**Table 4.4:** The first few Laguerre polynomials, $L_q(x)$.

| |
|---|
| $L_0 = 1$ |
| $L_1 = -x + 1$ |
| $L_2 = x^2 - 4x + 2$ |
| $L_3 = -x^3 + 9x^2 - 18x + 6$ |
| $L_4 = x^4 - 16x^3 + 72x^2 - 96x + 24$ |
| $L_5 = -x^5 + 25x^4 - 200x^3 + 600x^2 - 600x + 120$ |
| $L_6 = x^6 - 36x^5 + 450x^4 - 2400x^3 + 5400x^2 - 4320x + 720$ |

**Table 4.5:** Some associated Laguerre polynomials, $L_{q-p}^p(x)$.

| | |
|---|---|
| $L_0^0 = 1$ | $L_0^2 = 2$ |
| $L_1^0 = -x + 1$ | $L_1^2 = -6x + 18$ |
| $L_2^0 = x^2 - 4x + 2$ | $L_2^2 = 12x^2 - 96x + 144$ |
| $L_0^1 = 1$ | $L_0^3 = 6$ |
| $L_1^1 = -2x + 4$ | $L_1^3 = -24x + 96$ |
| $L_2^1 = 3x^2 - 18x + 18$ | $L_2^3 = 60x^2 - 600x + 1200$ |

They are not pretty, but don't complain—this is one of the very few realistic systems that can be solved at all, in exact closed form. As we will prove later on, they are mutually orthogonal:

$$\int \psi_{nlm}^* \, \psi_{n'l'm'} \, r^2 \sin\theta \, dr \, d\theta \, d\phi = \delta_{nn'}\delta_{ll'}\delta_{mm'}. \qquad [4.90]$$

---

***Problem 4.10** Work out the radial wave functions $R_{30}$, $R_{31}$, and $R_{32}$, using the recursion formula (Equation 4.76). Don't bother to normalize them.

---

***Problem 4.11**

**(a)** Normalize $R_{20}$ (Equation 4.82), and construct the function $\psi_{200}$.

**(b)** Normalize $R_{21}$ (Equation 4.83), and construct $\psi_{211}$, $\psi_{210}$, and $\psi_{21-1}$.

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****Problem 4.12**

**(a)** Using Equation 4.88, work out the first four Laguerre polynomials.

**(b)** Using Equations 4.86, 4.87, and 4.88, find $v(\rho)$ for the case $n = 5$, $l = 2$.

**Table 4.6:** The first few radial wave functions for hydrogen, $R_{nl}(r)$.

$$R_{10} = 2a^{-3/2}\exp(-r/a)$$

$$R_{20} = \frac{1}{\sqrt{2}}a^{-3/2}\left(1 - \frac{1}{2}\frac{r}{a}\right)\exp(-r/2a)$$

$$R_{21} = \frac{1}{\sqrt{24}}a^{-3/2}\frac{r}{a}\exp(-r/2a)$$

$$R_{30} = \frac{2}{\sqrt{27}}a^{-3/2}\left(1 - \frac{2}{3}\frac{r}{a} + \frac{2}{27}\left(\frac{r}{a}\right)^2\right)\exp(-r/3a)$$

$$R_{31} = \frac{8}{27\sqrt{6}}a^{-3/2}\left(1 - \frac{1}{6}\frac{r}{a}\right)\left(\frac{r}{a}\right)\exp(-r/3a)$$

$$R_{32} = \frac{4}{81\sqrt{30}}a^{-3/2}\left(\frac{r}{a}\right)^2\exp(-r/3a)$$

$$R_{40} = \frac{1}{4}a^{-3/2}\left(1 - \frac{3}{4}\frac{r}{a} + \frac{1}{8}\left(\frac{r}{a}\right)^2 - \frac{1}{192}\left(\frac{r}{a}\right)^3\right)\exp(-r/4a)$$

$$R_{41} = \frac{\sqrt{5}}{16\sqrt{3}}a^{-3/2}\left(1 - \frac{1}{4}\frac{r}{a} + \frac{1}{80}\left(\frac{r}{a}\right)^2\right)\frac{r}{a}\exp(-r/4a)$$

$$R_{42} = \frac{1}{64\sqrt{5}}a^{-3/2}\left(1 - \frac{1}{12}\frac{r}{a}\right)\left(\frac{r}{a}\right)^2\exp(-r/4a)$$

$$R_{43} = \frac{1}{768\sqrt{35}}a^{-3/2}\left(\frac{r}{a}\right)^3\exp(-r/4a)$$

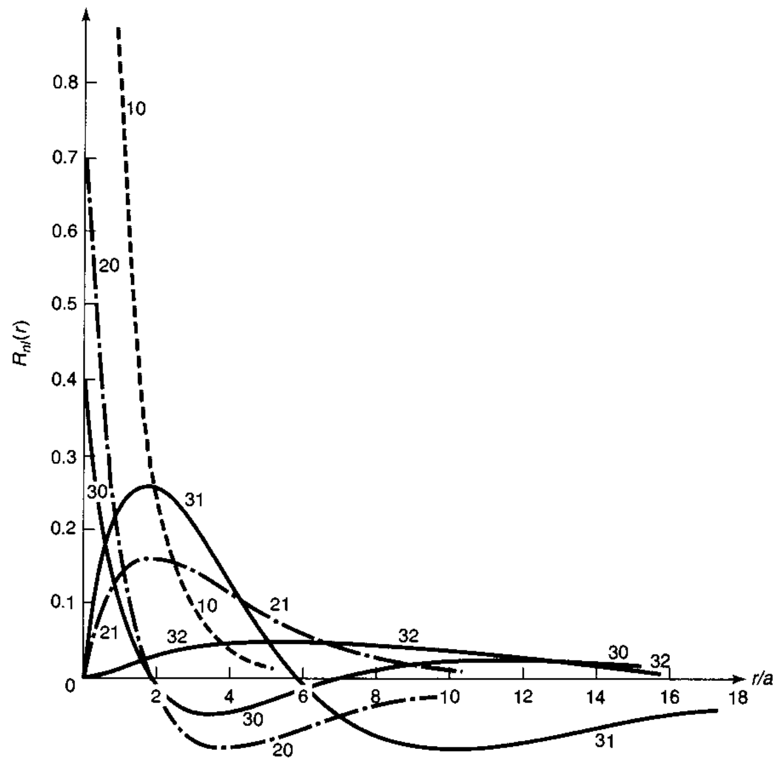


**Figure 4.4:** Graphs of the first few hydrogen radial wave functions, $R_{nl}(r)$.