

MM2-Final - Septiembre 2014

UNIVERSIDAD COMPLUTENSE DE MADRID
FACULTAD DE CIENCIAS FÍSICAS

15 de Septiembre de 2014

Examen de Métodos Matemáticos II – Grupo D

Nombre y Apellidos:

DNI y firma:

Consejo muy útil: Siempre que sea razonable es conveniente comprobar los resultados.

1. [2 puntos] Sea la función $f(x) = \begin{cases} \sin x, & 0 < x < \pi, \\ 0, & \pi < x < 2\pi \end{cases}$. Una extensión periódica de la misma viene dada por

$$f(x)_{\text{ext}} = a + b \sin x - \frac{2}{\pi} \left(\frac{1}{1 \cdot 3} \cos 2x + \frac{1}{3 \cdot 5} \cos 4x + \frac{1}{5 \cdot 7} \cos 6x + \dots \right).$$

- a) Indicar cuál es el período de $f(x)_{\text{ext}}$ y dibujar al menos tres períodos de la misma. b) Calcular las constantes a y b . c) Deducir, mediante la *identidad de Parseval* la suma de la serie

$$\frac{1}{1^2 \cdot 3^2} + \frac{1}{3^2 \cdot 5^2} + \frac{1}{5^2 \cdot 7^2} + \dots$$

2. [2 puntos] Sea la ecuación

$$x^2 y'' + x^3 y' - 2(1+x^2)y = 0.$$

- a) Escribir el polinomio indicial en $x = 0$ y calcular sus raíces. b) Encontrar una solución analítica no trivial en dicho punto. c) ¿Son todas las soluciones analíticas en $x = 0$?

3. [2 puntos] Resolver por separación de variables

$$\begin{cases} u_{tt} - u_{xx} = t \sin x, & 0 < x < \pi, \quad t > 0 \\ u(x, 0) = u_t(x, 0) = 0 \\ u(0, t) = u(\pi, t) = 0. \end{cases}$$

4. [2 puntos] Calcular por separación de variables la única solución acotada del problema del plano que cumple

$$\begin{cases} \Delta u = 0, & r < 1, \quad -\pi/4 < \theta < \pi/4, \\ u_r(1, \theta) = f(\theta), \\ u(r, -\pi/4) = u(r, \pi/4) = 0. \end{cases}$$

cuando i) $f(\theta) = 2 \cos 2\theta$, ii) $f(\theta) = 1$

5. [2 puntos] Sea la ecuación

$$xu_x + yu_y = 2u.$$

Determinar la solución general por el método de las características y i) la solución que cumple el dato $u(x, 1-x) = 2x-1$, ii) ¿cuántas soluciones de la ecuación cumplen $u(x, x) = 0$?

Prob continuo de ④: otra manera.

$$\begin{cases} \theta'' + \lambda \theta = 0 \\ \theta(-\frac{\pi}{4}) = \theta(\frac{\pi}{4}) = 0 \end{cases}$$

$$\lambda = \omega^2: \quad \theta = A \sin \omega \theta + B \cos \omega \theta$$

when $\theta = -\frac{\pi}{4}, \frac{\pi}{4}$:

$$\begin{cases} 0 = A \sin \omega \frac{\pi}{4} + B \cos \omega \frac{\pi}{4} \\ 0 = -A \sin \omega \frac{\pi}{4} + B \cos \omega \frac{\pi}{4} \end{cases}$$

sont no-trivial iff $\det = 0$:

$$\det = 0 \quad \begin{cases} \sin \omega \frac{\pi}{4} = 0 \quad \text{or} \dots \\ \cos \omega \frac{\pi}{4} = 0 \quad \text{or} \dots \end{cases}$$

$\sin \omega \frac{\pi}{4} = 0$ if $\omega = \frac{4}{\pi} [0, \pi, 2\pi, 3\pi, 4\pi, \dots]$
 $= 4, 8, 12, 16, \dots$
 $= 2 \cdot 2n \quad \text{and } n=1, 2, 3, \dots$

$\cos \omega \frac{\pi}{4} = 0$ if $\omega = \frac{4}{\pi} [\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \dots]$
 $= 2, 6, 10, 14, 18, \dots$

L
 $= 2(2n-1) \quad \text{and } n=1, 2, 3, \dots$

Fijaciones:

$$g_n = \{ \sin 4n\theta \}, n=1, 2, 3, \dots \quad \text{when } \lambda_n = (2 \cdot 2n)^2$$

$$h_n = \{ \cos 2(2n-1)\theta \}, n=1, 2, 3, \dots \quad \text{when } \lambda_n = [2(2n-1)]^2$$

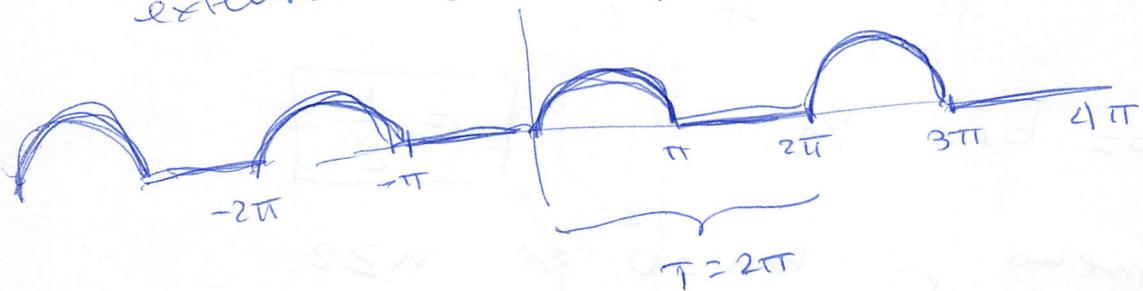
No hay más se fijó... ¡otra vez sin piezas
 papeles atrapados!!

[Finalizar gráficos anteriores.]

$$\text{① } f_{\text{ext}}(x) = a + b \sin x - \frac{2}{\pi} \left[\frac{\cos 2x}{1 \cdot 3} + \frac{\cos 4x}{3 \cdot 5} + \frac{\cos 6x}{5 \cdot 7} + \dots \right]$$

a) Just looking at $\omega = 1, T = 2\pi.$ \boxed{x}

This means that $f_{\text{ext}}(x)$ is the periodic extension from 0 to π



Se supone que $b \neq 0.$ Si paridad en forma $x=0 \rightarrow$
el a será (b paridad en forma $x=0 \rightarrow$
al ser a ceros ... o sea, $a \neq 0.$ In
several $\omega = \frac{a_0}{2} + \sum [a_n \cos nx + b_n \sin nx],$

$$f_{\text{ext}} = \frac{a_0}{2} + \sum [a_n \cos nx + b_n \sin nx],$$

a is $\frac{a_0}{2}, b = b_1$ and probably
[certainly!!!] each $b_n, n \geq 2$ is zero.

Calculation of $a:$

\boxed{x} Si b fuera 0 [que no es] se podría pensar
que $\omega = 2,$ then $T = \pi \dots$ pero con

$$f(x) = \begin{cases} \sin x & 0 \leq x < \pi \\ 0 & \pi \leq x < 2\pi \end{cases}$$

we see $\omega = 1$ si o si.

esos esos esos

$$a = \frac{a_0}{2} = \frac{1}{2\pi} \int_0^{2\pi} dx f(x)$$

$$= \frac{1}{2\pi} \int_0^{\pi} dx \sin x = -\frac{1}{2\pi} [\cos x]_0^{\pi} = \frac{1}{\pi}$$

$$\boxed{a = \frac{1}{\pi}}$$

$$\frac{b_1}{2} = \frac{1}{2\pi} \int_0^{2\pi} R(x) \cdot \sin x = \frac{1}{2\pi} \int_0^{\pi} dx \sin^2 x$$

$$= \frac{1}{2\pi} \int_0^{\pi} dx \frac{1}{2} [1 - \cos 2x]$$

$$= \frac{1}{4\pi} \cdot \pi = \frac{1}{4}$$

Integrando de 0 a $\frac{\pi}{2}$

$$b = b_1 = \frac{1}{2}$$

$$\boxed{b = \frac{1}{2}}$$

Comprobemos que $b_n = 0$ si $n \geq 2$:

$$\begin{aligned} \frac{b_n}{2} &= \frac{1}{2\pi} \int_0^{\pi} dx \sin x \cdot \sin nx \\ &= \frac{1}{2\pi} \int_0^{\pi} dx \frac{1}{2} \left[\cos \left(\frac{n-1}{n+1} x \right) - \cos \left((n+1)x \right) \right], \quad n=2, 3, \dots \\ &= \frac{1}{4\pi} \left[\frac{\sin(n-1)x}{n-1} - \frac{\sin((n+1)x)}{n+1} \right]_0^{\pi} \\ &= \frac{1}{4} [0 - 0] = 0 \end{aligned}$$

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Then

$$f_{\text{exact}}(x) = \frac{1}{\pi} + \frac{1}{2} \sin x - \frac{2}{\pi} \left(\frac{\cos x}{1 \cdot 3} + \frac{x \cos x}{3 \cdot 5} + \frac{x^2 \cos x}{5 \cdot 7} + \dots \right)$$

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c) Parseval's identity. Start with first and calculate

$$f_{ext}^2 = \frac{1}{\pi^2} + \frac{1}{4} \sin^2 x + \frac{4}{\pi^2} \left[\frac{\omega^2 2x}{1^2 \cdot 3^2} + \frac{\omega^2 4x}{3^2 \cdot 5^2} + \frac{\omega^2 6x}{5^2 \cdot 7^2} + \dots \right]$$

+ double products.

Integrate in a period:

$$\begin{aligned} \int_0^{2\pi} dx f_{ext}^2 &= \frac{1}{\pi^2} 2\pi + \frac{1}{4} \int_0^{2\pi} \sin^2 x dx \\ &\quad + \frac{4}{\pi^2} \left[\frac{1}{1^2 \cdot 3^2} \int_0^{2\pi} dx \omega^2 2x \right. \\ &\quad \left. + \frac{1}{3^2 \cdot 5^2} \int_0^{2\pi} dx \omega^2 4x + \dots \right]. \end{aligned}$$

Notice that

$$\int_0^{2\pi} dx f_{ext}^2 = \int_0^{\pi} dx \sin^2 x = \int_0^{\pi} \frac{dx}{2} (1 - \omega^2 2x) = \frac{\pi}{2}$$

$$\int_0^{2\pi} \sin^2 x dx = \int_0^{2\pi} \frac{dx}{2} (1 - \omega^2 2x) = \pi.$$

$$\int_0^{2\pi} dx \omega^2 2x = \int_0^{\pi} dx \omega^2 4x = \int_0^{\pi} dx \omega^2 6x = \dots$$

$$= \pi$$

$$\omega^2 2x = \frac{\pi}{2} + \frac{1}{2} \omega^2 4x \dots \text{ signal concave down...}$$

so we have [⊗ need]

$$\frac{\pi^2}{4} = \frac{\pi^2}{2} = \frac{\pi}{4} + \frac{\pi}{4} + \frac{4}{\pi^2} \pi \left(\underbrace{\frac{1}{1^2 \cdot 3^2} + \frac{1}{3^2 \cdot 5^2} + \frac{1}{5^2 \cdot 7^2} + \dots}_{\text{series periodic.}} \right)$$

$$\Rightarrow \frac{1}{1^2 \cdot 3^2} + \frac{1}{3^2 \cdot 5^2} + \frac{1}{5^2 \cdot 7^2} + \dots = \frac{\pi}{4} \left(\frac{\pi}{2} - \frac{2}{\pi} - \frac{\pi}{4} \right) = \frac{\pi^2}{16} - \frac{1}{2}.$$

Result :
$$\frac{1}{1^2 \cdot 3^2} + \frac{1}{3^2 \cdot 5^2} + \frac{1}{5^2 \cdot 7^2} + \dots = \frac{\pi^2}{16} - \frac{1}{2}$$

as can be checked with Maple, or any book.

(2) $x^2 y'' + x^3 y' - 2[1+x^2]y = 0 \quad (*)$

$x=0$ is a regular singular point of (*). Around $x=0$ the Euler's equation of (*) is

$$x^2 y'' + 0 - 2y = 0$$

[Notice $x^2 y'' + x[\underbrace{x^3}_0] y' - 2[\underbrace{1+x^2}_1] y = 0$

The indicial polynomial is

$$r(r-1) - 2 = 0 \quad \text{or} \quad (r-2)(r+1) = 0$$

with solutions

$$\begin{cases} r=2 \\ r=-1 \end{cases}$$

thus, around $x=0$, the equation (*) behaves as

$$y \sim c_1 x^2 + \frac{c_2}{x} \quad \text{is analytic at } x=0$$

We study the analytic solution ($r=2$):

$$y = \sum_{n=0}^{\infty} a_n x^{n+2} \quad [a_0 \neq 0]$$

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$$\gamma' = \sum_0^{\infty} (n+2) a_n x^{n+1}$$

$$\gamma'' = \sum_0^{\infty} (n+2)(n+1) a_n x^n.$$

$$0 = x^2 \gamma'' + x^3 \gamma' - 2(1+x^2)$$

$$= \sum_0^{\infty} x^{n+2} \left[(n+2)(n+1) a_n - 2a_n \right] = n(n+3)a_n$$

$$+ \sum_0^{\infty} x^{n+4} \left[(n+4)a_n - 2a_n \right]$$

$$\Rightarrow 0 = \sum_0^{\infty} n(n+3)a_n x^{n+2} + \sum_0^{\infty} n a_n x^{n+4}$$

$$= x^2 [0 \cdot 3 a_0]$$

$$+ x^3 [1 \cdot 4 a_1]$$

$$+ x^4 [2 \cdot 5 \cdot a_2 + 0 \cdot a_0]$$

$$+ x^5 [3 \cdot 6 \cdot a_3 + 1 \cdot a_1]$$

$$+ x^6 [4 \cdot 7 \cdot a_4 + 2 \cdot a_2]$$

$$+ \dots +$$

$$+ x^{n+4} [(n+2)(n+5)a_{n+2} + n a_n] + \dots$$

from here : $a_{n+2} = \frac{-n a_n}{(n+2)(n+3)}, n=0, 1, 2, \dots$

also $a_1 = 0$ [from]

thus:

a_0 = free (siempre multiplo de por 0, es libre)

$$a_1 = 0$$

$$a_2 = 0$$

$$a_3 = -\frac{a_1}{3 \cdot 6} = 0$$

$$a_4 = -\frac{2}{4 \cdot 9} a_2 = 0$$

$$a_5 = (-\dots) a_3 = 0$$

solution:

$$\boxed{\gamma = a_0 x^2}$$

(la solucion general es
 $\gamma = x^2, \forall a_0 \neq 0$)

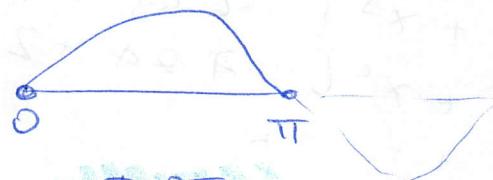
$$[check \rightarrow x^2 \frac{\partial f}{\partial x} + x^3 \frac{\partial f}{\partial x} - 2f(x) = 0]$$

There are solutions... we go to page ...

$$(3) \begin{cases} u_{tt} - u_{xx} = t \sin x & 0 \leq x \leq \pi, t \in \mathbb{R} \\ u(x, 0) = u_t(x, 0) = 0 \\ u(0, t) = u(\pi, t) = 0 \end{cases}$$

$$u = X \cdot T$$

$$\begin{cases} X'' + \lambda X = 0 \\ X(0) = X(\pi) \end{cases}$$



$$T = 2\pi, \omega = 1, \text{ sinn } n \cdot x, n = 1, 2, 3, \dots$$

Prob
converges

$$u = \sum_{n=1}^{\infty} T_n(t) \sin nx, \quad \text{undetermined coefficients. Use } u_{tt} - u_{xx} = t \sin x \text{ to find them.}$$

$$\sum_{n=1}^{\infty} (T_n'' + n^2 T_n) \sin nx = t \sin x,$$

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gives (compose wefficients of sin x)

$$\begin{cases} T_1'' + T_1 = t \\ T_1(0) = T_1'(0) = 0 \end{cases}; \quad \begin{cases} T_n'' + n^2 T_n = 0 \\ T_n(0) = T_n'(0) = 0 \end{cases}$$

$n \geq 2$

solution:



\downarrow
a ojo...



$$T_1 = A \cos t + B \sin t + t$$

$$T_1' = -A \sin t + B \cos t + 1$$

$$B = -1 \text{ para } T_1'(0) = 0$$

$$A = 0 \text{ para } T_1(0) = 0$$

or

$$T_1 = t - \sin t$$

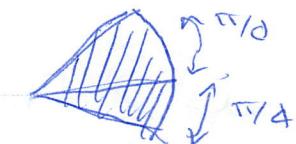
solution: $T_1 = t - \sin t$
+ D asut
per aules condicions

$$\boxed{T_1 \equiv 0}$$

solution: $u = (t - \sin t) \sin x$

Unigue.

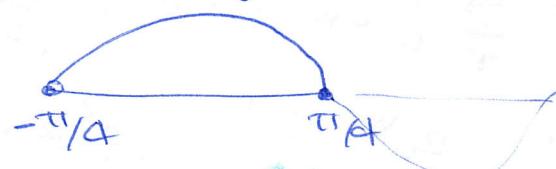
④ $\begin{cases} u_r = 0 & r < 1 \\ u_r(r, \theta) = f(\theta) \\ u_r(r, -\frac{\pi}{4}) = u_r(r, \frac{\pi}{4}) = 0 \end{cases}$



$$u = R \cdot \theta$$

prob contorno: $\begin{cases} \theta'' + \lambda \theta \\ \theta(-\frac{\pi}{4}) = \theta(\frac{\pi}{4}) = 0 \end{cases}$

b 1º forma no eix de simetria.



$$\Theta_n = \left\{ \sin 2n \left(\theta + \frac{\pi}{4} \right) \right\}, \quad n=1,2,3,\dots$$

solucion del problema de contorno

Ver deixa de enunciado sobre "este problema de contorno".

Also

$$\int_{-\pi/4}^{\pi/4} \sin 2m(\theta + \frac{\pi}{4}) \cdot \sin 2m(\theta + \frac{\pi}{4}) = 0 \quad \text{if } m \neq 0$$

$m, n = 1, 2, 3, \dots$

and

$$\int_{-\pi/4}^{\pi/4} d\theta \sin^2 2m(\theta + \frac{\pi}{4}) = \int_{-\pi/4}^{\pi/4} \frac{1}{2} \theta (1 - \cos 4m(\theta + \frac{\pi}{4}))$$

\uparrow
zero $m = m$

0 the integral

$$= \frac{1}{2} [\frac{\pi}{4} + \frac{\pi}{4}] = \frac{\pi}{4}$$

orthogonality relations

$$: \frac{1}{4} \left[\sin 4m(\theta + \frac{\pi}{4}) \right]_{-\pi/4}^{\pi/4}$$

These are some power functions.

$$\int_{-\pi/4}^{\pi/4} d\theta \sin 2m(\theta + \frac{\pi}{4}) \sin 2m(\theta + \frac{\pi}{4})$$

$$= \frac{\pi}{4} \delta_{mm}. \quad m, n = 1, 2, \dots$$

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Up to now we have met

$$u = \sum_1^\infty R_m(r) \sin 2m(\theta + \frac{\pi}{4}),$$

then introduced in $\Delta u = 0$ gives

$$0 = \sum_1^\infty \left[R_m'' + \frac{R_m'}{r} - \frac{4u^2}{r^2} R_m \right] \sin 2m(\theta + \frac{\pi}{4})$$

$$\text{or } R_m'' + \frac{R_m'}{r} - \frac{4u^2}{r^2} R_m = 0, \quad m = 1, 2, \dots$$

$$\text{or } R_m = C_1 r^{2m} + \frac{C_2}{r^{2m}},$$

and

$$u = \sum_1^\infty \left(C_1 r^{2m} + \frac{C_2}{r^{2m}} \right) \sin 2m(\theta + \frac{\pi}{4})$$

No da anterior
en $r \rightarrow 0$, $u \rightarrow 0$, a
mas: $C_2 = 0$ todo.

No os perdais ... la solución del problema es

$$\left\{ \begin{array}{l} u_{rr}=0 \\ u_r=0 \\ u(r_1-\frac{\pi}{4})=u(r_1+\frac{\pi}{4})=0 \end{array} \right. \quad \text{usado}$$

$$u = \sum_{n=1}^{\infty} 2m r^{2n} \sin 2m \left(\theta + \frac{\pi}{4} \right)$$

u acoste de
en r=0

Falta imponer $u_r(1,0) = f(0)$.

$$u_r = \sum_{n=1}^{\infty} 2m n a_n r^{2n-1} \sin 2n \left(\theta + \frac{\pi}{4} \right)$$

s.t.

$$f(0) = \sum_{n=1}^{\infty} 2m a_n \sin 2n \left(0 + \frac{\pi}{4} \right)$$

Now: i) $f(\theta) = 2 \cos 2\theta$, ii) $f(\theta) = 1$
esq

i) $2 \cos 2\theta = \sum_{n=1}^{\infty} 2m a_n \sin 2n \left(\theta + \frac{\pi}{4} \right)$

$n=1$: $\sin 2(\theta + \frac{\pi}{4}) = \sin(2\theta + \frac{\pi}{2}) = \cos 2\theta$

$n=2$: $\sin 4(\theta + \frac{\pi}{4}) = \sin(4\theta + \pi) = -\sin 4\theta$

$n=3$: $\sin 6(\theta + \frac{\pi}{4}) = \sin(6\theta + \frac{3\pi}{2}) = -\cos 6\theta$

$n=4$: $\sin 8(\theta + \frac{\pi}{4}) = \sin(8\theta + 2\pi) = \sin 8\theta$

$n=5$: $\sin 10(\theta + \frac{\pi}{4}) = \sin(10\theta + 5\pi) = -\sin 10\theta$

$$\Rightarrow 2 \cos 2\theta = 2a_1 \cos 2\theta - 4a_2 \sin 4\theta - 6a_3 \cos 6\theta$$

segun senos
que son...
que son perpendiculares
entre si

or $a_2 = a_3 = \dots = 0$, $a_1 = 1$

Solución: $u = r^2 \cos 2\theta = r^2 (\omega^2 \theta - r^2 \theta) = |x^2 - y^2|$

Se puede componer $f(\theta) = r^2 \cos 2\theta$ es la suma
 (es la síntesis o suma, ademas) de $\begin{cases} u=0 \\ r=1 \end{cases}$
 $u(r, \theta) = 2 \cos 2\theta$
 $u(1, -\frac{\pi}{4}) u(1, \frac{\pi}{4}) = 0.$

(ii) $f(\theta) = 1$

fuera una parte constante de $\{ \cos 2\theta, -\sin 4\theta, -\cos 6\theta, \sin 8\theta, \dots \}$

Hay que de sencillar el serie para ser única.

$$1 = \sum_{m=0}^{\infty} 2m \underline{a_m} \sin 2m(\theta + \frac{\pi}{4})$$

multiplique en $\sin 2m(\theta + \frac{\pi}{4})$:

$$\sin 2m(\theta + \frac{\pi}{4}) = \sum_{n=0}^{\infty} 2m a_m \sin 2m(\theta + \frac{\pi}{4}) \sin 2n(\theta + \frac{\pi}{4})$$

integre en $[-\frac{\pi}{4}, \frac{\pi}{4}]$, use la ortogonalidad
 sección add obtener.

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} d\theta \sin 2m(\theta + \frac{\pi}{4}) = 2m \cdot \frac{\pi}{4} a_m,$$

$$\Rightarrow m a_m \cdot \frac{\pi}{2} = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} d\theta \sin 2m(\theta + \frac{\pi}{4})$$

$$= -\frac{1}{2m} [\cos 2m(\theta + \frac{\pi}{4})]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= -\frac{1}{2m} [\cos \pi - 1]$$

$$= -\frac{1}{2m} [(-1)^m - 1]$$

así se vece a_m
 (o sea como usted
 lo dice)

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$$\begin{cases} a_n = 0 & , n = 2, 4, 6, \dots \\ a_n = \frac{2}{\pi n^2} & , n = 1, 3, 5, \dots \end{cases}$$

then

$$u = \sum_{n=1}^{\infty} \frac{2}{\pi} \frac{\sin 2(2n-1)(\theta + \frac{\pi}{4})}{(2n-1)^2}$$

The solution is unique. $\left[: r^{4n-2}, \sin(4n-2)(\theta + \frac{\pi}{4}) = r^{2(n-1)}(2\theta + \frac{\pi}{2}) \right]$

$$(5) \quad xux + u_1 u_2 = 2u \quad \rightarrow \text{no note el centro } x \text{ por } .$$

$$(x_1, u_1, 2u) (u_x, u_1, -1) = 0$$

$$\begin{cases} \frac{dx}{x} = \frac{du}{u} = \frac{du}{2u} \\ \frac{du}{dx} = \frac{u}{x} : \quad u = C_1 x \\ \frac{du}{dx} = \frac{2u}{x} \quad , \quad u = C_2 x^2 \end{cases}$$

$$\text{General soluci: } u = x^2 f\left(\frac{u}{x}\right).$$

En la des. anterior
se de falte.

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$$\begin{cases} u = 1-x \\ u = 2x-1 \end{cases}$$

$$2x-1 = x^2 f\left(\frac{1}{x}-1\right)$$

$$\frac{2}{x} - \frac{1}{x^2} = f\left(\frac{1}{x}-1\right).$$

$$2z - z^2 = f(z-1)$$

$$z = 1/x:$$

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Now change $z \rightarrow z+1$

$$\begin{aligned}2(z+1) - (z+1)^2 &= (z+1)[2 - (z+1)] \\&= (z+1)(1-z) \\&= 1 - z^2 \\&= f(z).\end{aligned}$$

$$f(z) = 1 - z^2 \quad \text{and} \quad u = x^2 [1 - (\frac{y}{x})^2] = x^2 - y^2$$

$$u = x^2 - y^2$$

(simple todo, es que es genial)

$$(ii) u(x, x) = 0$$

$$0 = x^2 f(1) \quad " \quad f(1) = 0 \quad \text{to simple infinite possibilities.}$$

Una vez son:

$$f(x) = 0 \quad [\text{la identidad } 0]$$

$$f(x) = (x-1)$$

$$f(x) = (x-1)^2$$

$$f(x) = (x-1)x$$

:

etc...



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ARTES	INSTITUTO DE INVESTIGACIONES	1993
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