Off-equilibrium spin glass dynamics with the Janus computer

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Introduction

Spin glasses: experiments vs. simulations

- Experiments focus on off-equilibrium dynamics.
- The system is rapidly cooled to a subcritical temperature, $T < T_c$, let to equilibrate a time t_w and probed at $t + t_w$.
- The evolution of the coherence length is very slow at $T < T_c$.

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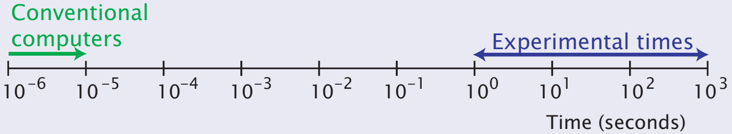

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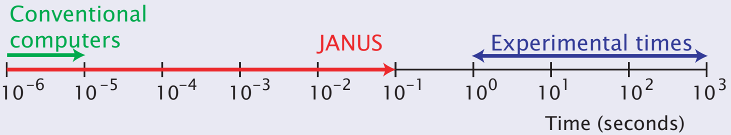

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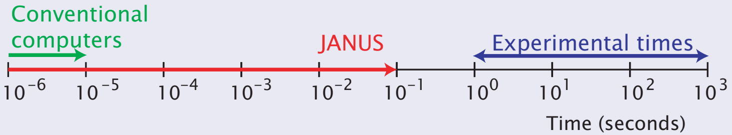
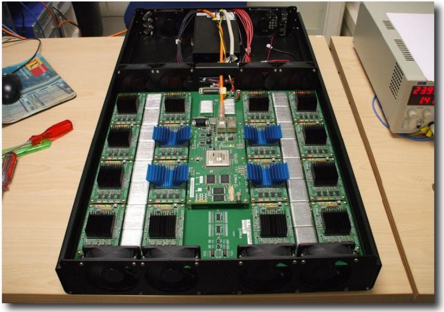


The Janus computer

- Janus is a custom built computing system:
  - Massively parallel
  - Made of FPGAs
  - Made of modules
- Designed with spin glasses in mind, but reconfigurable.

Our simulations



The model and our parameters

- $\mathcal{H} = -\sum_{\langle \mathbf{x},\mathbf{y} \rangle} J_{\mathbf{xy}} \sigma_{\mathbf{x}} \sigma_{\mathbf{y}}, \quad P(J_{\mathbf{xy}}) = \delta(J_{\mathbf{xy}}^2 - 1).$
- $L = 80$ systems for ~ 100 samples ($T = T_c, 0.8, 0.7, 0.6$).
- We follow the system for 10^{11} Monte Carlo steps (~ 0.1 s).
- PC wall-clock: $\gtrsim 3$ years, Janus: 24 days.

The coherence length: definition

- We consider the autocorrelation of $q_{\boldsymbol{x}}(t_\mathrm{w}) = \sigma_{\boldsymbol{x}}^{(1)}(t_\mathrm{w})\sigma_{\boldsymbol{x}}^{(2)}(t_\mathrm{w})$.

$$C_4(\boldsymbol{x}, t_\mathrm{w}) = \overline{L^{-3}\sum_{\boldsymbol{x}} q_{\boldsymbol{x}}(t_\mathrm{w})q_{\boldsymbol{x}+\boldsymbol{r}}(t_\mathrm{w})} \xrightarrow[T<T_\mathrm{c}]{} C_4 \simeq r^{-a}\mathrm{e}^{-[r/\xi(t_\mathrm{w})]^b}$$

- At $T < T_\mathrm{c}$ the value of a matters:
  - Coarsening dynamics: $a = 0$.
  - RSB: $a > 0$.
- T_c: $a = 1 + \eta = 0.625(10)$ (Hasenbusch et al 2008).

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- We would like an Ansatz-independent determination of ξ and a:

$$I_k(t_\mathrm{w}) = \int_0^\infty \mathrm{d}r\, r^k C_4(r, t_\mathrm{w})$$

then, if $C_4 \simeq r^{-a} f(r/\xi)$,

$$\xi_{k,k+1}(t_\mathrm{w}) = I_{k+1}(t_\mathrm{w})/I_k(t_\mathrm{w}) \propto \xi(t_\mathrm{w}), \quad (\text{for } \xi \ll L)$$

The coherence length and the replicon

- $C_4(r, t_\mathrm{w}) \sim r^{-a} f(t/\xi(t_\mathrm{w})) \implies I_1(t_\mathrm{w}) \propto \xi_{k,k+1}^{2-a}(t_\mathrm{w})$.

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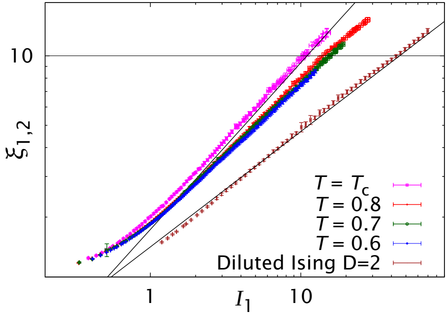

We plot $\xi_{1,2}$ vs. I_1:

$$a(0.8) = 0.442(11)$$
$$a(0.7) = 0.397(12)$$
$$a(0.6) = 0.359(13)$$

$$a(T_c) = 0.585(12)$$

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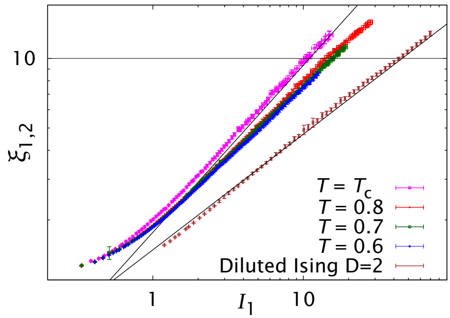

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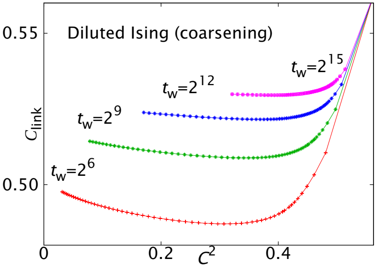
- $T < T_c$:
  - Incompatible with coarsening dynamics ($a = 0$).
  - Agreement with equilibrium results (Marinari & Parisi, 2001).
- $a(T_c)$ 2.5 SD below the FSS estimate $a = 0.625(10)$.

The link correlation function

- We are now interested in the spin and link correlation functions,

$$C(t,t_w) = \overline{L^{-3}\sum_{\boldsymbol{x}} \sigma_{\boldsymbol{x}}^{t+t_w}\sigma_{\boldsymbol{x}}^{t_w}}, \quad C_{\mathrm{link}}(t,t_w) = \overline{L^{-3}\sum_{\langle \boldsymbol{x},\boldsymbol{y}\rangle} \sigma_{\boldsymbol{x}}^{t+t_w}\sigma_{\boldsymbol{x}}^{t_w}\sigma_{\boldsymbol{y}}^{t+t_w}\sigma_{\boldsymbol{y}}^{t_w}}.$$

- We eliminate t as an independent variable and study $C_{\mathrm{link}}(C^2)$.
- Coarsening dynamics: $C^2 < q_{\mathrm{EA}}^2 \Rightarrow C_{\mathrm{link}}(C^2) = \text{constant}$.
- RSB: C_{link} is not constant (example: in SK $C_{\mathrm{link}} = C^2$)



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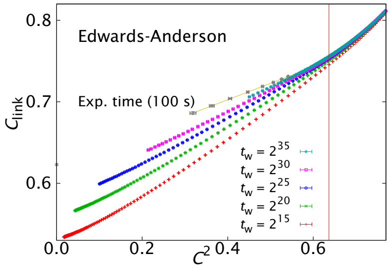

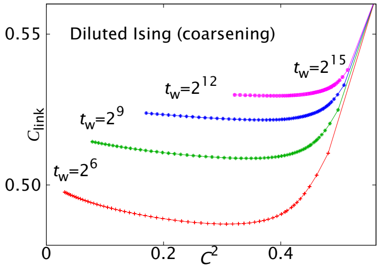

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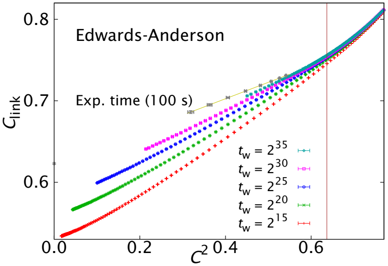

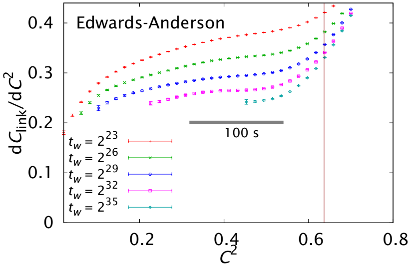

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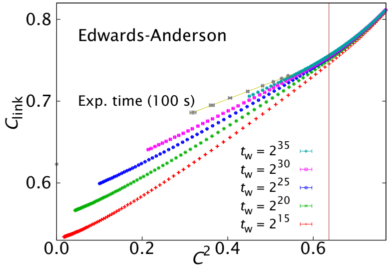

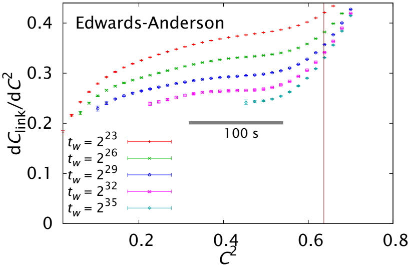

- We find non-coarsening dynamics even for experimental times.

Conclusions and outlook

Summary of our results

- Janus halves the logarithmic time gap between simulations and experiment.
- Our simulations indicate non-coarsening dynamics.
- Further results (PRL **101**, 157201 (2008); arXiv:0811.2864)
  - Non-equilibrium overlap equivalence.
  - Non-equilibrium scaling functions reproducing equilibrium results for finite systems.
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Next steps

- We are running equilibrium simulations with parallel tempering to study $T < T_c$.
- $P(q)$, ultrametricity, temperature chaos, etc.