Helmholtz potential study of the Diluted Antiferromagnet in a Field

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The DAFF

- Diluted Anti-Ferromagnet in a Field (DAFF): standard experimental model for ordered phases competition in condensed mater physics, in the presence of quenched disorder.

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$$s_i = \pm 1, \quad P(\epsilon_i) = p\delta(\epsilon_i - 1) + (1-p)\delta(\epsilon_i).$$

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- Very difficult to study numerically:
  - Severe critical slowing down: $\log \tau \propto \xi^{\theta}$.
  - Self-averaging violations $\longleftrightarrow$ averages dominated by rare events.

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$$\hat{b} = 1 - \frac{1/2 - 1/N}{\hat{m} - m}, \qquad \hat{b}_{\text{s}} = 1 - \frac{1/2 - 1/N}{\hat{m}_{\text{s}} - m_{\text{s}}}$$

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- The Helmholtz potential Ω can be reconstructed from the tethered magnetic field as a function of $(\hat{m}, \hat{m}_s)$

$$\nabla\Omega(\hat{m}, \hat{m}_s) = \left(\frac{\partial\Omega}{\partial\hat{m}}, \frac{\partial\Omega}{\partial\hat{m}_s}\right) = \left(\overline{\langle\hat{b}\rangle}_{\hat{m},\hat{m}_s}, \overline{\langle\hat{b}_s\rangle}_{\hat{m},\hat{m}_s}\right)$$

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   - Easier: saddle-point equations (ensemble equivalence)

$$Z(h) = \int \mathrm{d}\hat{m}\mathrm{d}\hat{m}_s \, \mathrm{e}^{-N[\Omega(\hat{m},\hat{m}_s)-\beta h\hat{m}]}$$

$$\nabla[\Omega(\hat{m}, \hat{m}_s) - \beta h\hat{m}] = 0 \quad \Longrightarrow \quad \begin{cases} \overline{\langle b\rangle}_{\hat{m}^*,\hat{m}_s^*} = \beta h \\ \overline{\langle b_s\rangle}_{\hat{m}^*,\hat{m}_s^*} = 0 \end{cases}$$

$$\lim_{N\to\infty} \overline{\langle O\rangle}(h) = \lim_{N\to\infty} \overline{\langle O\rangle}_{\hat{m}^*(h),\hat{m}_s^*(h)}$$

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- Much faster equilibration than canonical parallel tempering (no tunnelling between metastable states)
- Restores self-averaging (we average Ω rather than F).

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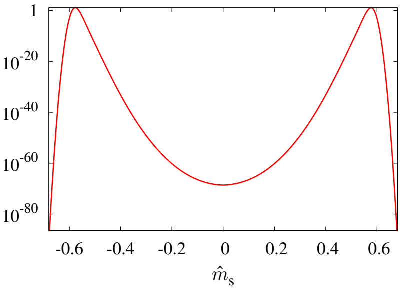

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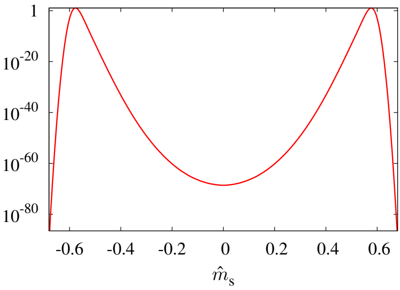

- Large tunnelling barrier between the positive and negative peak,
- No peak at $\hat{m}_s = 0$
- Consistent with a second-order transition
- Explains metastability in canonical simulations (jumps between two ordered phases).

Scale invariance

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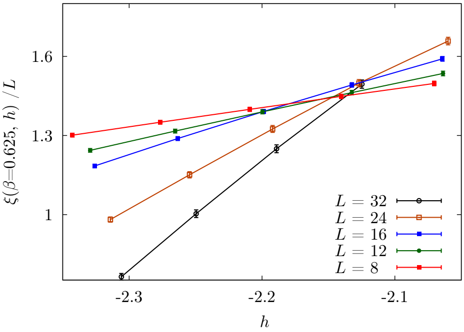
- We plot ξ/L against the applied magnetic field for fixed $T = 1.6$



$$\partial_h \xi \propto L^{1+1/\nu} \quad \Longrightarrow \quad \nu = 0.90(15)$$

$$\overline{\langle m_s^2 \rangle} \propto L^{-2\beta/\nu} \quad \Longrightarrow \quad \beta/\nu = 0.0102(6)$$

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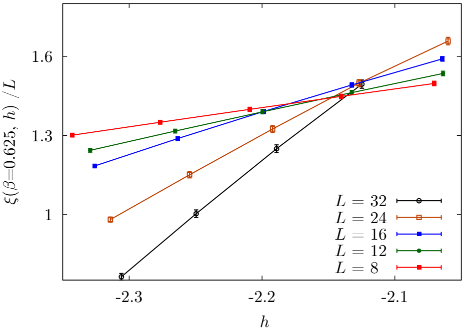
- We can use standard Finite-Size Scaling methods to compute β, ν:



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- Canonical ensemble: ΔF between the ordered and disordered phase goes as $\Delta F \propto L^{\theta - D}$ (Vink et al. PRE **82** 051134, 2010)

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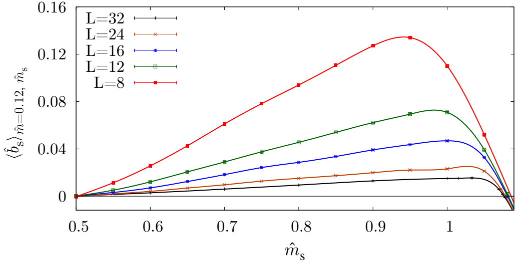

$$\partial\Omega / \partial\hat{m}_s = \overline{\langle b_s \rangle}_{\hat{m},\hat{m}_s}$$

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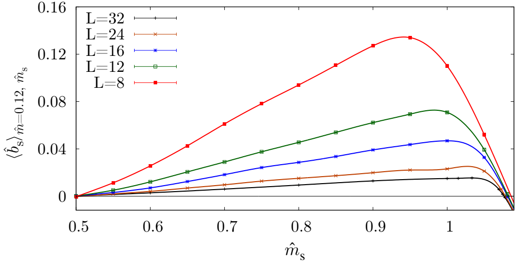

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- $\theta < D - 1 \implies$ the metastable states do not define stable phases

Conclusions

- Using the tethered formalism, we have studied the critical behaviour of the DAFF
- Our Tethered Monte Carlo method
  - Is capable of handling rugged free-energy landscapes
  - Restores self-averaging
- We observe clear signs of a second-order phase transition
- We obtain the three independent critical exponents, including the hyperscaling violations exponent
- We observe a (slow) divergence of the specific heat, consistent with experiments (not shown in this talk).
- Our results, $\theta = 1.47(2)$, $\beta = 0.0102(6)$, are consistent with the analytical conjecture (Schwartz and Soffer 1986)

$$\theta = D/2 - 2\beta/\nu \approx 1.48$$