UNIVERSIDAD COMPLUTENSE DE MADRID
FACULTAD DE CIENCIAS FISICAS

15 de Junio de 2016(9)

Examen de Métodos Matemáticos II – Grupo C

Nombre y Apellidos:

DNI y firma:

Consejo muy útil: Siempre que sea **razonable** es conveniente comprobar los resultados.

1. [2 puntos] La parábola $f(x) = Ax^2 + Bx + C$ definida en el intervalo $(0, \pi)$ se extiende periódicamente en la recta real mediante la serie de cosenos

$$f_{ext}(x) = \cos x + \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} + \frac{\cos 4x}{4^2} + \cdots .$$

   Hallar las cosntantes A, B, C.

   Nota: Puede usted usar todo lo que quiera: teoremas de convergencia puntual de Dirichlet, derivar la serie término a término, integrarla, calcular coeficientes con fŕomulas de Euler, etc. Escriba claramente lo que utiliza pues el problema se resuelve de muchas maneras. Como siempre, el período lo indica el desarrollo dado.

2. [1.75 puntos] Sea la ecuación

$$x(1-x)y'' - 2(x+2)y' + 6y = 0.$$

   a) Escribir el polinomio indicial en $x = 0$ y calcular sus raíces.

   b) Escribir el teorema de Frobenius para este caso concreto (copie de su hoja de fórmulas o mejor aún si lo sabe de memoria) y hallar una solución que **no se anule en el origen**. ¿Qué radio de convergencia tiene esta solución?

   c) A la vista del resultado obtenido en b), ¿qué se concluye de la solución general de la ecuación, que lleva o que no lleva logaritmos?

3. [2.5 puntos] Resolver por separación de variables

$$\begin{cases} u_t - u_{xx} = 0, \quad x \in (0, \pi),\ t > 0 \\ u(x, 0) = 0, \\ u(0, t) = 1,\ u_x(\pi, t) = 0. \end{cases}$$

4. [2 puntos] Calcular por separación de variables la única solución acotada del problema del plano que cumple

$$\begin{cases} \Delta u = \cos\theta, \quad 1 < r < 2, \quad 0 \le \theta < 2\pi \\ u_r(1, \theta) = u(2, \theta) = 0. \end{cases}$$

   Dibujar el recinto de integración.

5. [1.75 puntos] Sea la ecuación

$$2xu_x - yu_y = u - xy^2.$$

   Determinar la solución general por el método de las características y la solución que cumple el dato $u(x, 2) = 0$.

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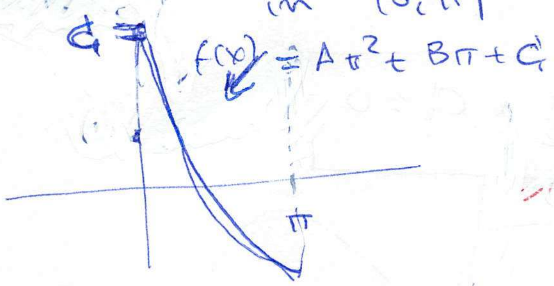
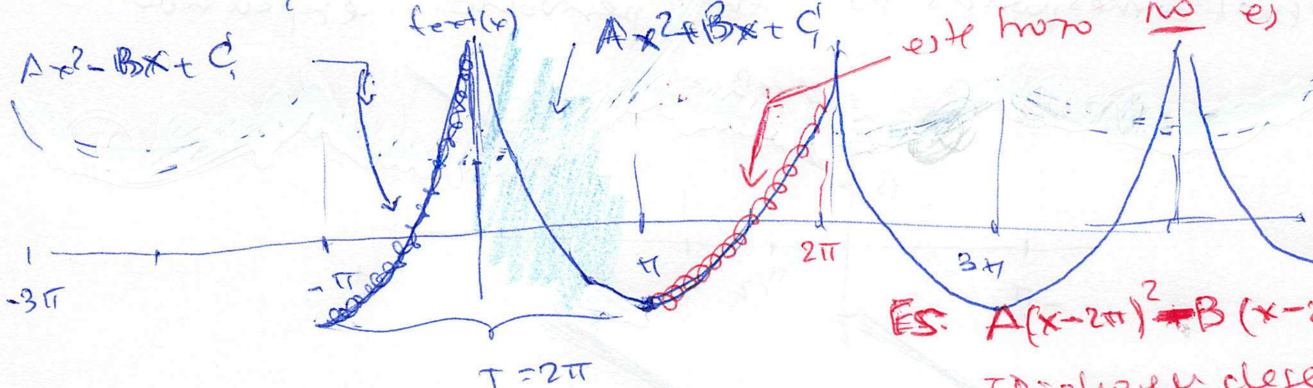
(1) in $[0,\pi]$



igual no es esa parábola, pero da igual...

$$f_{ext}(x) = \cos x + \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} + \frac{\cos 4x}{4^2} + \ldots$$ (*)

$\omega = 1$ then $T = 2\pi$ and consequently,



Es: $A(x-2\pi)^2 + B(x-2\pi) + C_1$

[Dicho en clase 800.000 veces ...!!]

Obviously, if you know the sums

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots,$$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$$

you know C_1 and $A\pi^2 + B\pi + C_1$, because according to Dirichlet convergence theorems at

$$x = 0, \quad C_1 = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$$

$$x = \pi, \quad A\pi^2 + B\pi + C_1 = -1 + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} - \ldots$$

Assuming that we do not have those sums at hand, observe that in the expansion

$$\cos x + \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} + \ldots$$

is missing the constant term $\frac{a_0}{2}$, then

$$0 = \frac{a_0}{2} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \boxed{} = \frac{1}{\pi}\int_0^{\pi} \left(Ax^2 + Bx + C_1\right) dx$$

$$= \frac{1}{\pi}\left(A\frac{\pi^3}{3} + B\frac{\pi^2}{2} + C\pi\right)$$

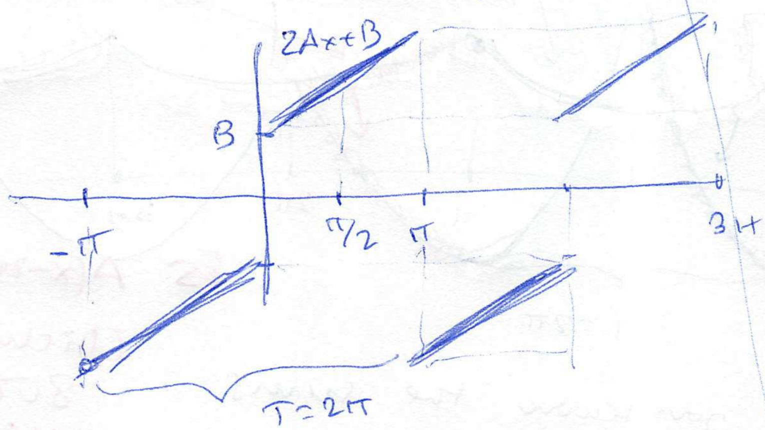
or

$$\boxed{A\frac{\pi^2}{3} + B\frac{\pi}{2} + C = 0}.$$

Let us derive (*),

(**) $$2Ax+B = -\left(\sin x + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \dots\right), \quad x\in(0,\pi),$$

that corresponds to the periodic expansion



There we know that Dirichlet says that at $x=\frac{\pi}{2}$

$$2A\frac{\pi}{2}+B = -\underbrace{\left(1+0-\frac{1}{3}+0+\frac{0}{5} - \dots\right)}_{\text{this is the "famous } \frac{\pi}{4}\text{"}} = -\frac{\pi}{4},$$

then

$$A\pi+B = -\pi/4$$

We are going to obtain this relation in a moment too.

A manner to calculate A, B in (**) is with Euler formulas,

$$2Ax+B = \sum_{1}^{\infty} b_n \sin nx$$

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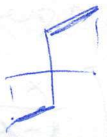
where

$$\frac{b_n}{2} = \frac{1}{2\pi}\int_{-\pi}^{\pi}$$  $$\sin nx = \frac{1}{\pi}\int_0^{\pi}(2Ax+B)\sin x,$$

and we know that

$$b_1=-1,\quad b_2=-\frac{1}{2},\quad b_3=-\frac{1}{3},\ \dots\quad b_n=-\frac{1}{n},$$

just looking at (**).

$$\Bigg\lceil \int x\sin ax = -\frac{x}{a}\cos ax + \frac{1}{a^2}\sin ax$$

$$\int \sin ax = \qquad -\frac{1}{a}\cos ax.$$

And

$$\int (2Ax+B)\sin ax = -\frac{2}{a}(Ax+B)\cos ax + \frac{2A}{a^2}\sin ax,$$

that gives

$$\int_0^{\pi}(2Ax+B)\sin nx = -\frac{2}{n}\Big[(A\pi+B)\underbrace{(-1)^n}_{=\cos n\pi} - B\Big]$$

$n=1,2,3\dots$

$\lfloor$

Since

$$\frac{b_n}{2} = -\frac{2}{n\pi}\Big[(A\pi+B)(-1)^n - B\Big] \begin{cases} n \text{ even: } -\dfrac{2A}{n}, \\ n \text{ odd: } \dfrac{2}{n\pi}\big[A\pi+2B\big] \end{cases}$$

we have: se sabe de (**)

$$b_2=-\frac{1}{2} \quad \text{gives} \quad -\frac{1}{4} = -\frac{2A}{2} \quad \text{or} \quad \boxed{A=1/4}$$

$$b_1=-1, \quad -\frac{1}{2} = \frac{2}{\pi}\Big[\frac{\pi}{4}+B\Big] \quad \text{or} \quad \boxed{B=-\frac{\pi}{2}}$$

(notice that we knew the relation $A\pi+B=-\frac{\pi}{4}$, that is related to any odd n)

With A, B find C and obtain

$$\boxed{A=1/4,\quad B=-\frac{\pi}{2},\quad C=\pi^2/6}$$

② $x(1-x)y'' - 2(x+2)y' + 6y = 0$ (***)

The point $x=0$ is a regular singular point since

$$x^2 y'' - 2x\left[\frac{x+2}{1-x}\right]y' + 6\left[\frac{x}{1-x}\right]y = 0$$

is well described around $x=0$ by the Euler equation

$$x^2 y'' - 4xy' + 0y = 0.$$

The indicial polynomial is

$$0 = r(r-1) - 4r = r(r-5), \text{ roots } \begin{cases} r_1 = 5 \\ r_2 = 0 \end{cases}$$

and the solution of (***) around $x=0$ behaves as

$$y = c_1 x^5 + c_2 .$$

[Una persona sensata que haga este problema empiece con la solución $\sum_0^\infty a_n x^{n+5}$ with $a_0 \neq 0$, la saca y luego sigue con $\sum_0^\infty b_m x^m$ with $b_0 \neq 0$. Yo lo he hecho así y he visto que ésta última solución se reduce a un polinomio de orden 2 y por eso no lo repetimos. Por eso se lo pido a ustedes, porque alguien (yo) lo ha hecho antes.]

[This is for ~~me~~ = student in an exam. You can go directly to (*)

$$y = \sum_0^\infty a_n x^{n+5}, \quad a_0 \neq 0$$

$$y' = \sum_0^\infty (n+5) a_n x^{n+4}$$

$$y'' = \sum_0^\infty (n+5)(n+4) a_n x^{n+3},$$

that inserted in (***) is the relation

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$$0=\sum_0^\infty n(n+5)\,a_n x^{n+4}-\sum_0^\infty (n+8)(n+3)\,a_n x^{n+5}$$

$$\begin{aligned}
&= x^4[0\cdot 5\cdot a_0 - \\
&+ x^5[1\cdot 6\, a_1 - 8\cdot 3\, a_0] \\
&+ x^6[2\cdot 7\, a_2 - 9\cdot 4\, a_1] \\
&+ x^7[3\cdot 8\, a_3 - 10\cdot 5\, a_2] \\
&+ x^8[4\cdot 9\, a_4 - 11\cdot 6\, a_3] \\
&+ x^9[5\cdot 10\, a_5 - 12\cdot 7\, a_4] \\
&+ x^{10}[6\cdot 11\, a_6 - 13\cdot 8\, a_5] \\
&+ x^{11}[7\cdot 12\, a_7 - 14\cdot 9\, a_6] \\
&\;\vdots \\
&+ x^{n+5}[(n+1)(n+6)\,a_{n+1} - (n+8)(n+3)\,a_n] + \dots
\end{aligned}$$

a_0 = free

$a_1 = 4a_0$

$a_2 = \frac{9\cdot 4}{2\cdot 7}a_1 = \frac{9\cdot 2}{7}a_1 = \frac{9\cdot 2\cdot 4}{7}a_0 = \frac{72}{7}a_0$

$a_3 = \frac{10\cdot 5}{3\cdot 8}a_2 = \frac{5\cdot 5}{3\cdot 4}a_2 = \frac{150}{7}a_0$

$a_4 = \frac{275}{7}a_0$

$\vdots$

Solution:

$$y_1 = x^5\left(1+4x+\frac{72}{7}x^2+\frac{150}{7}x^3+\frac{275}{7}x^4+\dots\right)$$

Recurrence formula for the coefficients

$$a_{n+1}=\frac{(n+3)(n+8)}{(n+1)(n+6)}\,a_n,\quad n=0,1,2,\dots$$

Obviously,

$$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\frac{(n+1)(n+6)}{(n+3)(n+8)}=1$$

(claro, hasta la singularidad $x=1$ desde $x=0$).

(*) $y = \sum_0^\infty b_n x^n$ with $b_0 \neq 0$

$n+5 \to n$ or $n \to n-5$

(éste es lo que pide el examen!) ∴ si sale sin problema (con $b_0 \neq 0$) es que NO hay logaritmos en la solución.

$$0 = \sum_0^\infty (n-5)\, n\, b_n x^{n-1} - \sum_0^\infty (n-2)(n+3) b_n x^n$$

$$\begin{aligned}
&= x^{-1}[-5\cdot 0\, b_0] \\
&+ x^0[-4\cdot 1\, b_1 - (-2)\cdot 3\, b_0] \\
&+ x[(-3)\cdot 2\, b_2 - (-1)\cdot 4\, b_1] \\
&+ x^2[(-2)\cdot 3\, b_3 - (0)\cdot 5\, b_2] \\
&+ x^3[(-1)\cdot 4\, b_4 - 1\cdot 6\, b_3] \\
&+ x^4[(0)\cdot 5\, b_5 - 2\cdot 7\, b_4] \\
&+ x^5[1\cdot 6\, b_6 - 3\cdot 8\, b_5] \\
&+ \cdots
\end{aligned}$$

$$b_{n+1} = \frac{(n-2)(n+3)}{(n-4)(n+1)} b_n$$

$b_0 =$ free

$b_1 = \frac{2\cdot 3}{1\cdot 4} b_0 = \frac{3}{2} b_0$

$b_2 = \frac{1\cdot 4}{2\cdot 3} b_1 = b_0$

$b_3 = 0$

$b_4 = 0$

$$y_2 = b_0\left(1 + \frac{3}{2}x + x^2\right)$$

$R = \infty$

polinomio de orden 2

$b_5 =$ free

← recalculating the previous solution

$b_6 = \frac{3\cdot 8}{1\cdot 6} b_5 = 4 b_5$

$b_7 = \frac{4\cdot 9}{2\cdot 7} b_6 = \frac{72}{7} b_5$

$\vdots$

y_1 & y_2 are linearly independent. Are both solutions of (**). Then, the general solution has no logarithms.

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(3)
$$\begin{cases} u_t - u_{xx} = 0, & 0<x<\pi,\ t>0 \\ u(x,0)=0 \\ u(0,t)=1,\ u_x(\pi,t)=0 \end{cases}$$

$w = u - (ax+b)$, $w_x = u_x - a$

$0 = 1 - b$ $\boxed{\begin{matrix} b=1 \\ a=0 \end{matrix}}$

change: $\boxed{w = u - 1}$. with this change,

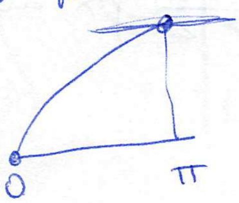
$$\begin{cases} w_t - w_{xx} = 0 \\ w(x,0) = -1 \\ w(0,t) = w_x(\pi,t) = 0 \end{cases}$$

try $w = X \cdot T$ with $\frac{T'}{T} = \frac{X''}{X} = -\lambda$

$$\begin{cases} X'' + \lambda X = 0 \\ X(0) = X'(\pi) = 0 \end{cases}, \quad \begin{cases} T' + \lambda T = 0 \\ \dots\dots \end{cases}$$

↓ prob de contorno (el importante) ↓ acompañante

λ's positives only.



$T = 4\pi$

$\omega = \frac{1}{2}$

$\{\sin \frac{n}{2} x\}$, $n = 1,2,3,\dots$ but not all

$\sin \frac{x}{2}$ yes

$\sin x$ no (does not satisfy $X'(\pi)=0$)

$\sin \frac{3x}{2}$ yes

$\sin 2x$ no

$X_n = \{\sin \frac{2n-1}{2} x\}$, $n = 1,2,3\dots$

$\lambda_n = \left(\frac{2n-1}{2}\right)^2$

Besides: $\int_0^\pi dx \, \sin\frac{2n-1}{2}x \, \sin\frac{2m-1}{2}x = \frac{\pi}{2}\delta_{nm}$

(orthogonality relation of the boundary problem)

$$\begin{cases} T_n' + \left(\frac{2n-1}{2}\right)^2 T_n = 0 \\ \dots \end{cases} \Rightarrow T_n = e^{-\left(\frac{2n-1}{2}\right)^2 t}$$

Fundamental solutions:

$$w_n = T_n X_n = e^{-\left(\frac{2n-1}{2}\right)^2 t} \sin\frac{2n-1}{2}x.$$

Our solution, a "linear combination of w_n's"

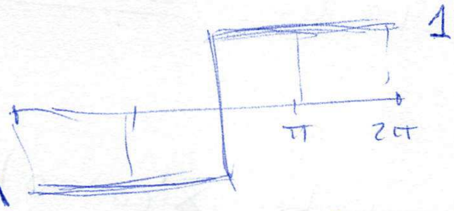
↑ serie here

$$\boxed{w = \sum_1^\infty b_n \, e^{-\left(\frac{2n-1}{2}\right)^2 t} \sin\frac{2n-1}{2}x}$$

with b_n the coefficients of

$$-1 = \sum_1^\infty b_n \sin\left(\frac{2n-1}{2}\right)x$$

I calculate b_n for $1 = \sum_1^\infty b_n \sin\left(\frac{2n-1}{2}\right)x$ and after change a sign.



use $\int_0^\pi dx \, \sin\frac{2n-1}{2}x \, \sin\frac{2m-1}{2}x = \frac{\pi}{2}\delta_{nm}$

to obtain

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$$b_n = \frac{2}{\pi}\int_0^{\pi} dx \ \sin\frac{2n-1}{2}x$$

$$= -\frac{2}{\pi}\cdot\frac{2}{2n-1}\left[\cos\frac{2n-1}{2}x\right]_0^{\pi}$$

$$= -\frac{4}{\pi(2n-1)}\left[0-1\right]$$

$$= \frac{4}{\pi(2n-1)} \ ,$$

then

$$1 = \sum_1^{\infty} \frac{4}{\pi(2n-1)} \sin\frac{2n-1}{2}x$$

and

$$u = w + 1$$
$$= 1 - \frac{4}{\pi}\sum_1^{\infty} \frac{e^{-\left(\frac{2n-1}{2}\right)^2 t}}{2n-1} \cdot \sin\frac{2n-1}{2}x$$

[observe that when $t\to\infty$, $u = 1$, as expected

u stationary
$\bar{u}'' = 0$
$\bar{u}(0) = 1$
$\bar{u}'(\pi) = 0$
$\Rightarrow \bar{u} = 1$

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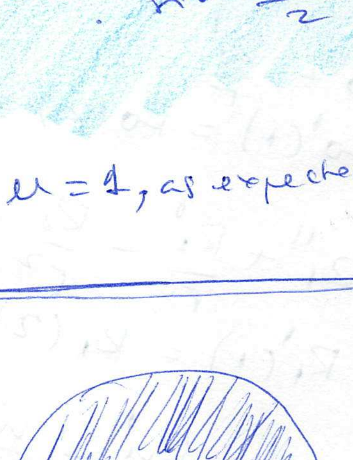
(4) $0<\theta<2\pi$

$$\begin{cases} \Delta u = \cos\theta \\ u_r(1,\theta) = u(2,\theta) = 0 \end{cases}$$



The equation is non homogeneous but the boundary conditions are periodic in the angle:

$u =$ " $R\Theta$ "

$$\begin{cases} \Theta'' + \lambda\Theta = 0 \\ \Theta(0) = \Theta(2\pi) \\ \Theta'(0) = \Theta'(2\pi) \end{cases} \Rightarrow \begin{array}{l} \lambda_0 = 0, \ \Theta_0 = 1 \\ \lambda_n = n^2 \ \{\sin n\theta, \cos n\theta\} \\ n = 1,2,3\ldots \end{array}$$

then

$$u = \underline{R_0(r)} + \sum_1^{\infty} \underline{R_n} \cos n\theta + \sum_1^{\infty} \underline{S_n} \sin n\theta$$

function of r — function of r

$$\Delta u = u_{rr} + \frac{u_r}{r} + \frac{u_{\theta\theta}}{r^2} = \cos\theta$$

we insert u as above in this equation to determine $R_0(r)$, $R_n(r)$, $S_n(r)$.

$$\Delta u = R_0'' + \frac{R_0'}{r} + \sum_1^{\infty}\left(R_n'' + \frac{R_n'}{r} - \frac{n^2 R_n}{r^2}\right)\cos n\theta + \sum_1^{\infty}\left(S_n'' + \frac{S_n'}{r} - \frac{n^2 S_n}{r^2}\right)\sin n\theta$$

$$= \cos\theta,$$

Then:

prob 1
$$\begin{cases} R_0'' + \dfrac{R_0'}{r} = 0 \\ R_0'(1) = R_0(2) = 0 \end{cases}$$

prob 2
$$\begin{cases} R_1'' + \dfrac{R_1'}{r} - \dfrac{R_1}{r^2} = 1 \\ R_1'(1) = R_1(2) = 0 \end{cases}$$

todos menos el $\underline{1}$

prob 3
$$\begin{cases} R_n'' + \dfrac{R_n'}{r} - \dfrac{n^2 R_n}{r^2} = 0 \\ R_n'(1) = R_n(2) = 0 \end{cases} \qquad n = 2, 3, 4, \ldots$$

$\underline{\text{Todos}}$

prob 4
$$\begin{cases} S_n'' + \dfrac{S_n'}{r} - \dfrac{n^2 S_n}{r^2} = 0 \\ S_n'(1) = S_n(2) = 0 \end{cases} \qquad n = 1, 2, 3, \ldots$$

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Let us solve probl. 2, 3 & 4:

prob 1: Solution (cl...): $R_0 = \underline{c_0} + \underline{d_0} \log r$ ← constants

$R_0'(1) = R_0(2) = 0 \Rightarrow c_0 = d_0 = 0 \Rightarrow \boxed{R_0 = 0}$

prob 2: $R_1 = c_1 r + \dfrac{d_1}{r} + \dfrac{r^2}{3}$ ← a ojo una solución particular de lo no homogénea

$R_1'(1) = R_1(2) = 0$

$\Rightarrow d_1 = 0$

$c_1 = -2/3$ $\Rightarrow \boxed{R_1 = -\frac{2}{3}r + \frac{r^2}{3}}$

prob 3: $R_n = c_2 r^n + \dfrac{d_2}{r^n}$

$R_n'(1) = R_n(2) = 0 \Rightarrow c_2 = d_2 = 0$ para todo $n = 2, 3, \ldots$

$\Rightarrow \boxed{\begin{array}{l} R_n = 0 \\ n = 2, 3 \ldots \end{array}}$

prob 4: Igual que prob 3

$\boxed{S_n = 0, \quad n = 1, 2, 3 \ldots}$

sólo queda esto...

Then, $\boxed{u = \left(\dfrac{r^2}{3} - \dfrac{2r}{3}\right)\cos\theta}$ $\boxed{\text{solución única}}$

Now I check if it is true:

$u(2,\theta) = 0$ ✓

$u_r(1,\theta) = \left(\dfrac{2r}{3} - \dfrac{2}{3}\right)\cos\theta$ at $r=1$: $= 0$ ok

$u_r = \left(\dfrac{2r}{3} - \dfrac{2}{3}\right)\cos\theta$

$u_{rr} = \dfrac{2}{3}\cos\theta$

$u_{\theta\theta} = -\left(\dfrac{r^2}{3} - \dfrac{2r}{3}\right)\cos\theta$

$\Delta u = u_{rr} + \dfrac{u_r}{r} + \dfrac{u_{\theta\theta}}{r^2}$

$= \left[\dfrac{2}{3} + \dfrac{2}{3} - \cancel{\dfrac{2}{3r}} - \dfrac{1}{3} + \cancel{\dfrac{2}{3r}}\right]\cos\theta$

$= \cos\theta$. Bingo!!

(5) $2xu_x - yu_y = u - xy^2$

$(u_x, u_y, -1)\cdot(2x, -y, u - xy^2) = 0$

characteristics equations: $\frac{dx}{2x} = -\frac{dy}{y} = \frac{du}{u-xy^2}$

$\left\{ \frac{dx}{2x} = -\frac{dy}{y} \right.$ or $\frac{dy}{dx} = -\frac{y}{2x}$; $y = c_1 x^{-1/2}$

or $\boxed{xy^2 = c_1}$ 1st characteristic

$-\frac{dy}{y} = \frac{du}{u-xy^2} = \frac{du}{u-c_1}$; this is $\frac{du}{dy} = -\frac{u}{y} + \frac{c_1}{y}$

solution: $u = \frac{c_2}{y} + c_1$

solution of the homogeneous equation $\left(\frac{du}{dy} = -\frac{u}{y}\right)$

particular solution of the non-homogeneous

thus $\boxed{u = \frac{c_2}{y} + xy^2}$ 2nd characteristic.

general solution $\boxed{u = \frac{1}{y} f(xy^2) + xy^2}$

Cauchy datum: $u(x,2) = 0$

$0 = \frac{1}{2} f(4x) + 4x$; $f(4x) = -8x$ or $f(x) = -2x$.

And, $\boxed{u = -2xy + xy^2}$ unique solution.

checking: $u_x = -2y + y^2$
$u_y = -2x + 2xy$

$2xu_x - yu_y - u + xy^2 = -4xy + \cancel{2xy^2} + \cancel{2xy} - \cancel{2xy^2} + \cancel{2xy} - \cancel{xy^2} + \cancel{xy^2}$

$\equiv 0$

OK

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