) \sin n\theta.$$

a) Condition $u_r(2, \theta) + u(2, \theta) = \omega \sin \theta$ is equivalent to

$$\underline{c_0} + \underline{d_0} \left(\log 2 + \frac{1}{2} \right)$$

$$+ \left(3a_1 + \frac{c_1}{4} \right) \cos \theta$$

$$+ 8a_2 \cos 2\theta$$

$$+ \left(20a_3 - \frac{c_3}{16} \right) \cos 3\theta \dots$$

↓ lo pongo así un momento para verlo claro, en cuanto lo vea, pido a la proye siguiente ...

$$+ \left(3b_1 + \frac{d_1}{4} \right) \sin \theta$$

$$+ 8b_2 \sin 2\theta$$

$$+ \left(20b_3 - \frac{d_3}{16} \right) \sin 3\theta \dots = \omega \sin \theta$$

or to

$$c_0 + d_0 \left(\log 2 + \frac{1}{2} \right) = 0$$

$$3a_1 + \frac{c_0}{4} = 1, \quad a_2 = 0, \quad 3b_1 + \frac{d_1}{4} = 0, \quad b_2 = 0$$

$$(n+2) a_n = \frac{(n-2)c_n}{2^{2n}}, \quad n = 3, 4, 5, \dots$$

$$(n+2) b_n = \frac{(n-2)d_n}{2^{2n}}$$

whose solution is

$$c_0 = -d_0 \left(\log 2 + \frac{1}{2} \right)$$

$$a_n = \frac{1}{3} - \frac{c_0}{12}, \quad a_2 = 0, \quad b_1 = -\frac{d_1}{12}, \quad b_2 = 0$$

$$c_n = \frac{n-2}{n+2} \frac{c_0}{2^{2n}}, \quad n = 3, 4, 5, \dots$$

$$b_n = \frac{n-2}{n+2} \frac{d_1}{2^{2n}}, \quad \text{at } r=0$$

We require that u is bounded at $\theta = 0$ (by hand),

$$\begin{aligned} d_0 &= c_1 = c_2 = c_3 = \dots \\ &= b_1 = b_2 = b_3 = \dots = 0 \end{aligned}$$

and only survives

$$c_1 = \frac{1}{3}.$$

The solution is

$$u = \frac{1}{3} \cos \theta$$

b) $u_r(3\theta) - u(2\theta) = \omega \theta$ is equivalent to

$$\omega \theta = -c_0 + d_0 \left(\log 2 + \frac{1}{2} \right)$$

$$+ \left(a_1 + \frac{3}{4} c_1 \right) \cos \theta$$

$$+ \frac{6c_2}{2} \cos 2\theta$$

$$+ \left(-4a_3 + \sum_{i=1}^3 c_i \right) \cos 3\theta$$

$$+ \dots + \left(b_1 + \frac{3}{4} d_1 \right) \sin \theta$$

$$+ \frac{d_2}{2} \sin 2\theta + \left(-4b_3 + \sum_{i=1}^3 d_i \right) \sin 3\theta + \dots$$

notice: no cond on a_2 whatever
 (a_2, b_2) are absent here

or to

$$c_0 = \dots d_0 (\log 2 + \frac{1}{2})$$

$$a_1 = -1 - \frac{3}{4} c_1, b_1 = -\frac{3}{4} d_1$$

$$c_2 = 0$$

$$d_2 = 0$$

$$a_n = \frac{n+2}{n-2} \cdot \frac{1}{2^{2n}} \cdot c_n \quad n = 3, 4, 5, \dots$$

$$b_n = \frac{n+2}{n-2} \cdot \frac{1}{2^{2n}} \cdot d_n \quad n = 3, 4, 5, \dots$$

there is no condition
on a_2 nor on b_2
because they are
absent, so

a_2, b_2
are free.

By hand,

$$d_0 = c_1 = c_2 = c_3 = \dots$$

$$= d_1 = d_2 = d_3 = \dots = 0$$

To ensure that u is bounded at $r=0$ and
only remain the constants a_2, b_2 and
 $a_1 = -1$, that is

$$u = -r \cos \theta + a_2 r^2 \cos^2 \theta + b_2 r^2 \sin^2 \theta,$$

as mentioned. a_2, b_2 arbitrary (free)
constants.

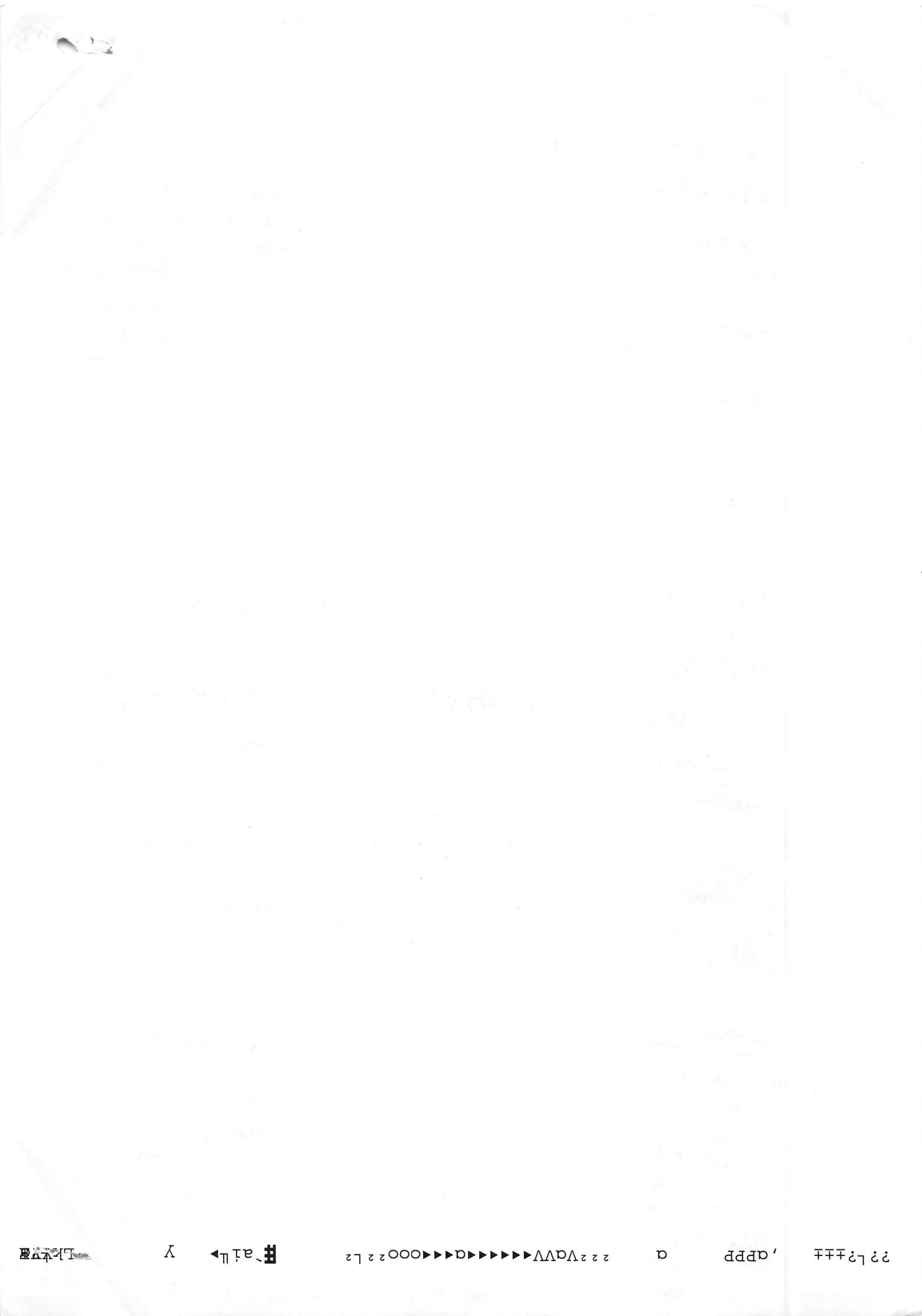
Un ejercicio curioso... como se le $\sin 2\theta$
me rotaría sin esperarlo... tampoco se
esperaba que se diese $\cos 2\theta$ pero $\sin 2\theta$ era
bien... bien... un sorpresa!!

anotaciones del examen

1- El ejercicio 1 estaba hecho en clase (casi
 todos los puntos)

2- El ejercicio 2 estaba en la hoja de problemas,
en red, compruébelo. Dice en clase que lo
hicieran como "entrenamiento" para el
examen de Junv. Me hacen poco caso.

3- Para el apartado e de 1, no hace falta
ser sólido de Fourier, sólo integrar,
así que era un reflejo de 0.5 puntos.



תְּהִלָּה

יְהִלָּה

וְאֵלֶּה

אֱמֹרָה

תְּהִלָּה