Connected spatial correlation functions and the order parameter of the $3D$ Ising spin glass

David Yllanes for the Janus Collaboration[1]

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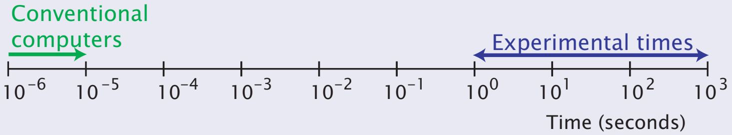
[1] R. Alvarez Baños, A. Cruz, L.A. Fernandez, J.M. Gil-Narvion, A. Gordillo-Guerrero, M. Guidetti, A. Maiorano, F. Mantovani, E. Marinari, V. Martin-Mayor, J. Monforte-Garcia, A. Muñoz Sudupe, D. Navarro, G. Parisi, S. Perez-Gaviro, J.J. Ruiz-Lorenzo, S.F. Schifano, B. Seoane, A. Tarancon, R. Tripiccione and D. Yllanes

Spin glasses: experiments vs. simulations

- Spin glasses are characterised by very sluggish dynamics.
- Off-equilibrium $\longrightarrow$ large gap between simulations and experiments.
- Equilibrium $\longrightarrow$ At low T, PCs can only thermalise very small lattices.



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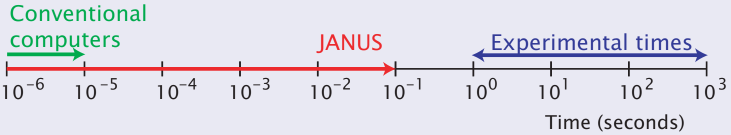
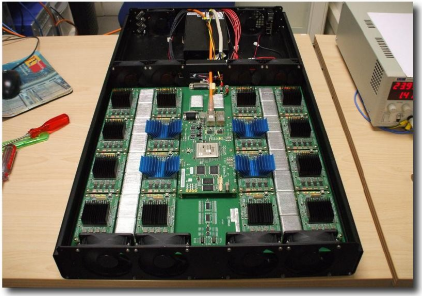


- Janus bridges the gap.

The Janus computer

- Janus is a custom-built computing system based on FPGAs:
  - Massively parallel
  - Reconfigurable
  - Modular

Our simulations

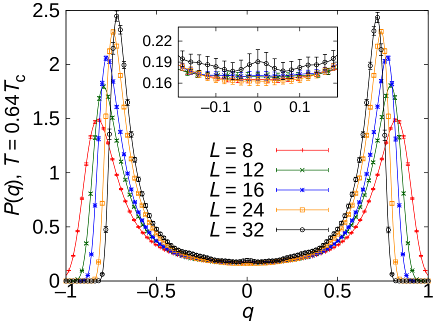


L	T_{min}	$N_{\mathrm{MC}}^{\mathrm{min}}$	$N_{\mathrm{MC}}^{\mathrm{max}}$
8	0.150	5×10^6	8.30×10^8
12	0.414	1×10^7	1.53×10^{10}
16	0.479	4×10^8	2.79×10^{11}
24	0.625	1×10^9	1.81×10^{12}
32	0.703	4×10^9	7.68×10^{11}

The model and our parameters

- $\mathcal{H} = -\sum_{\langle \boldsymbol{x},\boldsymbol{y}\rangle} J_{\boldsymbol{xy}}\sigma_{\boldsymbol{x}}\sigma_{\boldsymbol{y}}, \quad P(J_{\boldsymbol{xy}}) = \delta(J_{\boldsymbol{xy}}^2 - 1).$
- We thermalise $L = 32$ down to $T = 0.703 \simeq 0.64\,T_{\mathrm{c}}$.
- 4000 samples for $L \leq 24$ and 1000 samples for $L = 32$.
- Parallel tempering with sample-dependent simulation times.
- A total of 1.1×10^{20} spin updates.

The probability distribution of the order parameter



- Our order parameter is the overlap q

$$q = \frac{1}{V}\sum_x q_x = \frac{1}{V}\sum_x \sigma_x^{(1)}\sigma_x^{(2)}$$

- Its pdf $P(q)$ has peaks at $\pm q_{\mathrm{EA}}$.

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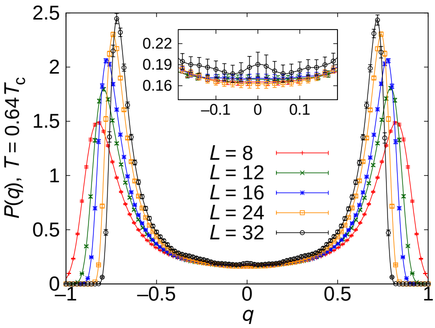

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RSB Non-zero probability density in $|q| < q_{\mathrm{EA}}$.

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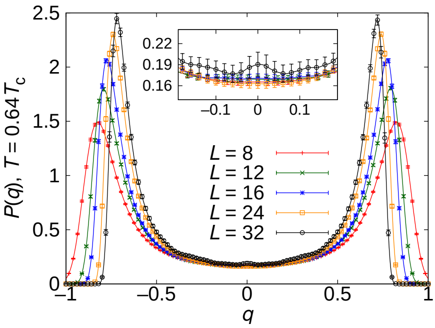

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- q_{EA} very difficult to compute.

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- In real space, one has to perform a subtraction that complicates the analysis.

Correlations in Fourier space

- We consider instead the Fourier transform of $C_4(\boldsymbol{r}|q)$

$$\hat{C}_4(\boldsymbol{k}|q^2 < q_{\mathrm{EA}}^2) \propto k^{\theta(q)-D} + \dots \qquad \hat{C}_4(\boldsymbol{k}|q^2 > q_{\mathrm{EA}}^2) \propto \frac{1}{k^2 + \xi_q^{-2}}$$

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- From very general RG arguments we can derive a scaling law:

$$\theta(q_{\text{EA}}) = 2/\hat{\nu}.$$

The computation of q_{EA} (I)

- According to Finite-Size Scaling,

$$F_q \simeq L^{D-\theta(q_{\mathrm{EA}})} G\big(L^{1/\hat{\nu}}(q - q_{\mathrm{EA}})\big)$$

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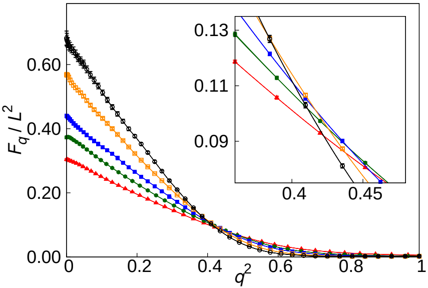
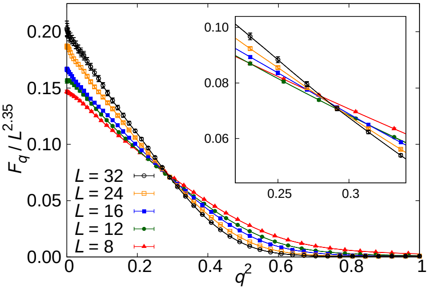
- In the large-L limit, these quantities diverge for $|q| < q_{\mathrm{EA}}$ but vanish for $|q| > q_{\mathrm{EA}}$.





The computation of q_{EA} (II)

- We consider pairs $(L, 2L)$.
- The crossing points $q_{L,y}$ scale as

$$q_{L,y} = q_{\mathrm{EA}} + A_y L^{1/\hat{\nu}} \qquad (**)$$

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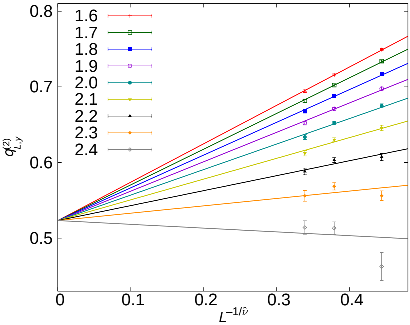

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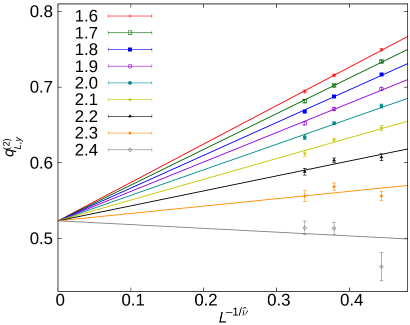

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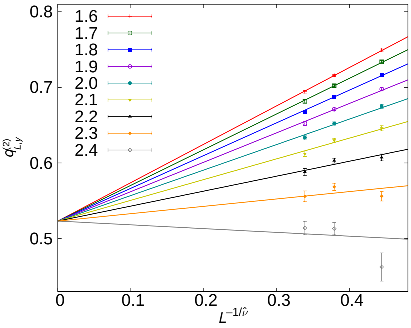

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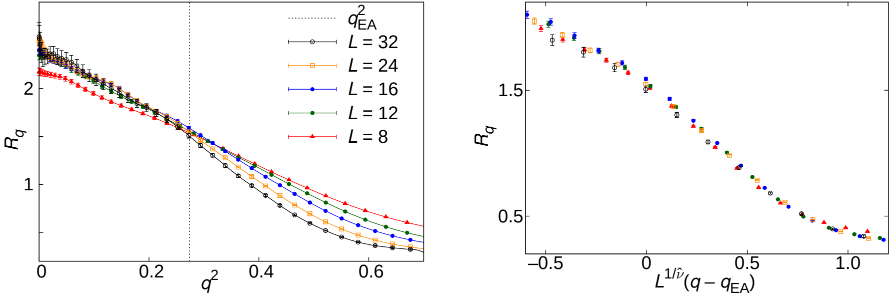
- From the dimensionless ratio $F_q/F_q^{(2)}$ we can obtain scaling plots:



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- There is a scaling law $\theta(q_{\mathrm{EA}}) = 2/\hat{\nu}$.
- In the RSB setting, we can formulate the conjecture (compatible with our numerical results)

$$\theta(0) = 1/\hat{\nu}$$