QUANTUM COSMOLOGY: GAMES WITHOUT FRONTIERS

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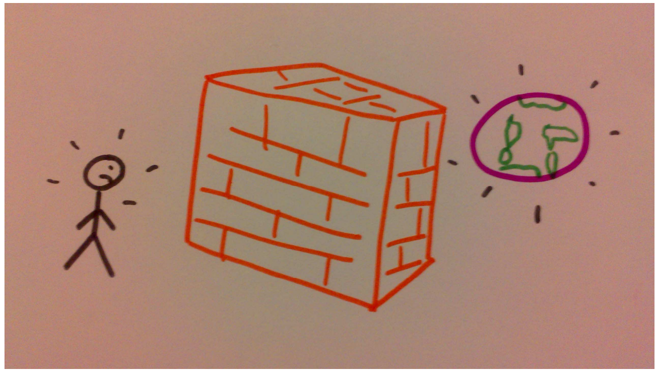
INTRODUCTION



- In classical spacetimes with horizons the different regions are disconnected

- Is it so quantum mechanically?
  - Quantization of the spacetime itself

- We quantize the system in a manner that allow us to find if these correlations exists ➔ Minisuperspace model
  - Kantowski-Sachs type with cosmological constant ➔ Schwarzschild-de Sitter spacetimes

- Quantization of the regions as different subspaces of a same Hilbert space

CLASSICAL MODEL

- Metric that depends on two variables (A,b)

$$\sigma^{-2}\mathrm{d}s^2 = -\frac{N(r)^2}{A(r)}\mathrm{d}r^2 + A(r)\mathrm{d}t^2 + b(r)^2\mathrm{d}\Omega_2^2$$

→ Generically A=0 corresponds to a horizon

- It will be convenient for our analysis to introduce $c = Ab$

- Then the Einstein-Hilbert action

$$S = -\int_0^\infty \mathrm{d}r \left(\frac{\dot{b}\dot{c}}{N} + N\mathring{B}(b) \right)$$

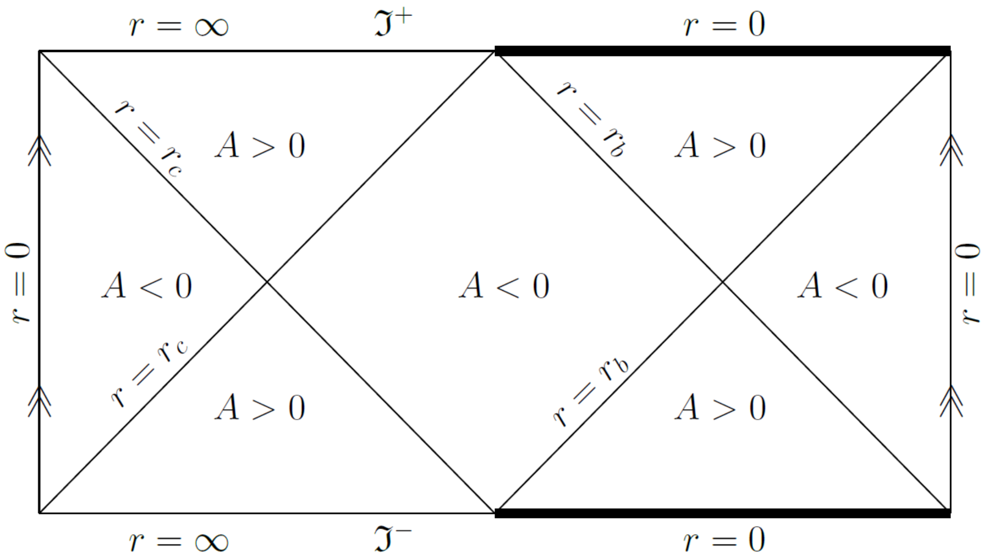
with

$$\begin{cases} B(b) = \frac{\lambda}{3}b^3 - b \\ \mathring{B}(b) = \partial_b B(b) = \lambda b^2 - 1 \end{cases}$$

- From the point of view of the metric, the solution corresponds to the Schwarzschild-de Sitter metric

$$b(r) = r, \qquad A(r) = -1 + 2m/r + \lambda r^2/3$$

➢ Penrose diagrams ➔ different cases depending on the value of m



when $0 < m < 1/\sqrt{9\lambda}$

➢ In order to perform a canonical quantization, we are interested in making a Hamiltonian formulation of the system

➢ The variation with respect to the lapse function give rise to the Hamiltonian constraint NC=0, with

$$C = -p_b p_c + \mathring{B}(b)$$

CANONICAL QUANTIZATION

- We follow an extension of Dirac quantization procedure [Ashtekar & Tate, 1994]
- Construct a kinematical operator algebra
    - Starting from the canonical variables in the phase space of the system (closed under Poisson brackets)
- Represent the algebra by operators acting on a kinematical complex vector space
    - An auxiliar space in which we can represent the constraint ➔ We endow with a Hilbert space structure
- Select the physical states by imposing the constraint operator ➔ space of physical states will be the kernel of this constraint
    - Physical Hilbert space structure: Inner product

➢ The physical states $$\Phi(b,c) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \mathrm{d}p \phi(p) e^{i[pc+B(b)/p]}$$

➢ Inner product $$\langle \Phi_1, \Phi_2 \rangle = \int_{\mathbb{R}} \mathrm{d}c \Phi_1(b,c)^* \Phi_2(b,c)$$

- Note that the inner product does not depend on b
- Resembles a kind of transformation from the Heisenberg picture

$$\Phi(b,c) = \hat{U}(b)\Phi(0,c) \qquad \text{with} \quad \hat{U}(b) = e^{iB(b)/\hat{p}}$$

➢ We can define a family of observables as $\hat{c}_b = c$

- This observable gives the value of c for each value of b

➢ It is possible to make a projection for negatives and positives values of c ➔ We have two subspaces of the total Hilbert space

RESULTS

- If a $b=b_0$ we observe only the region $c<0$ (classically our region)

⬇

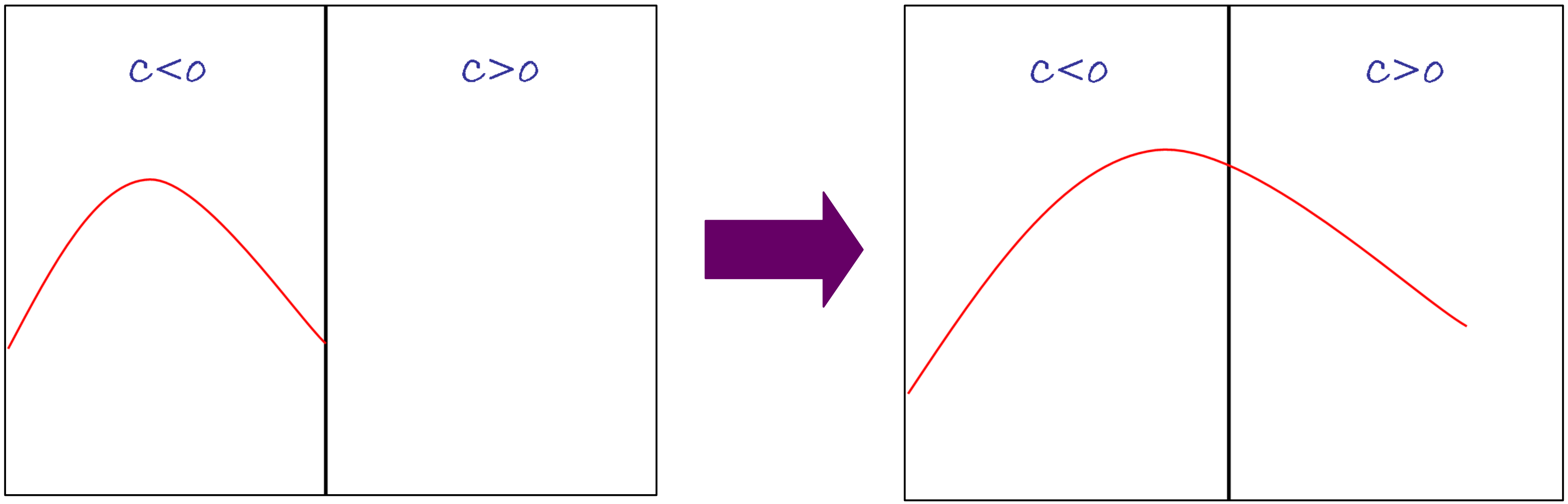
Choosing states with null projection on the positive region

- Is this restriction robust?

  - Compatibility of measures of c at different values $b=b_0$, b_1 ➔ We found that the observables are not mutually compatible

  - Restriction to a region of the universe is not stable

$b = b_0 \longrightarrow \hat{P}_+^0 \Phi = 0$ $\qquad$ $b = b_1 \longrightarrow \hat{P}_+^1 \Phi \neq 0$



- Depends on the value of b ➔ unestable under b-evolution

➢ If the observable is unitary ➔ unitarity is not respected in each subspace separately $\longrightarrow \exp(i\hat{c}_b^0)$

Mixing regions by quantum effects is a generic result in this quantization!

CONCLUSIONS

- Canonical quantization of the Schwarzschild-de Sitter spacetime by means of Kantowski-Sachs minisuperspace model ➔ physical structure is consistent only if we consider the whole system
  - Mixing regions
  - Unitarity of the global system but not for each subspace

- A distintive feature of our analysis is that we restrict it exclusively on the quantum behavior of the geometry

- In a future study we could introduce a quantum field in the quantized background studied here

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Thank you for your attention