UNIVERSIDAD COMPLUTENSE DE MADRID
FACULTAD DE CIENCIAS FISICAS

9 de Junio de 2015

Examen de Métodos Matemáticos II – Grupo C

Nombre y Apellidos:

DNI y firma:

Consejo muy útil: Siempre que sea **razonable** es conveniente comprobar los resultados.

1. [2 puntos] La función $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \sin x & 0 \le x < \pi \end{cases}$ se extiende periódicamente con desarrollo

$$\frac{1}{\pi} + b\sin x - \frac{2}{\pi}\left(\frac{\cos 2x}{1\cdot 3} + \frac{\cos 4x}{3\cdot 5} + \frac{\cos 6x}{5\cdot 7} + \cdots\right).$$

a) Dibujar al menos **tres** períodos de esta extensión y calcular el valor de la constante b. Use este valor de b en los apartados siguientes.

b) Utilizando los teoremas de convergencia puntual de series trigonométricas calcular la suma de la serie

$$\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \frac{1}{7\cdot 9} + \cdots .$$

d) Integrar término a término el desarrollo de $f(x)$ y determinar la constante de integración. Del resultado calcular la suma de

$$\frac{1}{1\cdot 2\cdot 3} - \frac{1}{5\cdot 6\cdot 7} + \frac{1}{9\cdot 10\cdot 11} - \frac{1}{13\cdot 14\cdot 15} + \cdots ,$$

justificando nuevamente el valor obtenido con los teoremas de convergencia puntual.

Nota: Ya se ve que no hay que usar identidad de Parseval.

2. [1.75 puntos] Sea la ecuación

$$2xy'' - y' + (x-2)y = 0.$$

a) Escribir el polinomio indicial en $x = 0$ y calcular sus raíces.

b) Encontrar los **cuatro** primeros términos no nulos de una solución **no analítica** en dicho punto y su regla de recurrencia (no hace falta resolver ésta, sólo escribirla).

c) Determinar si el punto del infinito es un punto ordinario, un punto regular singular o un punto irregular.

3. [2.5 puntos] Resolver por separación de variables y encontrar el valor de $u(x,1)$ en el problema

$$\begin{cases} u_{tt} - u_{xx} = 0, \quad x \in (0,2),\ t > 0 \\ u(x,0) = (x-2)^2,\ u_t(x,0) = 0, \\ u(0,t) = 4,\ u(2,t) = 0. \end{cases}$$

4. [2 puntos] Calcular por separación de variables la solución del problema del plano

$$\begin{cases} \Delta u = \dfrac{\cos 3\theta}{r^2}, \quad 1 < r < 2, \quad 0 < \theta < \pi \\ u(1,\theta) = u(2,\theta) = 0, \\ u_\theta(r,0) = u_\theta(r,\pi) = 0. \end{cases}$$

Dibujar el recinto de integración.

5. [1.75 puntos] Sea la ecuación

$$x^3 u_x - u_y = 2x^2 u.$$

Determinar la solución general por el método de las características e i) la solución que cumple el dato $u(1,y) = 2y$, ii) ¿cuántas soluciones de la ecuación cumplen $u(x, \frac{1}{2x^2}) = x^2$? iii) determinar si la curva $2\log x + c_2 = \frac{1}{u}$ es una característica de $x^3 u_x - u_y = 2x^2 u^2$.

MM2 June 2015

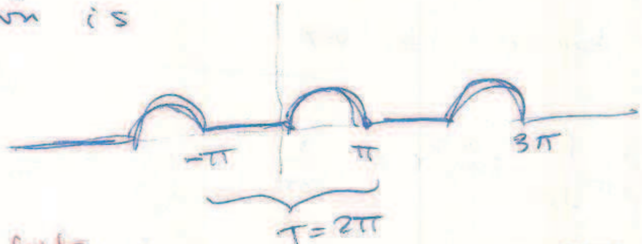
(1) $f(x)=\begin{cases}0 & -\pi\le x<0\\ \sin x & 0\le x\le\pi\end{cases}$

$$f_{ext}(x) \sim \frac{1}{\pi} + b\sin x - \frac{2}{\pi}\left(\frac{\cos 2x}{1\cdot 3}+\frac{\cos 4x}{3\cdot 5}+\frac{\cos 6x}{5\cdot 7}+\dots\right)$$

$\omega = 1$, then $T = 2\pi$

the coefficient b is not zero since $f(x) =$ has not a given parity.

the extension is



no hace falta en el examen. lo hago yo porque quiero...

checking:

$$\frac{a_0}{2} = \frac{1}{2\pi}\int_{-\pi}^{\pi} dx = \frac{1}{2\pi}\int_0^{\pi} \sin x\, dx = \frac{-1}{2\pi}\left[\cos x\right]_0^{\pi} = \frac{1}{\pi},$$

as expected.

$$\frac{a_n}{2} = \frac{1}{2\pi}\int_0^{\pi} dx \sin x \cos nx = \frac{1}{2}\left[\sin(1+n)x + \sin(1-n)x\right]$$

integrals are: $-\frac{\cos(1+n)x}{n+1} - \frac{\cos(1-n)x}{1-n}$

$$= \frac{1}{4\pi}\left[\frac{1}{1+n}\left(1-(-1)^{1+n}\right) + \frac{1}{1-n}\left(1-(-1)^{1-n}\right)\right]$$

$$= \frac{1}{4\pi}\left[\frac{1}{1+n}\left(1+(-1)^{n}\right) + \frac{1}{1-n}\left(1+(-1)^{n}\right)\right]$$

$$= \frac{1}{4\pi}\left[1+(-1)^n\right]\underbrace{\left[\frac{1}{1+n}+\frac{1}{1-n}\right]}_{\frac{2}{1-n^2}}$$

$$= \begin{cases} 0 & \text{if } n \text{ odd} \\ -\dfrac{1}{\pi(n^2-1)} & \text{if } n \text{ even.} \end{cases}$$

or two. See text.

$$\frac{b_n}{2} = \frac{1}{2\pi}\int_{-\pi}^{\pi} dx \;\frown\; \sin nx = \frac{1}{2\pi}\int_0^{\pi} dx \sin x \sin nx$$

$$n \neq 1 \quad = \frac{1}{4\pi}\int_0^{\pi} dx \left[-\cos(n+1)x + \cos(n-1)x \right]$$

$$\propto \left[\frac{\sin(n+1)x}{n+1}\right]_0^{\pi} = 0, \quad \left[\frac{\sin(n-1)x}{n-1}\right]_0^{\pi} = 0$$

$$\Rightarrow \quad b_n = 0 \quad \text{if } n \neq 1.$$

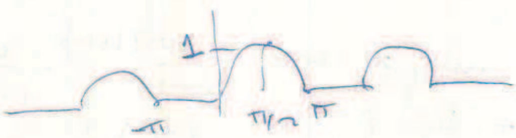
esto sí se pide...

$$\frac{b_1}{2} = \frac{1}{2\pi}\int_0^{\pi} \sin^2 x = \frac{1}{2\pi}\int_0^{\pi}\left(\frac{1}{2} - \frac{\cos 2x}{2}\right)dx$$

$$= \frac{1}{4} + 0 \quad \therefore \quad \boxed{b_1 = \frac{1}{2}} = b$$

result:

$$f(x): \;\frown\!\frown\!\frown \;\sim\; \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi}\left(\frac{\cos 2x}{1\cdot 3} + \frac{\cos 4x}{3\cdot 5} + \frac{\cos 6x}{5\cdot 7} \cdots\right)$$



Dirichlet this says that (tb vale $x = -\pi/\ldots$

$x = \pi/2$:

$$1 = \frac{1}{\pi} + \frac{1}{2} + \frac{2}{\pi}\underbrace{\left(\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} \cdots\right)}_{A}$$

$2x = \pi$
$4x = 2\pi$
$6x = 3\pi \ldots$

②

$\frac{1}{2} = \frac{1}{\pi} + \frac{2}{\pi}A$, then $\boxed{A = \frac{\pi}{4} - \frac{1}{2}}$

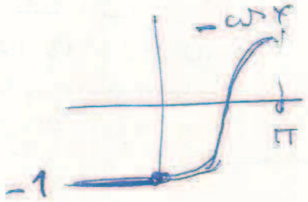
La serie pedida,

checked with Maple: $\frac{1}{1\cdot3} - \frac{1}{3\cdot5} + \frac{1}{5\cdot7} - \cdots = \frac{\pi}{4} - \frac{1}{2}$.

↳ en indefinido. En definido se seca un u := el último término (al igual u otra forma,

integrate first:

is



La pongo continua porque integrar regulariza las cosas... y mejora lo aprovecho...

(Howale discontinua pero con la C_1 bien calculada...)

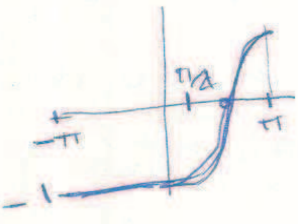
$$= C_1 + \frac{x}{\pi} - \frac{\cos x}{2} - \frac{2}{\pi}\left[\frac{\sin 2x}{1\cdot2\cdot3} + \frac{\sin 4x}{3\cdot4\cdot5} + \frac{\sin 6x}{5\cdot6\cdot7} + \cdots\right)$$

At $x=0$: Dirichlet's convergence th:

$-1 = C_1 + 0 - \frac{1}{2}$ " then $C_1 = -1/2$.

Solo integrales indefinidas llevan C_1, los definidos nunca

We have that



$$= \frac{x}{\pi} - \frac{1}{2}(1+\cos x) - \frac{2}{\pi}\left(\frac{\sin 2x}{1\cdot2\cdot3} + \frac{\sin 4x}{3\cdot4\cdot5} + \cdots\right)$$

$x = \pi/4$ | $1\cdot2\cdot3$ | $3\cdot4\cdot5$ | $5\cdot6\cdot7$ | $7\cdot8\cdot9$

$2x = \pi/2$ | 1 | 0 | -1 | 0

$4x = \pi$

$6x = 3\pi/2$

Dirichlet's theorem again.

Take $x = \pi/4$ & Dirichlet's theorem again.

$$-\frac{\sqrt{2}}{2} = \frac{1}{4} - \frac{1}{2}\left(1 + \frac{\sqrt{2}}{2}\right) - \frac{2}{\pi}\underbrace{\left(\frac{1}{1\cdot2\cdot3} - \frac{1}{5\cdot6\cdot7} + \frac{1}{9\cdot10\cdot11} - \frac{1}{13\cdot14\cdot15} + \cdots\right)}_{=B}$$

$$\frac{\sqrt2}{2}+\frac14-\frac12\left(1+\frac{\sqrt2}{2}\right)=\frac{2}{\pi}B \quad ,,$$

$$\left(\frac{\sqrt2}{4}-\frac14\right)\frac{\pi}{2}=B \quad ,, \quad \boxed{B=\frac{\pi}{8}(\sqrt2-1)}$$

y **no** $\frac{\pi}{8}(\sqrt2+1)$ como leo...

I checked with maple...

$$\frac{1}{1\cdot2\cdot3}-\frac{1}{5\cdot6\cdot7}+\frac{1}{9\cdot10\cdot11}-\frac{1}{13\cdot14\cdot15}+\dots=\frac{\pi}{8}(\sqrt2-1).$$

Visto en buenos alumnos:

Nota al corregir: $x/\pi-\frac12\cos x-\frac{2}{\pi}\left[\frac{\sin 2x}{1\cdot2\cdot3}+\frac{\sin 4x}{3\cdot4\cdot5}+\cdots\right]$ **no** es "$-\cos x$" en $(0,\pi)$. Basta substituir $x=\pi/2$ para verlo. Es "$\frac12-\cos x$".

(2) $2xy''-y'+(x-2)y=0$

$x=0$ is a singular point. The equation is also

$$2x^2y''-xy'+x(x-2)y=0.$$

and Euler at $x=0$ is

$$2x^2-xy'+0y=0,$$

whose indicial polynomial is

$$2r(r-1)-r=0 \quad \text{or} \quad r\begin{cases}3/2\\0.\end{cases}$$ (*)

The solution that is not analytic at $x=0$ is

$$y_1=\sum_0^\infty a_n x^{n+3/2}, \quad \text{with } a_0\neq0 \text{ por ser lo dice Frobenius}$$

$$y_1'=\sum_0^\infty (n+3/2)a_n x^{n+1/2}$$

$$y_1''=\sum_0^\infty (n+3/2)(n+1/2)a_n x^{n-1/2}$$

AY!!!

(*) Esa es la **no analítica** en $x=0$. La función $x^{3/2}$ **no** es analítica en $x=0$. Dame un par de veces... y lo verá. Y lo dije en clase.

$$0 = 2xy'' - y' + (x-2)y$$
$$= \sum_0^\infty 2(n+1/2)(n+3/2)\, a_n x^{n+1/2}$$
$$- \sum_0^\infty (n+3/2)\, a_n x^{n+1/2} + \sum_0^\infty a_n x^{n+5/2} - \sum_0^\infty 2a_n x^{n+3/2}$$

$$= \qquad (n+3/2)[2n+\cancel{1}-\cancel{1}] = (2n+3)n$$

$$= \sum_0^\infty n(2n+3)\, a_n x^{n+1/2} + \sum_0^\infty a_n x^{n+5/2} - \sum_0^\infty 2a_n x^{n+3/2}$$

$$= x^{1/2}[0] + x^{3/2}[1\cdot 5a_1 - 2a_0]$$
$$+ x^{5/2}[2\cdot 7a_2 - 2a_1 + a_0]$$
$$+ x^{7/2}[3\cdot 9a_3 - 2a_2 + a_1]$$
$$+ x^{9/2}[4\cdot 11a_4 - 2a_3 + a_2]$$
$$+ x^{11/2}[5\cdot 13a_5 - 2a_4 + a_3]$$
$$+ x^{13/2}[6\cdot 15a_6 - 2a_5 + a_4]$$
$$+ x^{15/2}[7\cdot 17a_7 - 2a_6 + a_5]$$
$$+ \cdots$$

$$a_n = \frac{2a_{n-1} - a_{n-2}}{n(2n+3)}.$$

regla de
recurrencia.
[no se puede resolver]

$n = \cancel{0},1,2,3\ldots$ & $a_{neg} \equiv 0$

o no preciso dividir entre 0

and $a_1 = \frac{2}{5}a_0$

$$a_2 = \frac{2a_1 - a_0}{2\cdot 7} = \frac{a_1}{7} - \frac{a_0}{2\cdot 7} = \frac{1}{7}\left(\frac{2}{5} - \frac{1}{2}\right)a_0 = -\frac{a_0}{70}$$

$$a_3 = \frac{1}{3\cdot 9}[2a_2 - a_1] = \frac{1}{3\cdot 9}\left[-\frac{a_0}{35} - \frac{2a_0}{5}\right] = -\frac{a_0}{9\cdot 7} = -\frac{a_0}{63}$$

$\vdots$

$a_4 =$ no se puede.

Then $y_1 = x^{3/2}\left(1 + \frac{2}{5}x - \frac{x^2}{70} - \frac{x^3}{63} + \cdots\right).$

$(x^{3/2})' \sim x^{1/2}$; $(x^{1/2})' \sim x^{-1/2}$ por eso no es analítica en $x=0$

At $|x| \to \infty$, $s \equiv \frac{1}{x} \to 0$

$s = \frac{1}{x}$, $\frac{d\eta}{dx} = \frac{d\eta}{ds}\cdot\frac{ds}{dx} = -\frac{1}{x^2}\dot{\eta} = -s^2\dot{\eta}$

$$\frac{d^2\eta}{dx^2} = -s^2\frac{d}{ds}\left(-s^2\dot{\eta}\right) = s^2\left[s^2\ddot{\eta} + 2s\dot{\eta}\right]$$
$$= s^4\ddot{\eta} + 2s^3\dot{\eta}$$

$$0 = 2x\eta'' - \eta' + (x-2)\eta = \frac{2}{s}\left(s^4\ddot{\eta} + 2s^3\dot{\eta}\right) + s^2\dot{\eta} + \left(\frac{1}{s} - 2\right)\eta$$

or

$$2s^4\ddot{\eta} + 5s^3\dot{\eta} + (1-2s)\eta = 0$$

Now... $2s^2\ddot{\eta} + 5s\dot{\eta} + \left(\frac{1}{s^2} - \frac{2}{s}\right)\eta = 0$, Irregular point.

$s=0$: is irregular.

(3)
$$\begin{cases} u_{tt} - u_{xx} = 0 , & x \in (0,2) , \quad t > 0 \\ u(x,0) = (x-2)^2 , & u_t(x,0) = 0 \\ u(0,t) = 4 , & u(2,t) = 0 \end{cases}$$

need a 0 there

$w = u - ax - b$: at $x=0$: $0 = 4 - b$ ∴ $b = 4$

$w = u - ax - 4$ at $x = 2$: $0 = 0 - 2a - 4$ ∴ $a = -$

change from u to w given by:
$$\boxed{w = u + 2x - 4.}$$

$$\begin{cases} w_{tt} - w_{xx} = 0 \\ w(x,0) = x(x-2) , \quad w_t(x,0) = 0 \\ w(0,t) = w(2,t) = 0 \end{cases}$$

$w = T \cdot X$

$\begin{cases} T'' + \lambda T = 0 \\ T'(0) = 0 \end{cases}$ & $\begin{cases} X'' + \lambda X \\ X(0) = X(2) = 0 \end{cases}$

$\uparrow$ Acompañante $\qquad$ $\uparrow$ probl contorno

No son condiciones de contorno mixtas... aunque calce $\frac{n\pi}{2}x$ en los senos. Valen todas.

$T = 4$, $\omega = \frac{\pi}{2}$

$X_n = \{ \sin \frac{n\pi}{2} x \}$

$n = 1, 2, \dots$

$\lambda_n = \frac{n^2\pi^2}{4}$

$$\int_0^2 dx\, X_n X_m = 1 \cdot \delta_{nm}$$

$\uparrow$ calculamos ...

$$\begin{cases} T_n'' + \frac{n^2\pi^2}{4} T_n = 0 \\ T_n'(0) = 0 \end{cases} \Rightarrow T_n = \cos \frac{n\pi}{2} t$$

soluciones factibles: $w_n = \cos \frac{n\pi}{2} t \sin \frac{m\pi}{2} x$

Form of the solution

$$w = \sum_1^\infty b_n \cos \frac{n\pi}{2} t \cdot \sin \frac{n\pi}{2} x. \qquad [*]$$

s.t.

$$x(x-2) = \sum_1^\infty b_n \sin n\frac{\pi}{2} x,$$

where

$$b_n = \int_0^2 dx\, x(x-2) \sin m\frac{\pi}{2} x$$

[we know this by "extensions" or "orthogonality conditions":

odd $x(x-2)$

Ext: $\frac{b_n}{2} = \frac{1}{4} \int_{-2}^{2} \sin \frac{\pi}{2} x = \frac{1}{2} \int_0^2 x(x-2) \sin \frac{n\pi}{2} x$

$\to x(x-2)$

orthogonality cond: $\int_0^2 dx \sin\frac{n\pi}{2}x \sin\frac{m\pi}{2}x = 1\cdot\delta_{nm}$

$$x(x-2)\sin\frac{m\pi}{2}x = \sum_1^\infty b_n \sin\frac{n\pi}{2}x \sin\frac{m\pi}{2}x$$

Integrate in $\int_0^2$:

$$\int_0^2 dx\, x(x-2)\sin\frac{m\pi}{2}x = b_m\cdot 1$$

same result as = the 1st method.

L

Since

$$\int x^2\sin ax = -\frac{x^2}{a}\cos ax + \frac{2}{a^2}x\sin ax + \frac{2}{a^3}\cos ax$$

$$\int x\sin ax = -\frac{x}{a}\cos ax + \frac{1}{a^2}\sin ax,$$

one has too:

$$\int x(x-2)\sin ax = -\frac{x}{a}(x-2)\cos ax + \frac{2(x-1)}{a^2}\sin ax + \frac{2}{a^3}\cos ax$$

with $a=\frac{n\pi}{2}$, $2a=n\pi$

no contribution to $\int_0^2$ because of the coefficient: $x(x-2)$

$\sin 2a = \sin 0 = 0$

only term to consider.

$$\int_0^2 x(x-2)\sin\frac{n\pi}{2}x = \frac{2}{a^3}\left[\cos n\pi - 1\right]$$

$$= \frac{2}{a^3}\left[(-1)^n - 1\right]$$

0 n even

[pero no por el problema de contorno]!

$-\frac{32}{n^3\pi^3}$ if n odd.

Introducing $b_n = -\frac{32}{n^3\pi^3}$ if $n=1,3,5\ldots$

in [*], the result is

$$w = -\sum_1^\infty \frac{32}{\pi^3}\frac{1}{(2n-1)^3}\cos(2n-1)\frac{\pi}{2}t\cdot\sin\frac{(2n-1)}{2}\pi x,$$

and

$$\boxed{\begin{aligned} u &= 4-2x+w \\ &= 4-2x-\frac{32}{\pi^3}\sum_{1}^{\infty}\frac{1}{(2n-1)^3}\cos(2n-1)\frac{\pi t}{2}\,\sin\frac{(2n-1)}{2}\pi\end{aligned}}$$

Los senos impares no los selecciona el problema de contorno, sino el dato inicial (i.e, la serie de Fourier

Also

$$\boxed{u(x,1) = 4-2x}$$

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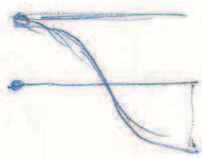
(4)
$$\begin{cases} \Delta u = \dfrac{\cos 3\theta}{r^2} \\ u(1,\theta) = u(2,\theta) = 0 \\ u_\theta(r,0) = u_\theta(r,\pi) = 0 \end{cases}$$

$1<r<2,\ 0<\theta<\pi$

$\Theta_0 = \{1\},\ \lambda_0 =$



$$u = R\Theta,\quad \begin{cases}\Theta'' + \lambda\Theta = 0 \\ \Theta'(0) = \Theta'(\pi) = 0\end{cases}$$

$T = 2\pi$
$\omega = 1$
$\{\cos n\theta\}$
$\lambda_n = n^2$
$n = 1,2,3,\dots$

Thus
$$\begin{cases}\lambda_0 = 1, & \Theta_0 = 1 \\ \lambda_n = n^2 & \Theta_n = \cos n\theta,\ n = 1,2,3,\dots\end{cases}$$

The solution of the problem is then
$$u = R_0(r) + \sum_{1}^{\infty} R_n(r)\cos n\theta,$$
with
$$\frac{\cos 3\theta}{r^2} = R_0'' + \frac{R_0'}{r} + \sum_{1}^{\infty}\left[R_n'' + \frac{R_n'}{r} - n^2\frac{R_n}{r^2}\right]\cos n\theta.$$

there are three problems to consider:
$$\begin{cases} R_0'' + \dfrac{R_0'}{r} = 0 \\ R_0(1) = R_0(2) = 0\end{cases};\quad \begin{cases} R_3'' + \dfrac{R_3'}{r} - 9\dfrac{R_3}{r^2} = \dfrac{1}{r^2} \\ R_3(1) = R_3(2) = 0\end{cases}\quad \text{and}$$

$$\begin{cases} R_n'' + \dfrac{R_n'}{r} - n^2\dfrac{R_n}{r^2} = 0 & , n = 1, 2, \cancel{3}, 4, \ldots \\ R_n(1) = R_n(2) = 0 \end{cases}$$

the solution of the first problem (R_0) is

$R_0 = c_0 + d_0 \log r$, but $R_0(1) = R_0(2) = 0$

gives $c_0 = d_0 = 0$. The second problem (R_3)

has solution:

$$R_3 = c_1 r^3 + \frac{c_2}{r^3} - \frac{1}{9}$$

$$\& \quad \left.\begin{array}{l} c_1 + c_2 - 1/9 = 0 \\ 8c_1 + \dfrac{c_2}{8} - \dfrac{1}{9} = 0 \end{array}\right\} \Rightarrow \begin{array}{l} c_1 = \dfrac{1}{81} \\ c_2 = \dfrac{8}{81} \end{array}$$

$$\Rightarrow R_3 = \frac{r^3}{81} + \frac{8}{81 r^3} - \frac{1}{9}.$$

the third problem: has solution:

$R_n = c_3 r^n + \dfrac{c_4}{r^n}$ but $c_3 = c_4 = 0$ due to $R_n(1) = R_n(2) = 0$.

then:

$$\begin{cases} \Delta u = \dfrac{\cos 3\theta}{r^2} \\ u(1,\theta) = u(2,\theta) = 0 \\ u_\theta(r,0) = u_\theta(r,\pi) = 0 \end{cases} \Rightarrow \boxed{u = \left(\frac{r^3}{81} + \frac{8}{81 r^3} - \frac{1}{9}\right)\cos 3\theta \quad \text{unique}}$$

(5) $x^3 u_x - u_y = 2x^2 u$:

$(u_x, u_y, -1)\cdot(x^3, -1, 2x^2u) = 0$:

$$\boxed{\frac{dx}{x^3} = -dy = \frac{du}{2x^2u}}$$

later.

1st characteristic curve: $\frac{1}{2x^2} = y - c_1$ or $\boxed{y = c_1 + \frac{1}{2x^2}}$

2nd: $\frac{dx}{x} = \frac{du}{2u}$ or $\frac{du}{dx} = 2\frac{u}{x}$ or $\boxed{u = c_2 x^2}$

General solution: $c_2 = f(c_1)$: $\boxed{u = x^2 f\left(y - \frac{1}{2x^2}\right)}$

i) $\left.\begin{array}{l} u(1,y) = 2y \\ u = x^2 f\left(y - \frac{1}{2x^2}\right) \end{array}\right\}$ simplify: $2y = f\left(y - \frac{1}{2}\right)$

: $y \to y + 1/2$ is $\boxed{2y + 1 = f(y)}$

then $u = x^2\left(2y - \frac{1}{x^2} + 1\right) = \boxed{2x^2y - 1 + x^2 = u}$

(checked that $u(1,y) = 2y$ and satisfies the equation)

ii) $\left.\begin{array}{l} u\left(x, \frac{1}{2x^2}\right) = x^2 \\ u = x^2 f\left(y - \frac{1}{2x^2}\right) \end{array}\right\} \Rightarrow$ $x^2 = x^2 f(0)$ or $\boxed{1 = f(0)}$

Infinite $f(x)$ satisfy $f(0) = 1$. Ex:

$f(x) \equiv 1$, $\boxed{u = x^2}$

$f(x) = x + 1$, $u = yx^2 - \frac{1}{2} + x^2 = \boxed{x^2(y+1) - \frac{1}{2} = u}$

etc...

iii) $x^3 u_x - u_y = 2x^2u^2$

$\frac{dx}{x^3} = -dy = \frac{du}{2x^2u^2}$: characteristic equations curve

$2\log x + c_2 - \frac{1}{u} = 0$ (should be to be a charact)

differentiate: $2\frac{dx}{x} + \frac{du}{u^2} = 2\frac{du}{u^2}$: $dx = \frac{x\,du}{2u^2}$

No. It is not a characteristic curve

But $2\log x + c_2 = -\frac{1}{u}$ it is a characteristic curve

du du, dx

(6)