Equilibrium vs. non-equilibrium in the $3D$ Ising spin-glass phase

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Statphys 24, Cairns, 22 July 2010.

[1] R. Alvarez Baños, A. Cruz, L.A. Fernandez, J.M. Gil-Narvion, A. Gordillo-Guerrero, M. Guidetti, A. Maiorano, F. Mantovani, E. Marinari, V. Martin-Mayor, J. Monforte-Garcia, A. Muñoz Sudupe, D. Navarro, G. Parisi, S. Perez-Gaviro, J.J. Ruiz-Lorenzo, S.F. Schifano, B. Seoane, A. Tarancon, R. Tripiccione and D. Yllanes

Aging correlation functions in the SG phase (I)

Simplest: two times, no spatial resolution

Cool to $T < T_c$, wait for time t_w, probe the system at time $t + t_w$

$$C(t, tw) = \frac{1}{N} \sum_{\boldsymbol{x}} \langle s_{\boldsymbol{x}}(t + t_w) s_{\boldsymbol{x}}(t_w) \rangle \,, \quad \chi(t, t_w) \approx \frac{1}{T}(1 - C(t, t_w)).$$

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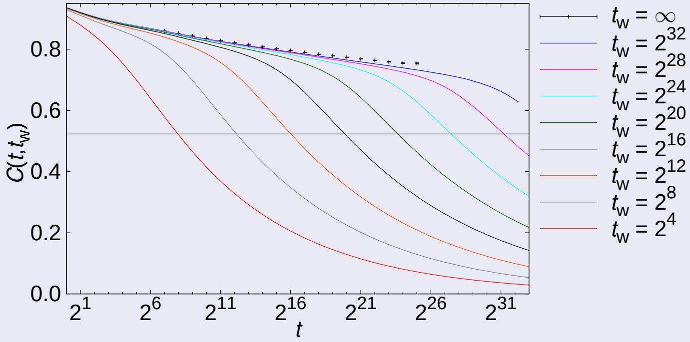

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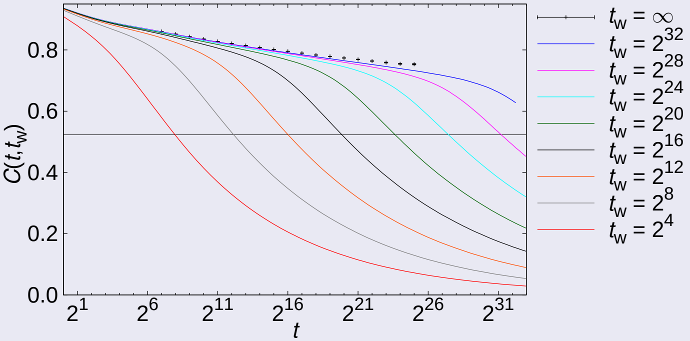
Change variables: $\{t, t_\mathrm{w}\} \longrightarrow \{C(t, t_\mathrm{w}), t_\mathrm{w}\}$



Aging correlation functions in the SG phase (II)

Spatial coherence: one time but two clones

$$C_4(\boldsymbol{r}, t_w) = \frac{1}{N}\sum_{\boldsymbol{x}} \langle [s^{(1)}_{\boldsymbol{x}}(t_w) s^{(1)}_{\boldsymbol{x}+\boldsymbol{r}}(t_w)][s^{(2)}_{\boldsymbol{x}}(t_w) s^{(2)}_{\boldsymbol{x}+\boldsymbol{r}}(t_w)] \rangle$$

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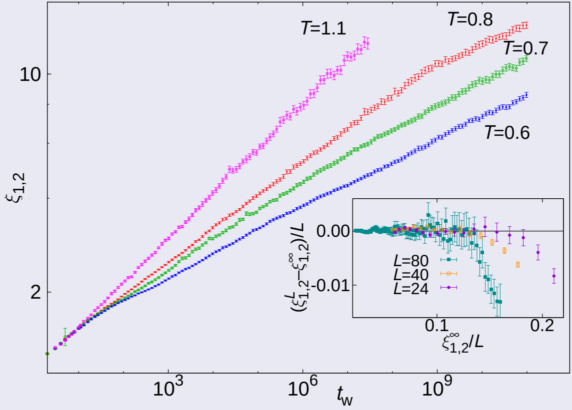
Coherence length $\xi(t_\mathrm{w})$ from long distance decay of $C_4(\boldsymbol{r}, t_\mathrm{w})$
($\xi(t_\mathrm{w}) \sim$ size of glassy magnetic domains)

Aging correlation functions in the SG phase (II)

Spatial coherence: one time but two clones

$$\xi(t_w) \sim t_w^{1/z(T)}\,, \quad z(T) = z_c \frac{T_c}{T}$$

Numerics (Janus 08'), and experimental (Orbach 99').



Aging correlation functions in the SG phase (III)

Aging dynamic heterogenities: two-times, two-sites correlations

$$C_{2+2}(\boldsymbol{r}; t, t_\mathrm{w}) = \frac{1}{N}\sum_{\boldsymbol{x}} \langle [s_{\boldsymbol{x}}(t+t_\mathrm{w}) s_{\boldsymbol{x}}(t_\mathrm{w})][s_{\boldsymbol{x}+\boldsymbol{r}}(t+t_\mathrm{w}) s_{\boldsymbol{x}+\boldsymbol{r}}(t_\mathrm{w})]\rangle - C^2(t, t_\mathrm{w})$$

Still only numerics in spin-glasses (but experimental for colloids and glass forming liquids!)

Aging correlation functions in the SG phase (III)

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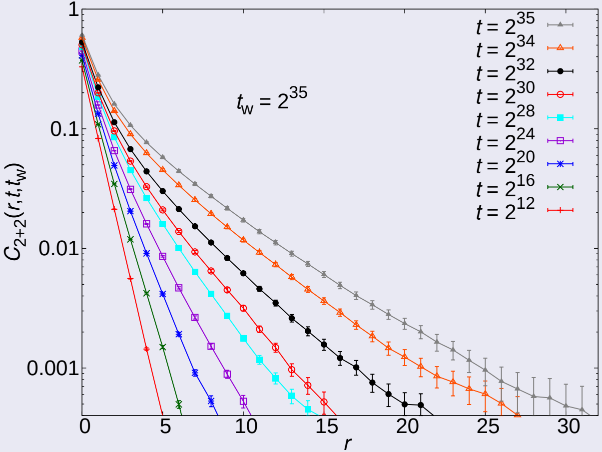
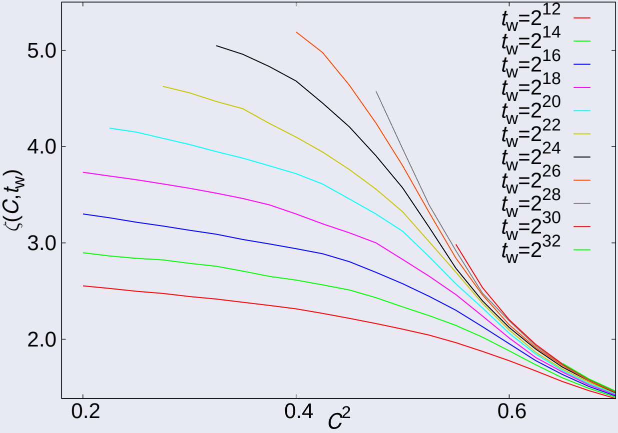
Correlation-length $\zeta(t, t_w)$ from long distance decay of $C_{2+2}(\boldsymbol{r}, t, t_w)$



Aging correlation functions in the SG phase (III)

Aging dynamic heterogenities: two-times, two-sites correlations

Something happens to $\zeta(t, t_w)$ when C goes below q_{EA}



Equilibrium vs. non-equilibrium

Time-length dictionary

Equilibrium: finite size at infinite t_w
Non-equilibrium: infinite system at finite t_w

Correspondence

Non-equilibrium	Equilibrium

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$C_{2+2}(\boldsymbol{r}; C, t_\mathrm{w})$	$C_4(\boldsymbol{r}\|q = C(t, t_\mathrm{w}))$
Aging dynamic heterogeneities	Crossover in $C_4(\boldsymbol{r}\|q)$ (D. Yllanes talk)

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Quantitative: at $T = 0.64 T_c$, choose t_w such that $L = 3.7\xi(t_w)$

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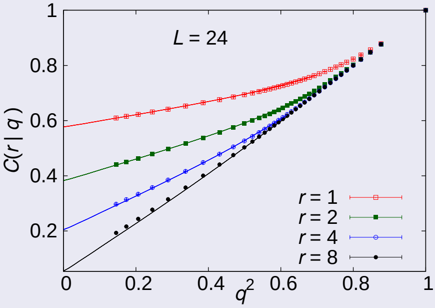

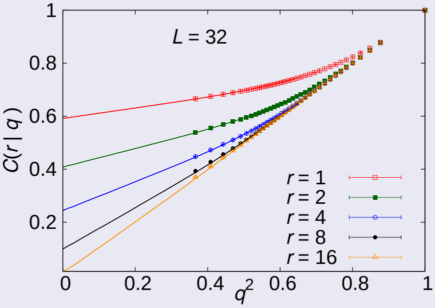

Consequences of time-length dictionary (I)

Finite-time scaling and aging heterogeneities

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- Equilibrium $C_4(\boldsymbol{r}|q)$ dependency on L and q: Finite-Size Scaling (D. Yllanes talk)
- Non-equilibrium: Finite-Time scaling (substitute L by $\xi(t_\mathrm{w})$)
- Size of heterogeneities, $\zeta(t, t_\mathrm{w})$, length scaling dimension

$$\zeta(t, t_\mathrm{w}) = \xi(t_\mathrm{w})\, \mathcal{F}\left([C^2 - q_\mathrm{EA}^2]\xi(t_\mathrm{w})^{1/\hat{\nu}}\right)$$

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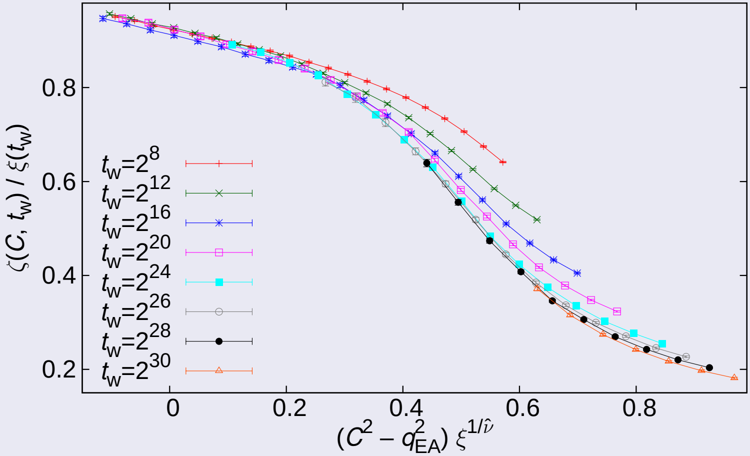
Finite-time scaling and aging heterogeneities

Parameters verbatim from equilibrium computation (D. Yllanes talk)



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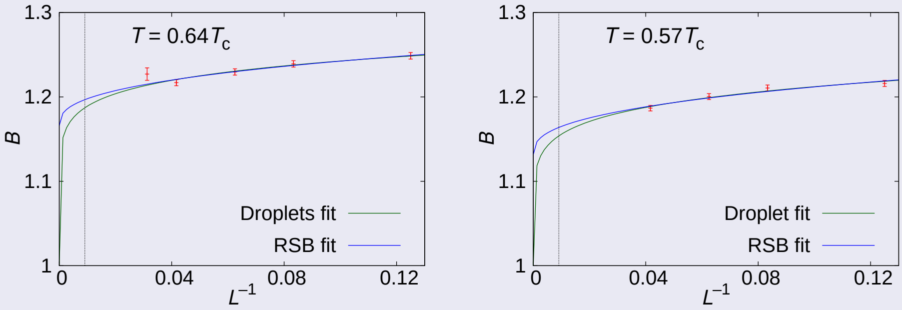
- Extrapolation from $L = 32$ to $L = 110$ milder than to $L = \infty$



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- Effective theory at experimentally relevant system size $L = 110$? $\longrightarrow$ Replica Symmetry Breaking.
- Fundamental theory of spin-glasses? Still unclear.