3-form inflation in a 5D braneworld

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Inflation driven by higher order spin fields

- Vector inflation has been investigated, however, seem to support anisotropy and to inflate, the vector needs a nonminimal coupling and seems to feature some instabilities.
  (Phys. Rev. D 40, 967 (1989))
- 2-form inflation resembles much the vector inflation with the same problems.
  (arXiv:0902.3667)
- 3-form fields inflation has also been studied and seem to present some interesting results.
  (arXiv:1404.0211)

We shall focus on 3-forms!

What is a 3-form?

- Its a rank 3 totally antisymmetric tensor,

$$A_{\mu\nu\rho} = -A_{\nu\mu\rho}$$

For example, the *Levi-Civita* symbol, ϵ_{ijk}, used in the cross-product,

$$(\vec{u} \times \vec{v})_i = \epsilon_{ijk} u_j v_k$$

is a 3-form.

3-form field model

- We start by considering a flat FLRW 4-dim cosmology, where the metric takes the form,

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

where $a(t)$ is the scale factor with t being the cosmic time.

- The general action for Einstein gravity and the 3-form is written as,

$$S = -\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{48}F^2 - V(A^2)\right]$$

where $\kappa^2 = 8\pi G$ and,

$$F_{\mu\nu\rho\sigma} = 4\nabla_{[\mu}A_{\nu\rho\sigma]} = \nabla_\mu A_{\nu\rho\sigma} - \nabla_\sigma A_{\mu\nu\rho} + \nabla_\rho A_{\sigma\mu\nu} - \nabla_\nu A_{\rho\sigma\mu}$$

Equations of motion

- Let χ be a comoving field associated with the 3-form, $A_{\mu\nu\rho}$.
- Assuming a homogeneous and isotropic universe (cosmological principle) the nonzero components of the comoving field, χ, are,

$$A_{ijk} = a^3(t)\epsilon_{ijk}\chi \Rightarrow A^2 = 6\chi^2$$

- The Euler-Lagrange equations, for the 3-form, lead to the equations of motion,

$$\nabla \cdot F = 12V'(A^2)A$$

or, in terms of the comoving field,

$$\ddot{\chi} + 3H\dot{\chi} + 3\dot{H}\chi + V_{,\chi} = 0$$

Friedmann and Raychaudhuri equations

- Varying the action with respect to the metric tensor we get the energy-momentum tensor,

$$T_{\mu\nu} = g_{\mu\nu}\mathcal{L} + \frac{1}{6}(F \circ F)_{\mu a} + 6V'(A^2)(A \circ A)_{\mu a}$$

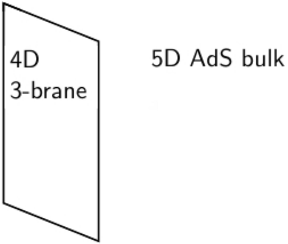
- Now, using Einstein equations, $G_{\mu\nu} = \kappa^2 T_{\mu\nu}$, we can calculate the Friedmann and Raychaudhuri equations,

$$\begin{aligned} H^2 &= \frac{\kappa^2}{3}\left\{\frac{1}{2}\left[(\dot{\chi} + 3H\chi)^2\right] + V\right\} \\ \dot{H} &= -\frac{\kappa^2}{2}V_{,\chi}\chi \end{aligned}$$

Randall-Sundrum II model

Going up to 5 dimensions...

- A single positive tension brane carrying the standard model fields is embedded in 5-dim Einstein gravity with a negative (bulk) cosmological constant and an infinite fifth dimension.



$$S = S_{EH} + S_{brane} = -\int d^5x\sqrt{-g^{(5)}}\left(\frac{R}{2\kappa_5^2} + \Lambda_5\right) - \int d^4x\sqrt{-g^{(4)}}\lambda$$

RSII Model

- The 5-dimensional Einstein equations lead to the Friedmann equation,

$$H^2 = \frac{\kappa_5^2}{3}\rho\left[1 + \frac{\rho}{2\lambda}\right]$$

where λ is the brane tension and κ_5^2 is the five dimensional gravitational constant.

- The motion equations and energy density for a single 3-form, already studied, are given by,

$$\ddot{\chi} + 3H\dot{\chi} + 3\dot{H}\chi + V_{,\chi} = 0$$

and

$$\rho_\chi = \frac{1}{2}(\dot{\chi} + 3H\chi)^2 + V$$

Dynamical system for RSII

- We now define the variables,

$$x \equiv \chi \qquad w \equiv \frac{\dot{\chi} + 3H\chi}{\sqrt{2\rho}} \qquad \Theta \equiv \left(1 + \frac{\rho}{2\lambda}\right)^{-1/2}$$

- The dynamical system for the equations of motion becomes,

$$\begin{cases} x' &= 3\left(\sqrt{\frac{2}{3}}\Theta w - x\right) \\ w' &= \frac{3}{2}\frac{V_{,x}}{V}(1 - w^2)\left[xw - \Theta\sqrt{\frac{2}{3}}\right] \end{cases}$$

with the following constraint equation for Θ,

$$\Theta^2 = \frac{1 - w^2}{1 - w^2 + \frac{V}{2\lambda}}$$

Critical points

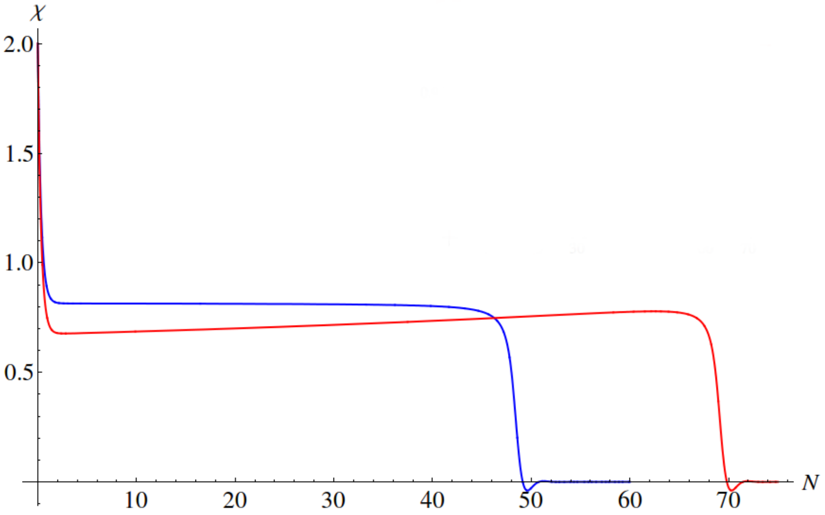
- The critical points of the system are,

	x	w	$\frac{V_{,x}}{V}$	Description
A	$\pm\sqrt{\frac{2}{3}}\Theta$	± 1	any	Kinetic domination
B	x_{ext}	$\sqrt{\frac{3}{2}}\frac{1}{\Theta}x_{ext}$	0	Potential extrema

- How does the system behaves as Θ changes?

Exponential potential $V = e^{\chi^2}$

$\Theta = 1 \quad (4-\text{dim})$ $\qquad$ $\Theta = \left(1 + \frac{\rho}{2\lambda}\right)^{-1/2} \quad (5-\text{dim})$



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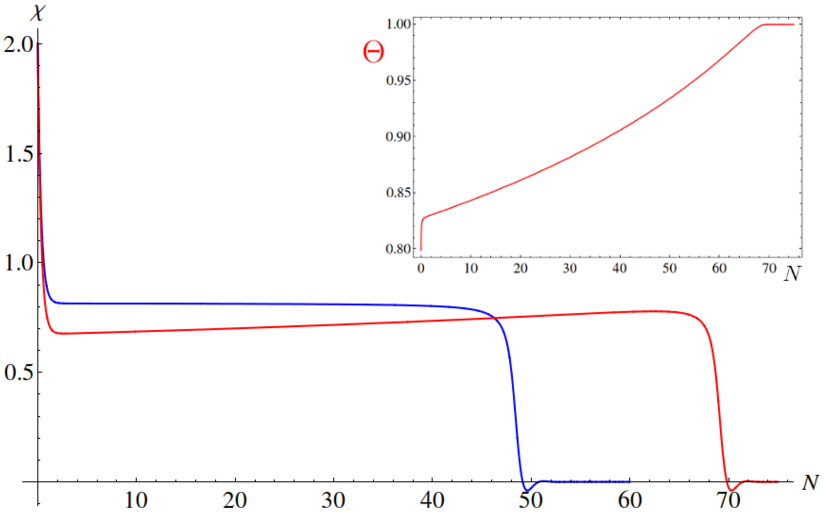

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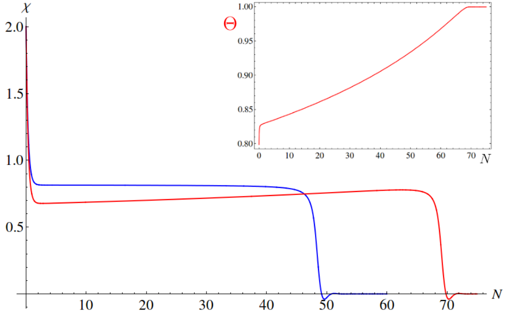
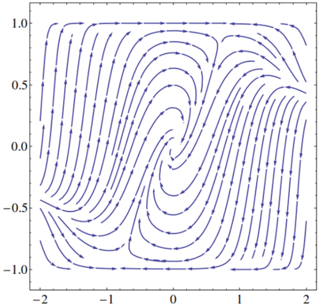
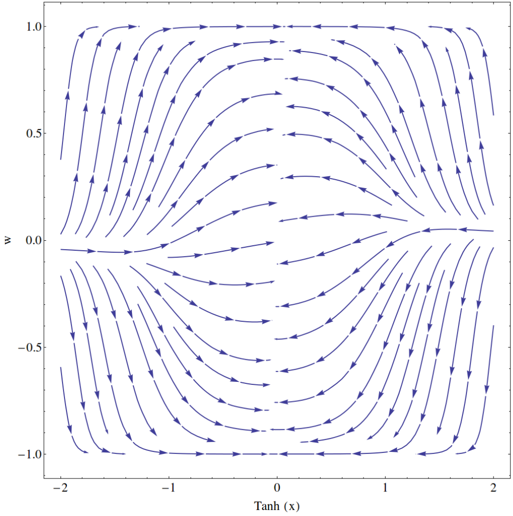




- How does Θ changes the phase space in 5-dim?

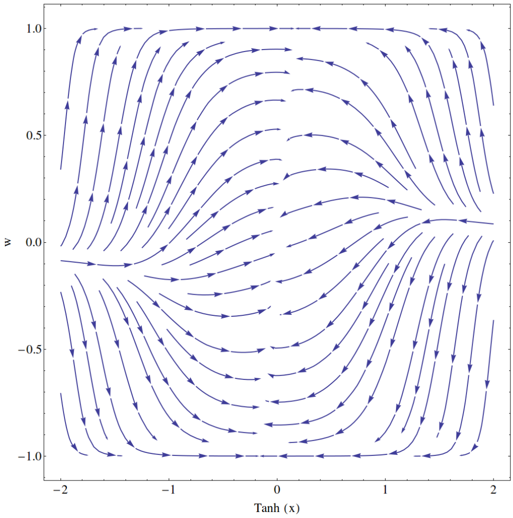
Exponential potential $V = e^{\chi^2}$

$\Theta = 0.1$



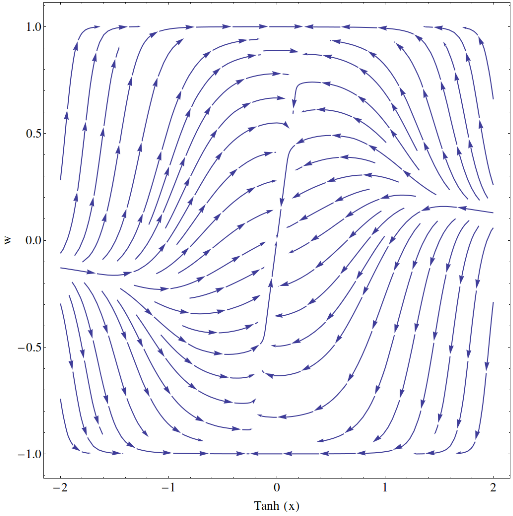
Exponential potential $V = e^{\chi^2}$

$\Theta = 0.2$



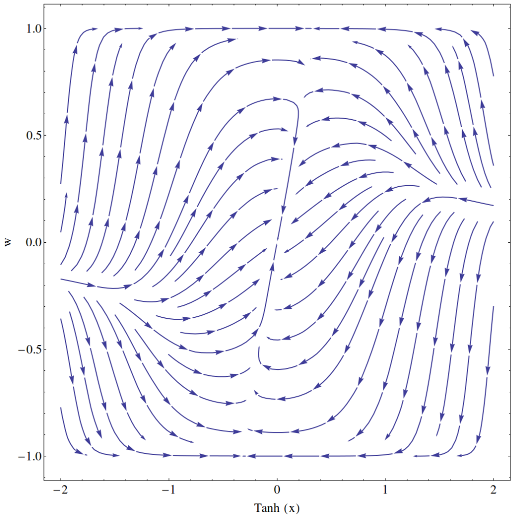
Exponential potential $V = e^{\chi^2}$

$\Theta = 0.3$



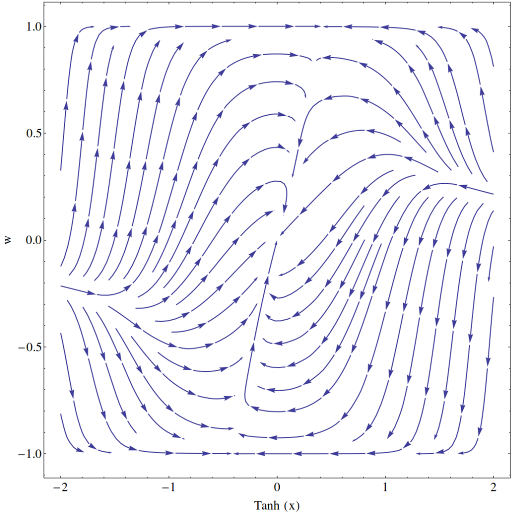
Exponential potential $V = e^{\chi^2}$

$\Theta = 0.4$



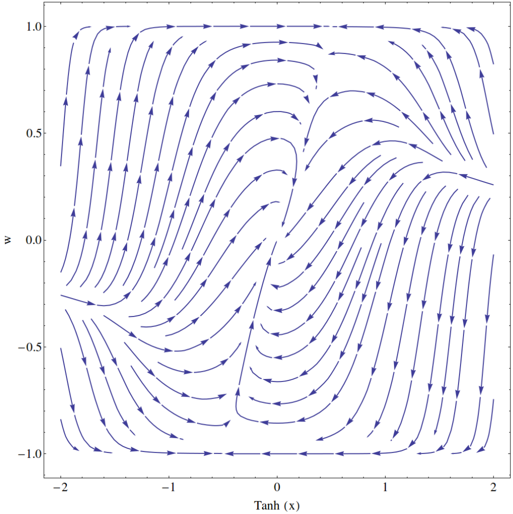
Exponential potential $V = e^{\chi^2}$

$\Theta = 0.5$



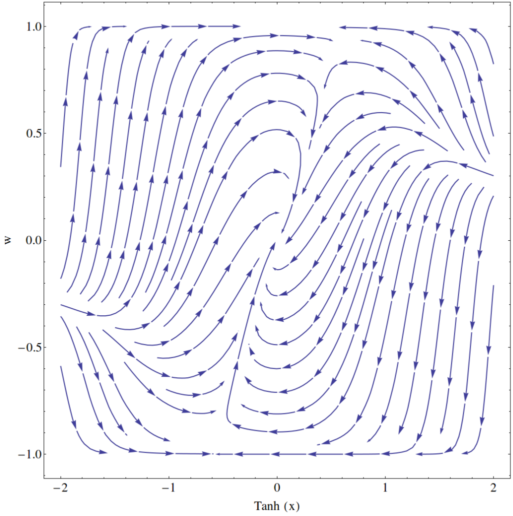
Exponential potential $V = e^{\chi^2}$

$\Theta = 0.6$



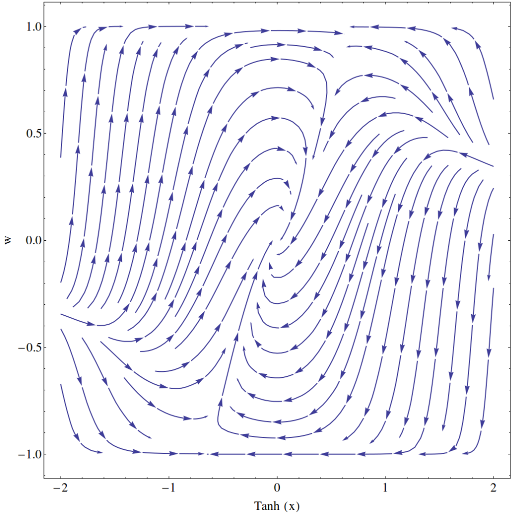
Exponential potential $V = e^{\chi^2}$

$\Theta = 0.7$



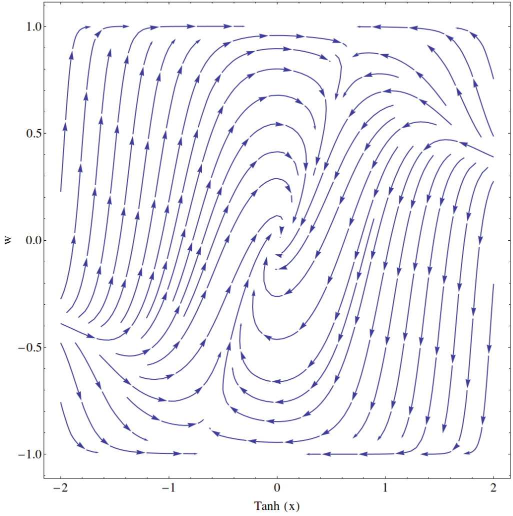
Exponential potential $V = e^{\chi^2}$

$\Theta = 0.8$



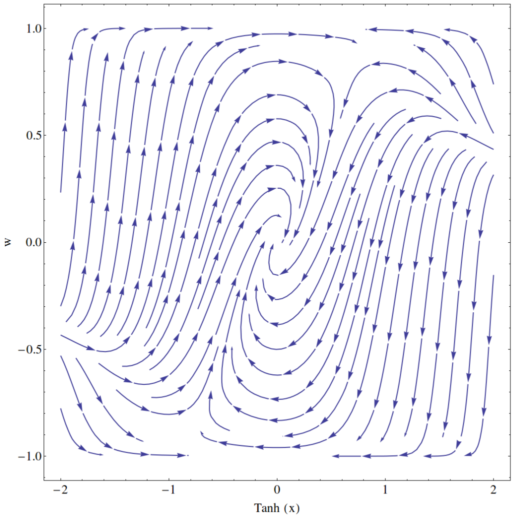
Exponential potential $V = e^{\chi^2}$

$\Theta = 0.9$



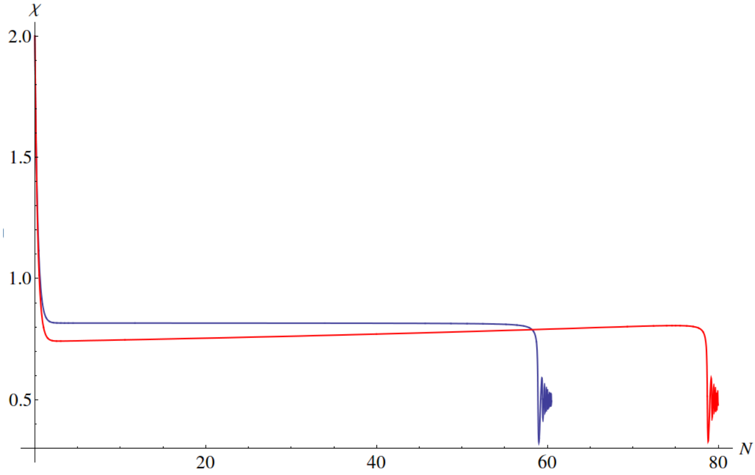
Exponential potential $V = e^{\chi^2}$

$\Theta = 1$



Landau-Ginzburg potential $V = (x^2 - c^2)^2$, $c = 0.5$

$\Theta = 1 \quad (4-\mathrm{dim})$ $\qquad$ $\Theta = \left(1 + \frac{\rho}{2\lambda}\right)^{-1/2} \quad (5-\mathrm{dim})$



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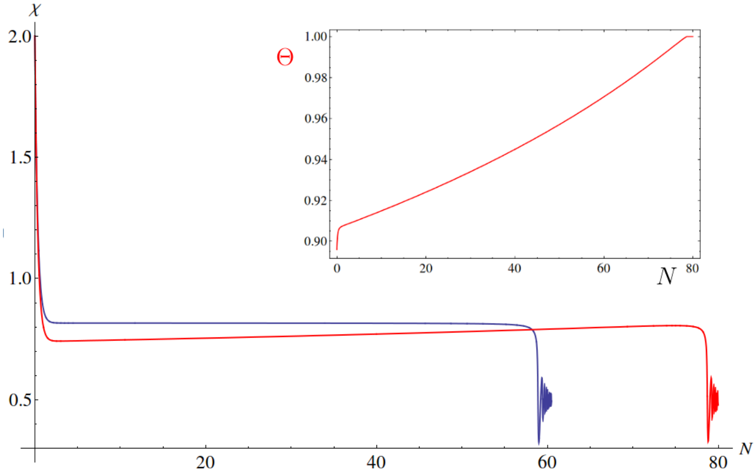


Stability through the effective potential

- Writing the equations of motion,

$$\ddot{\chi} + 3H\dot{\chi} + 3\dot{H}\chi + V_{,\chi} = 0$$

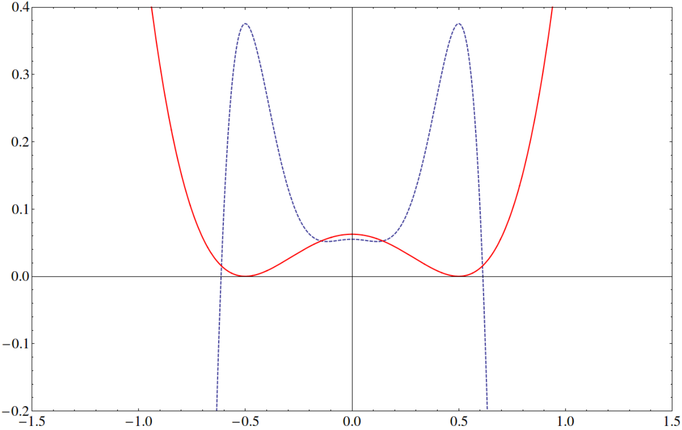
we can define the effective potential,

$$V_{eff,\chi} = 3\dot{H}\chi + V_{,\chi}$$

and study the stability of the critical points through its analysis instead of tracing the (x, w) phase space.

Landau-Ginzburg potential $V = (x^2 - c^2)^2$, $c = 0.5$

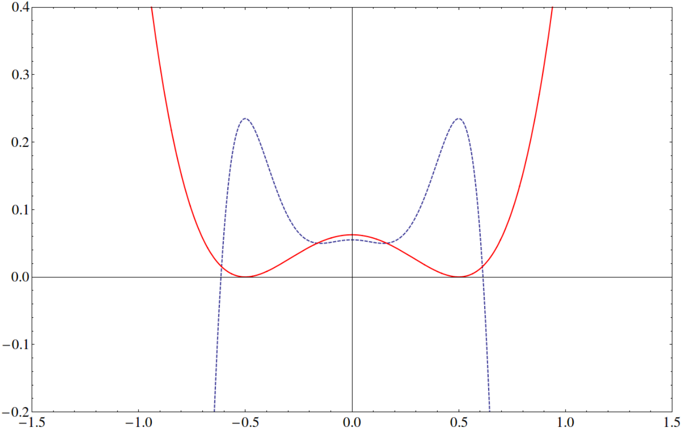
$\Theta = 0.2$



$V_{eff,\chi} = 3\dot{H}\chi + V_{,\chi}$ $\quad V = (x^2 - c^2)^2$

Landau-Ginzburg potential $V = (x^2 - c^2)^2$, $c = 0.5$

$\Theta = 0.25$



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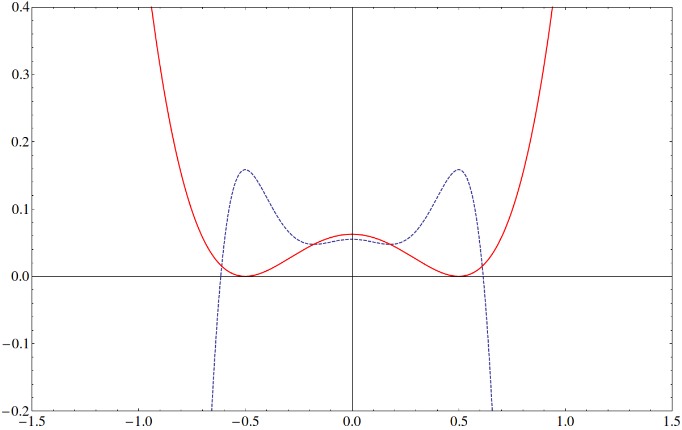

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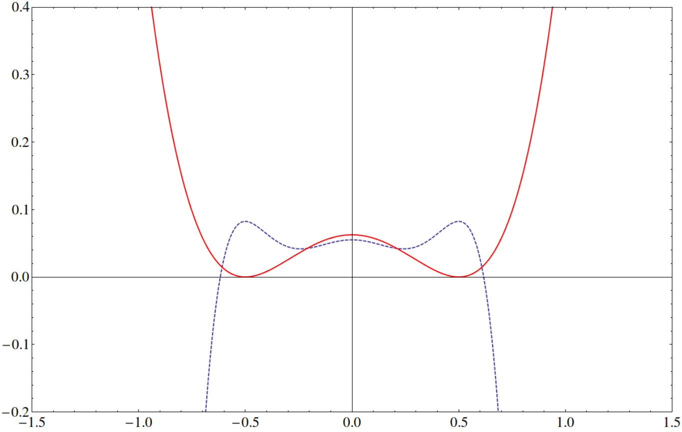
$\Theta = 0.4$



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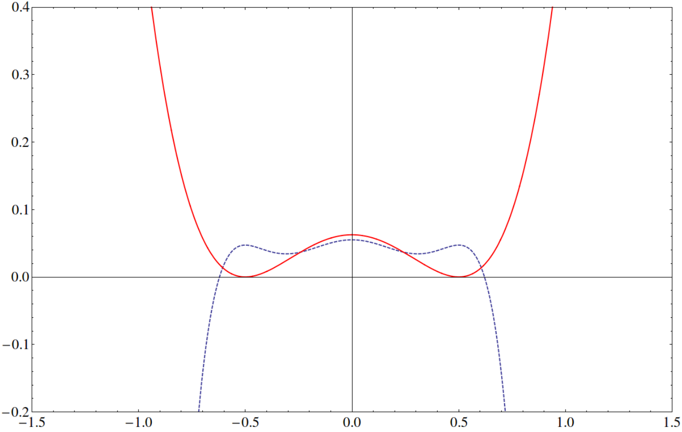

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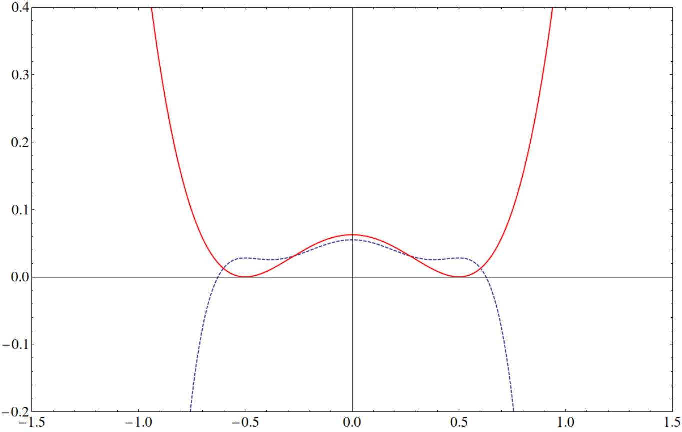

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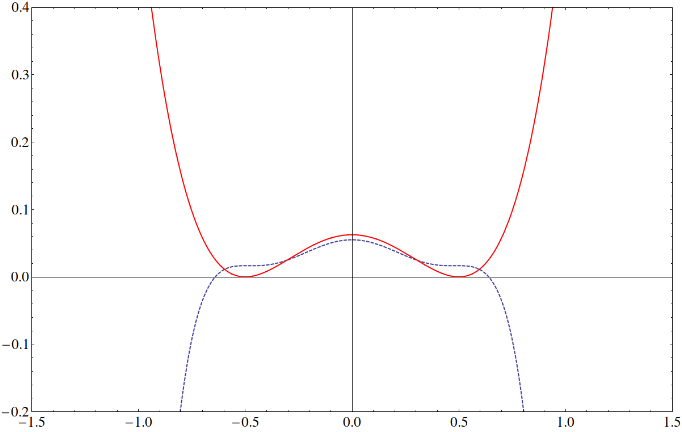

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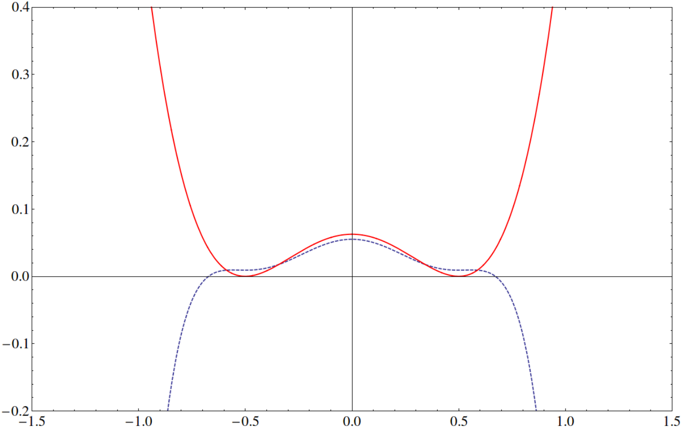

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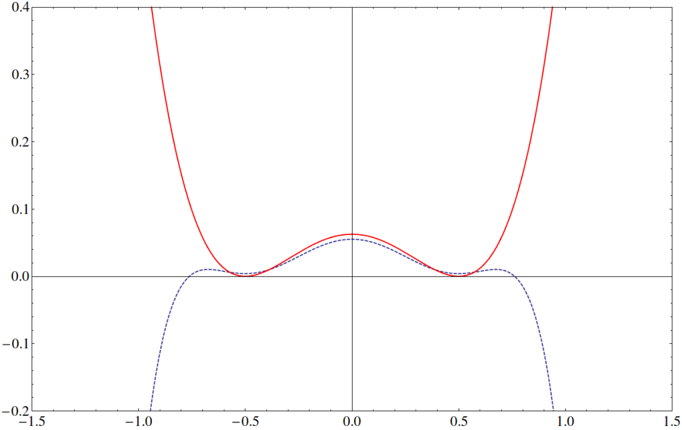

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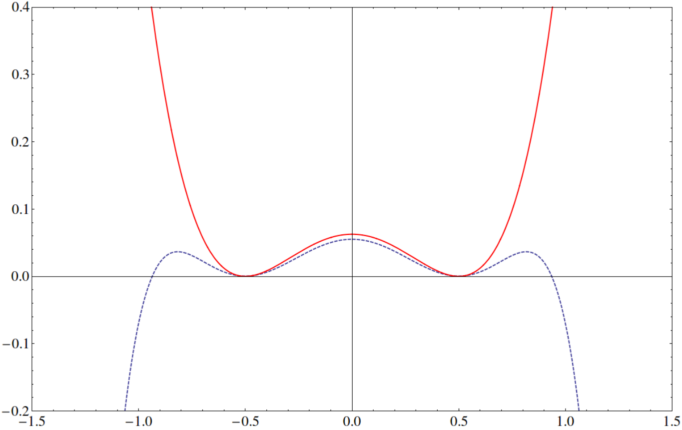
Landau-Ginzburg potential $V = (x^2 - c^2)^2$, $c = 0.5$

$\Theta = 1$



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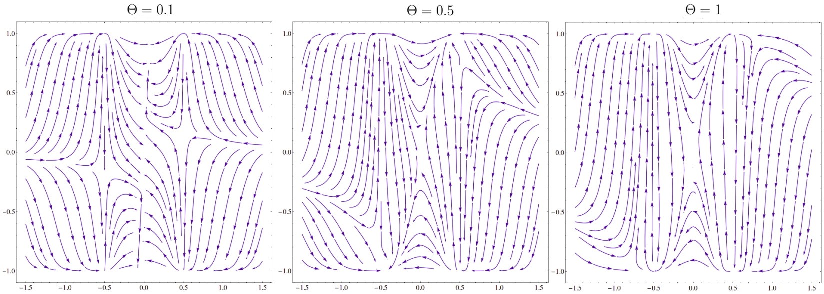

Summary

- 4 DIM
  - Less friction!
  - The field inflates always at the same critical point, $x = \pm\sqrt{2/3}$, then stops and ends in the attractor $x = x_{ext}$.
- 5 DIM
  - More friction!
  - The field inflates, as it goes through a journey ($x \approx 0$ to $x = \pm\sqrt{2/3}$), at instant critical points, $x = \pm\sqrt{2/3}\Theta$.

Future work

- Evolution of scalar perturbations.
- Evolution of tensor perturbations.
- Compare with data.

Thank you for the attention!