The de Almeida-Thouless line of the four-dimensional Ising spin glass

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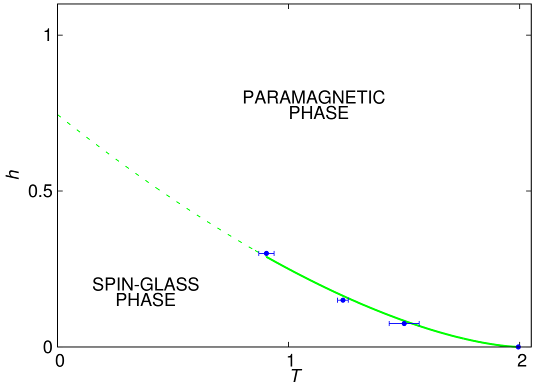
[1] R. A. Baños, A. Cruz, L.A. Fernandez, J.M. Gil-Narvion, A. Gordillo-Guerrero, M. Guidetti, D. Iñiguez, A. Maiorano, E. Marinari, V. Martin-Mayor, J. Monforte-Garcia, A. Muñoz Sudupe, D. Navarro, G. Parisi, S. Perez-Gaviro, J.J. Ruiz-Lorenzo, S.F. Schifano, B. Seoane, A. Tarancon, P. Tellez, R. Tripiccione and D. Yllanes

The de Almeida-Thouless line



A SG/PM transition in a magnetic field: surprising prediction by replica theory.

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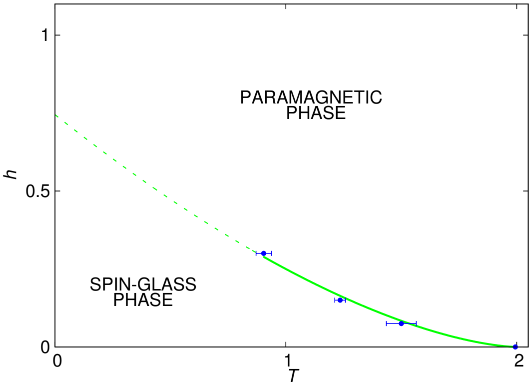

A SG/PM transition in a magnetic field: surprising prediction by replica theory. Which broken symmetry? not time-reversal symmetry!

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  Recent progress for small, positive ϵ (Parisi and Temesvari, 2011).
- Numerically: recent Finite Size Scaling studies did not find a dAT line (Young and Katzgraber 04; Jörg, Katzgraber, Krzakala 08).

Ingredients in our approach (I)

- We consider the Ising-Edwards-Anderson SG model [short-range interactions, $\sigma_x = \pm 1$, $J_{xy} = \pm 1$, four *real replicas* for connected $\hat{G}(\boldsymbol{p})$].

$$H = -\sum_{\langle x,y \rangle} J_{xy}\sigma_x\sigma_y \; - \; h\sum_x \sigma_x \,.$$

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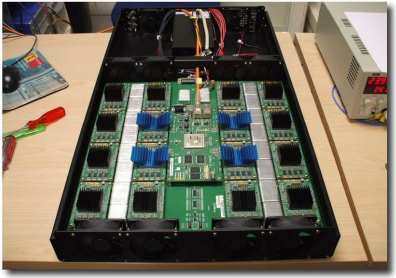
- $L = 16$ simulated on Janus:



The Janus computer

- Custom built, reconfigurable.
- 16 boards $\times$ 16 FPGAs.
- EA-Ising 4D: 86 ps/spin flip/FPGA
  Worst sample: $(2.6 \times 10^{10}$ sweeps$)\times$(32 temperatures).

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  Previous work emphasized second-moment correlation length

$$\xi_2 = \frac{1}{2\sin(\pi/L)}\sqrt{\frac{\hat{G}(0)}{\hat{G}(2\pi/L)} - 1}\,,$$

  $\hat{G}(\boldsymbol{p})$: SG correlation function in Fourier space.
  L: size of lattice with periodic boundary conditions.

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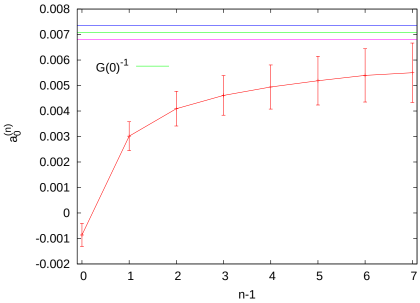

Lagrange polynomial extrapolation
$\boldsymbol{p} = (2\pi r/L, 0, 0, 0)\,,\ r \to 0$:

$$\frac{1}{G(\mathbf{p})} \sim a_0 + a_2\left(\not{p}^2\right) + a_4\left(\not{p}^2\right)^2 + \ldots$$

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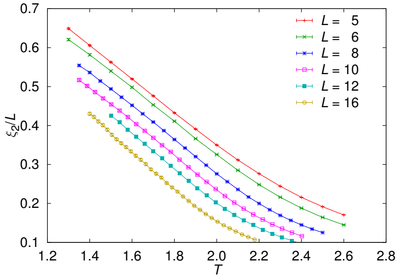
- In fact, ξ_2/L is not scale invariant at $h = 0.15$:



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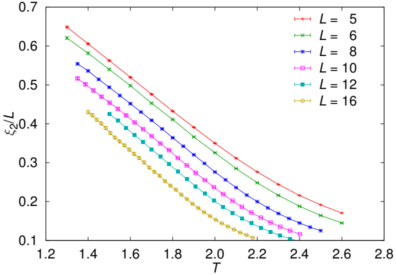

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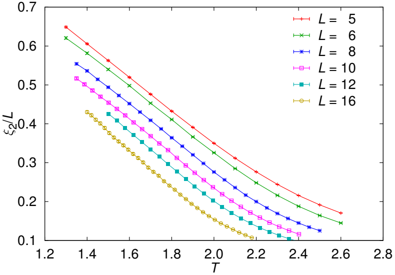

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  Even for Ising spins, SGs have soft (*Goldstone*) excitations.
- $h > 0 \longrightarrow$ longitudinal and transversal excitations.
  We do not know how to disentangle them.

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$$R_{12} = \frac{\hat{G}(2\pi/L, 0, 0, 0)}{\hat{G}(2\pi/L, 2\pi/L, 0, 0)}$$

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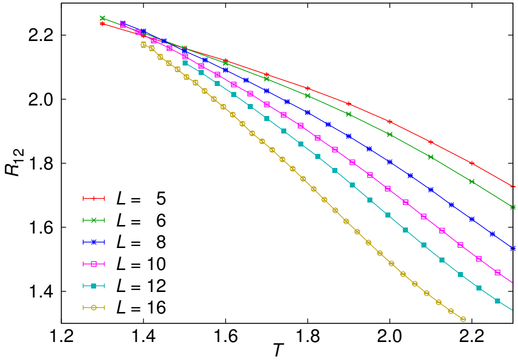
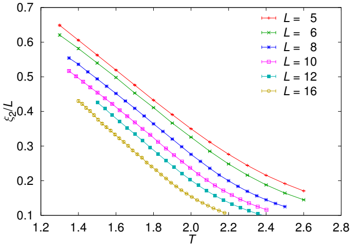
- Well behaved for FM Ising model, $R_{12}^{\text{Ising,2D}}(T_c) = 1.694\,024\ldots$
- Well behaved as well for the anomalous longitudinal propagator in the NLσM in a field (i.e. ferromagnetic Heisenberg model).

Results (I): Scale invariance at $h = 0.15$

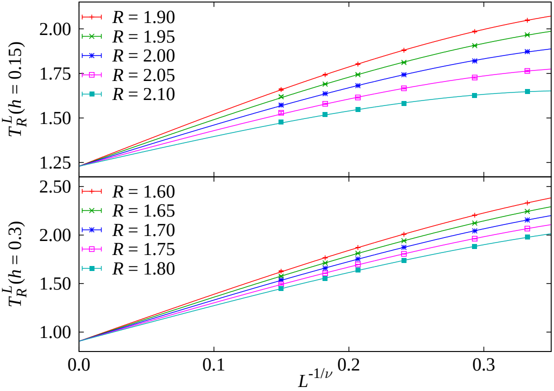




Very clear for R_{12} (ξ_2 misleading). Mind that $T_c(h=0) = 2.03(3)$.

$T^L_R(h)$ is chosen such that $R_{12}(T = T^L_R(h), h; L) = R$.

Joint fit: $T^L_R(h) \simeq T_c(h) + B_{R,h} L^{-1/\nu}[1 + C_{R,h} L^{-\omega}]$.



Conclusions

- Finite size scaling study of the $D = 4$ Ising spin-glass in a field.
- Janus gave access to large systems ($L = 16$) at low temperatures.
- Scale invariant R_{12}, not affected by anomalous longitudinal propagator.
- Found de Almeida-Thouless line, it follows Fisher-Sompolinski.
- Universality along the dAT line:

$$\nu = 1.46(7)[6] \quad , \quad \eta = -0.30(4)[1]$$

For $h = 0$: $\nu_0 = 1.025(15)$.