

15 Abril 2016

## Parcialillo de MM2, grupo C – 15/04/2016

Nombre y Apellidos:

Firma y DNI:

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**Consejo muy útil:** Siempre que sea **razonable** es conveniente comprobar los resultados.

1. [0.75 puntos] i) Resolver  $u_y + e^{x+y}u_x = u^2$  con el dato de Cauchy  $u(x, 0) = e^{-x}$ .  
ii) Escribir en forma canónica  $u_{yy} + e^{x+y}u_{xy} - u_y = 0$  y hallar su solución general.
2. [0.75 puntos] Resolver la ecuación diferencial

$$(4 - x^2)y'' + 2y = 0.$$

mediante una serie de potencias en un entorno de  $x = 0$  y calcular: a) la relación de recurrencia de los coeficientes, b) los **cuatro** primeros términos no nulos de cada solución linealmente independiente, c) el término general de cada solución, d) El radio de convergencia de las soluciones (de todas las maneras que usted sepa).

3. [0.25 puntos] En las ecuaciones que siguen especifique y clasifique los puntos singulares, escribiendo además la ecuación de Euler asociada cuando sea posible: (a)  $(1+x)y'' + 2xy' - 3y = 0$ , (b)  $x^3y'' + x^2y' + y = 0$ , (c)  $x(x-1)y'' + 6x^2y' + 3y = 0$  en  $x = 0$  sólo.

1º Paralelos de M2: 14/05/2016 Grupo G

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$$\begin{cases} u_1 + e^{x+\gamma} u_x = u^2 \\ u(x=0) = e^{-\gamma} \end{cases}$$

$$0.40 = \begin{cases} 0.20 \text{ para características} \\ \text{menos } [0.10 + 0.10] \\ 0.20 \text{ para general} \\ \gamma \text{ permanece.} \end{cases}$$

$$b) u_{\gamma\gamma} + e^{x+\gamma} u_{\gamma\gamma} - u_{\gamma} = 0 \quad \begin{matrix} \text{fórmula caótica} \\ \text{solución general} \end{matrix} \quad \begin{cases} 0.35 \text{ est.} \\ 0.15 \text{ para características} \\ 0.20 \text{ permanece} \\ e \text{ permanece} \\ & \text{y permanece.} \end{cases}$$

$$a) (u_x, u_{\gamma}, -1) (e^{x+\gamma}, 1, u^2) = 0$$

$$\frac{dx}{e^{x+\gamma}} = d\gamma = \frac{du}{u^2} : \text{Encuentra las características}$$

$$e^{-\gamma} dx = e^{\gamma} d\gamma \text{ or}$$

$$C_1 = e^{-\gamma} + e^{\gamma} \quad \begin{matrix} \text{(1st)} \\ \text{característica} \end{matrix}$$

$$d\gamma = \frac{du}{u^2} \text{ or}$$

$$u = \frac{1}{C_2 - \gamma} \quad \text{2da} \quad \text{or} \quad -\gamma + C_3 = +\frac{1}{u}$$

sólida general:

$$u = \frac{1}{f(e^{-\gamma} + e^{\gamma}) - \gamma}$$

$$-\gamma + f(e^{-\gamma} + e^{\gamma}) = \frac{1}{u}$$

con dato:

$$e^{-\gamma} = \frac{1}{f(e^{-\gamma} + 1)} \quad , \quad f(e^{-\gamma} + 1) = \frac{1}{e^{-\gamma}}$$

$$f(z+1) = \frac{1}{z} \quad \text{or} \quad f(z) = \frac{1}{z-1}$$

then

$$u = \frac{e^{-\gamma} + e^{\gamma} - 1}{1 - \gamma(e^{-\gamma} + e^{\gamma} - 1)}$$

$$\text{or} \quad \frac{1}{u} = -\gamma + \frac{1}{e^{-\gamma} + e^{\gamma} - 1}$$

Checking:  $\gamma = 0: \frac{e^{-\gamma}}{1} : \underline{\text{ok.}} \quad [\text{dato}]$ .

otras general es:  $\frac{1}{u} = f(e^{-\gamma} + e^{\gamma}) - \gamma$ .

$$\text{Then} \quad -\frac{1}{u^2} u_x = -f' e^{-\gamma} \rightarrow u_x = u^2 f' e^{-\gamma}$$

$$-\frac{1}{u^2} u_{\gamma} = f' e^{\gamma} - 1 \rightarrow u_{\gamma} = -u^2 f' e^{\gamma} + u^2$$

$$L \quad u_{\gamma} + e^{x+\gamma} u_x - u^2 = -x^2 f' e^{\gamma} + x^2 + e^{\gamma} / f' - x^2 = 0 \quad \underline{\text{ok}}$$

$f'$  = wrt  
the argument

b)  $u_{rr} + e^{x+1} u_{rx} - u_r = 0 \quad [x]$

$$r_r^2 + e^{x+1} r_x r_r = 0 \text{ is } r_r(r_r + e^{x+1} r_x) = 0$$

or  $\begin{cases} r_r = 0, & s = x \\ r_r + e^{x+1} r_x = 0, & \text{same as in } r = e^{\frac{x}{2}} + e^{-x} \end{cases}$

(a)

The equation is hypergeometric: has two characteristics in all the plane:

$$\left[ \begin{array}{cc} 0 & e^{x+1} \\ e^{x+1} & 1 \end{array} \right] = -\frac{(e^{x+1})^2}{4}, \boxed{urs+s=0} \quad \boxed{\text{canonical form}}$$

now  $r_x = -e^{-x}, \quad r_r = e^{\frac{x}{2}}$   
 $s_r = 1, \quad s_r = 0$

$$u_x = -e^{-x} \bar{u}_r + \bar{u}_s$$

$$u_r = \bar{u}_r e^{\frac{x}{2}}$$

$$u_{rr} = e^{\frac{x}{2}} \bar{u}_{rr} + e^{\frac{3x}{2}} \bar{u}_{rs}$$

$$u_{rx} = e^{\frac{x}{2}} [-\bar{u}_r e^{-x} + \bar{u}_{rs}]$$

$$0 = u_{rr} + e^{x+1} u_{rx} - u_r = \cancel{e^{\frac{x}{2}} \bar{u}_{rr} + e^{\frac{3x}{2}} \bar{u}_{rs} + e^{x+2}} [-\bar{u}_r e^{-x} + \bar{u}_{rs}]$$

$$= e^{x+2} [\bar{u}_{rs}]$$

canonical form of [x] is  $\boxed{\bar{u}_{rs} = 0}$

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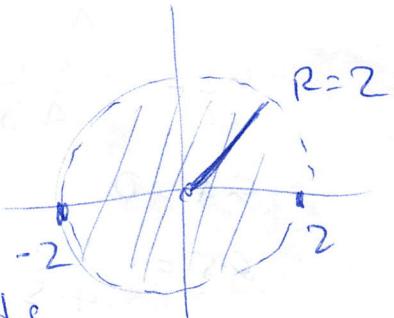


Solution:  $\bar{u} = f(r) + g(s)$ ,  $f, g$  arbitrary functions of  $C^2(\mathbb{S})$

$$\boxed{u = f(e^r + e^{-r}) + g(r)}$$

②  $(4-x^2)\gamma'' + 2\gamma = 0 \quad \text{in } x=0$

$$\gamma'' + \frac{2}{4-x^2}\gamma = 0$$



The theorem of ordinary points states that  $x=0$  is an ordinary point of the equation since  $P(x)=0$  and  $Q(x)=\frac{2}{4-x^2}$  are analytic functions at  $x=0$ . The solution of the ODE is around  $x=0$  a power series with convergence radius  $R=2$  at least! Then  $\geq 0.20$  per hödo est.

$$\gamma = \sum_0^\infty a_n x^n, \quad \gamma' = \sum_0^\infty n a_n x^{n-1}, \quad \gamma'' = \sum_0^\infty n(n-1) a_n x^{n-2}$$

introduced in the equation affords

$$0 = \sum_0^\infty 4n(n-1)a_n x^{n-2} - \sum_0^\infty (n+1)(n-2)a_n x^n \\ = x^{-2} [0 \cdot a_0] + x^{-1} [0 \cdot a_1] \\ + x^0 [4 \cdot 2 \cdot 1 a_2 - 1(-2)a_0]$$

there

$$+ \underbrace{2x [4 \cdot 3 \cdot 2 \cdot a_3 - 2(-1)a_1]}_{+ 3x^2 [4 \cdot 4 \cdot 3 \cdot 2 \cdot a_4 - 3 \cdot 2 \cdot 0 \cdot a_2]}$$

$$+ \underbrace{4x^3 [4 \cdot 5 \cdot 4 \cdot 3 \cdot a_5 - 4 \cdot 1 \cdot a_3]}_{+ 5x^4 [4 \cdot 6 \cdot 5 \cdot a_6 - 5 \cdot 2 \cdot a_4]} + \dots$$

Also resto  
mínimo fuer osto  
"factors and tende" sue hacia a mew. Por 100.000 veces! ...  
que se vea as "tendencia" sue hacia a mew. Por 100.000 veces! ...  
+ ... +  $(n+1)x^n [4(n+2)a_{n+2} - (n-2)a_n] + \dots$

Requirement sequence:

$$q_{n+2} = \frac{(n-2) q_n}{4(n+2)}, n=0, 1, 2, \dots$$

valid for

and

$q_0, q_1$  free

$$q_2 = -\frac{q_0}{4}$$

$$q_3 = -\frac{q_1}{4 \cdot 3}$$

$$q_4 = 0$$

$$q_5 = \frac{q_3}{4 \cdot 5} = -\frac{q_1}{4^2 \cdot 3 \cdot 5}$$

$$q_6 = q_8 = q_{10} = \dots = q_{\text{even}} = 0$$

$$q_7 = \frac{3q_5}{4 \cdot 7} = -\frac{q_1}{4^3 \cdot 5 \cdot 7},$$

0.25  
arriba  
el pape

To 0.20  
words denote  
for rule development  
factorized | opposite

The solution

$$\gamma = q_0 \left( 1 - \frac{x^2}{4} \right)$$

$\sum_m q_0 x^m$  is

reduced to a polynomial  
[you can check that  
 $4-x^2$  is a solution of  
the equation]

$$+ q_1 \left( x - \frac{x^3}{12} \right) + \frac{x^5}{240} - \frac{x^7}{2240} - \dots - \frac{x^{2n+1}}{4^n (2n-1)(2n+1)} - \dots ]$$

$$12 = 4 \cdot 1 \cdot 3$$

$$240 = 4^2 \cdot 3 \cdot 5$$

$$2240 = 4^3 \cdot 5 \cdot 7$$

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<u>0.20 + 0.20 + 0.75 + 0.1 = 0.75</u>				

$\frac{1}{4^2} =$   $\frac{1}{4^2}$   $\frac{1}{4^3}$   
resumir os le pape



Radii of convergence:

- La primera  $1 - \frac{x^2}{4}$  converge para todos  $x$ .  
Luego  $R = \infty$ .
- La otra secuencia (la fe va con  $a_1$ )

$$R = \lim_{n \rightarrow \infty} \left| \frac{a_n}{a_{n+1}} \right|$$

$$R = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_{n+2}} \right| \quad 0.10$$

$$R^2 = \lim_{n \rightarrow \infty} \left| \frac{a_n}{a_{n+2}} \right| = \lim_{n \rightarrow \infty} \left( \frac{4(n+2)}{n+2} \right) = 4$$

then,  $\boxed{R=2}$  as expected.

(at least  $R=2$  ... said the theorem)

③ (a)  $x=-1$  is a singular point of

$$(1+x)^{-1} + 2x^{-1} - 3x^3 = 0$$

choose to  $s = \text{green log}$   
 $s = x+1$ ,  $x = s-1$ ,  
and the equation is

$$s^{-1} + 2(s-1)^{-1} - 3(s-1)^3 = 0$$

Multiply by  $s$

$$s^2^{-1} + 2s(s-1)^{-1} - 3s(s-1)^3 = 0$$

Euler at  $s=0$  is  $s^2 - 2s = 0$ , then  
 $x=-1$  is a regular singular point

$$(b) x^3 \gamma'' + x^2 \gamma' + \gamma = 0$$

$x=0$  is singular. Type? 0.05

$$x^2 \gamma'' + x \gamma' + \frac{\gamma}{x} = 0$$

It is not possible to write a Euler equation at  $x=0$ , so  $x=0$  is an irregular point.

$$(c) x(x-1) \gamma'' + 6x^2 \gamma' + 3\gamma = 0$$

Study  $x=0$

$$x^2 \gamma'' + \frac{6x^3}{x-1} \gamma' + \frac{3x}{x-1} \gamma = 0$$

0.05 + 0.05

Euler at  $x=0$  is

$$x^2 \gamma'' + 0 \cdot \gamma' + 0 \cdot \gamma = 0 \quad (\gamma''=0, \text{ in a word}).$$

The point  $x=0$  is regular singular (regular regular).

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