Observable Gravitational Waves From Non-Minimal Inflation in SUGRA

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Based on:

- C.P., arXiv:1503.05887.

Outline

Non-Minimal Chaotic Inflation (Non-MCI)
- The non-SUSY Framework
- The Synergy Between $f_{\mathcal{R}}$ and $V_{\rm CI}$

Unitarity Constraint
- The Ultraviolet (UV) Cut-off Scale

Supergravity Embedding
- The General Framework
- Kinetically Modified non-MCI in SUGRA

Inflation Analysis
- Analytical Results
- Numerical Results

Conclusions

Coupling non-Minimally the Inflaton to Gravity

• Our Starting Point is The Action in the **Jordan Frame** Of A Scalar Field ϕ with Potential $V(\phi)$ non-Minimally Coupled to the Ricci Scalar Curvature, $\mathcal{R}$, Through A Frame Function $f_{\mathcal{R}}(\phi)$ (JF). This is:

$$\mathcal{S} = \int d^4x \sqrt{-\mathfrak{g}}\left(-\frac{1}{2} f_{\mathcal{R}}(\phi)\mathcal{R} + \frac{f_{\rm K}(\phi)}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right), \quad \text{Where}$$

$\mathfrak{g}$ is the Determinant Of The Background Metric and $f_{\mathcal{R}}(\langle\phi\rangle) \simeq 1$ (in Reduced Planck Units With $m_{\rm P} = 1$) to Guarantee the Ordinary **Einstein Gravity** At Low Energy. We Allow for a Kinetic Mixing Through the Function $f_{\rm K}(\phi)$.

[1] K. Maeda (1989), D.S. Salopek, J.R. Bond and J.M. Bardeen (1989); D.I. Kaiser (1995); T. Chiba and M. Yamaguchi (2008).

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• We can Write $\mathcal{S}$ in the **Einstein Frame** (EF) as follows

$$\mathcal{S} = \int d^4x\sqrt{-\widehat{\mathfrak{g}}}\left(-\frac{1}{2}\widehat{\mathcal{R}} + \frac{1}{2}\widehat{g}^{\mu\nu}\partial_\mu\widehat{\phi}\partial_\nu\widehat{\phi} - \widehat{V}\left(\widehat{\phi}\right)\right)$$

Performing a **Conformal Transformation**[1] According Which We Define the EF Metric:

$$\widehat{g}_{\mu\nu} = f_{\mathcal{R}}\, g_{\mu\nu} \quad\Rightarrow\quad \begin{cases}\sqrt{-\widehat{\mathfrak{g}}} = f_{\mathcal{R}}^2\sqrt{-\mathfrak{g}} \quad \text{and} \quad \widehat{g}^{\mu\nu} = g^{\mu\nu}/f_{\mathcal{R}}, \\ \widehat{\mathcal{R}} = \left(\mathcal{R} + 3\Box\ln f_{\mathcal{R}} + 3g^{\mu\nu}\partial_\mu f_{\mathcal{R}}\partial_\nu f_{\mathcal{R}}/2f_{\mathcal{R}}^2\right)/f_{\mathcal{R}}\end{cases}$$

and Introduce the **EF Canonically Normalized Field**, $\widehat{\phi}$, and Potential, $\widehat{V}$, Defined As Follows:

$$\left(\frac{d\widehat{\phi}}{d\phi}\right)^2 = J^2 = \frac{f_{\rm K}}{f_{\mathcal{R}}} + \frac{3}{2}\left(\frac{f_{\mathcal{R},\phi}}{f_{\mathcal{R}}}\right)^2 \quad \text{and} \quad \widehat{V}(\widehat{\phi}) = \frac{V\left(\widehat{\phi}(\phi)\right)}{f_{\mathcal{R}}\left(\widehat{\phi}(\phi)\right)^2}.$$

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• We Observe that $f_{\mathcal{R}}$ Affects **Both** J and $\widehat{V}_{\rm CI}$. Here we deliberate J from the $f_{\mathcal{R}}$-Dependence employing $f_{\rm K} \neq 1$.
• The Analysis of non-MCI in **the EF** Using The Standard Slow-Roll Approximation is Equivalent With **The Analysis in JF**.

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Inflationary Observables - Requirements

- The **Number of e-foldings**, $\widehat{N}_\star$, that the Scale $k_S = 0.05/\mathrm{Mpc}$ Suffers During nMI has to be Sufficient to Resolve the Horizon and Flatness Problems of Standard Big Bang:

$$\widehat{N}_\star = \int_{\widehat{\phi}_{\rm f}}^{\widehat{\phi}_\star} d\widehat{\phi}\, \frac{\widehat{V}}{\widehat{V}_{,\widehat{\phi}}} = \int_{\phi_{\rm f}}^{\phi_\star} d\phi\, J^2 \frac{\widehat{V}}{\widehat{V}_{,\phi}} \simeq 61.7 + \ln\frac{\widehat{V}(\phi_\star)^{1/2}}{\widehat{V}(\phi_{\rm f})^{1/3}} + \frac{1}{3}\ln T_{\rm rh} + \frac{1}{2}\ln\frac{f_{\mathcal{R}}(\phi_\star)}{f_{\mathcal{R}}(\phi_{\rm f})^{1/3}}$$

Where $\phi_\star$ $[\widehat{\phi}_\star]$ is The Value of ϕ $[\widehat{\phi}]$ When $k_\star$ Crosses Outside The Inflationary Horizon;
$\phi_{\rm f}$ $[\widehat{\phi}_{\rm f}]$ is the Value of ϕ $[\widehat{\phi}]$ at the end of non-MCI Which Can Be Found From The Condition

$$\max\{\widehat{\epsilon}(\phi_{\rm f}), |\widehat{\eta}(\phi_{\rm f})|\} = 1, \quad \text{With } \widehat{\epsilon} = \frac{1}{2}\left(\frac{\widehat{V}_{,\widehat{\phi}}}{\widehat{V}}\right)^2 = \frac{1}{2J^2}\left(\frac{\widehat{V}_{,\phi}}{\widehat{V}}\right)^2 \text{ and } \widehat{\eta} = \frac{\widehat{V}_{,\widehat{\phi}\widehat{\phi}}}{\widehat{V}} = \frac{1}{J^2}\left(\frac{\widehat{V}_{,\phi\phi}}{\widehat{V}} - \frac{\widehat{V}_{,\phi}}{\widehat{V}}\frac{J_{,\phi}}{J}\right).$$

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- The **Amplitude of the Power Spectrum** A_s of the Curvature Perturbations is To Be Consistent with PLANCK Data:

$$A_\mathrm{s}^{1/2} = \frac{1}{2\sqrt{3}\,\pi} \frac{\widehat{V}(\widehat{\phi}_\star)^{3/2}}{|\widehat{V}_{,\widehat{\phi}}(\widehat{\phi}_\star)|} = \frac{|J(\phi_\star)|}{2\sqrt{3}\,\pi} \frac{\widehat{V}(\phi_\star)^{3/2}}{|\widehat{V}_{,\phi}(\phi_\star)|} = 4.627 \cdot 10^{-5}$$

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- The (Scalar) Spectral Index, n_s, Its Running, α_s, And The Tensor-To-Scalar Ratio r are to be Consistent With the Fitting of the *Planck* Results by the **ΛCDM** Model (at 95% c.l.)[2]:

$$n_\mathrm{s} = 1-6\widehat{\epsilon}_\star + 2\widehat{\eta}_\star = 0.968\pm0.0045, \quad -0.0314 \le \alpha_\mathrm{s} = 2\left(4\widehat{\eta}_\star^2 - (n_\mathrm{s}-1)^2\right)/3-2\widehat{\xi}_\star \le 0.0046 \quad \text{and} \quad r = 16\widehat{\epsilon}_\star < 0.11,$$

Where $\widehat{\xi} = \widehat{V}_{,\widehat{\phi}}\widehat{V}_{,\widehat{\phi}\widehat{\phi}\widehat{\phi}}/\widehat{V}^2 = \widehat{V}_{,\phi}\,\widehat{\eta}_{,\phi}/\widehat{V}\,J^2 + 2\widehat{\eta}\widehat{\epsilon}$ And The Variables With Subscript $\star$ Are Evaluated at $\phi = \phi_\star$.

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- The Combined Bicep2/*Keck Array* and *Planck* Results Although Do Not Exclude Inflationary Models With Negligible r's, They Seem to Favor Those With r's of order 0.01 Since

$$r = 0.048^{+0.035}_{-0.032} \quad \Rightarrow \quad 0.01 \lesssim r \lesssim 0.085 \quad \text{at 68\%c.l.}$$

[2] *Planck Collaboration (2015);* Bicep2/*Keck Array and Planck Collaborations (2015)*

THE TWO REGIMES OF SYNERGISTIC NON-MCI

- NON-MCI HAS BEEN ORIGINALLY FORMULATED AS FOLLOWS: $V_{\rm CI} = \lambda\phi^4/4$, WITH $f_{\mathcal{R}} = 1 + c_{\mathcal{R}}\phi^2$ AND $f_{\rm K} = 1$.
- WE CAN GENERALIZE THE ABOVE CONSTRUCTION ESTABLISHING A SYNERGY BETWEEN $f_{\mathcal{R}}$ AND $V_{\rm CI}$ AS FOLLOWS[3]:

$$V_{\rm CI}(\phi) = \lambda^2\phi^n/2^{n/2} \quad \text{WITH} \quad f_{\mathcal{R}} = 1 + c_{\mathcal{R}}\phi^{n/2} \quad \text{AND} \quad f_{\rm K} = 1$$

[3] C. Pallis (2010); R. Kallosh, A. Linde and D. Roest (2013); A. Kehagias, A.M. Dizgah and A. Riotto (2013).

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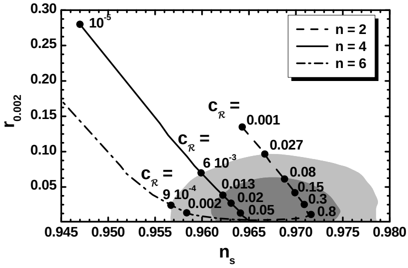
- The Resulting Model Exhibits The Following **Two Regimes:**
  - The **Weak** $c_{\mathcal{R}}$ Regime, With $c_{\mathcal{R}} \ll 1$ or $\phi > 1$ and $c_{\mathcal{R}}$-Dependent Observables Converging Towards Their Values In MCI,

    I.e., $n_{\rm s} \simeq 1 - (2+n)/2\widehat{N}_\star = 0.963, 0.947$ for $r \simeq 4n/\widehat{N}_\star \simeq 0.13, 0.28$ for $n = 2, 4$ Respectively ($\widehat{N}_\star = 55$).
  - The **Strong $c_{\mathcal{R}}$ Regime**, With $c_{\mathcal{R}} \gg 1$ and $\phi < 1$ and $c_{\mathcal{R}}$-Independent Observables:

$$n_{\rm s} \simeq 1 - 2/\widehat{N}_\star = 0.965 \text{ and } r \simeq 12/\widehat{N}_\star^2 = 0.0036.$$



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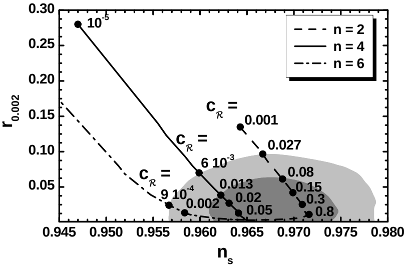

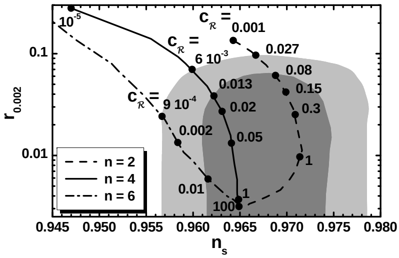

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Einstein Frame Computation

- We Analyze The Small-Field Behavior Of the Theory Expanding $\mathcal{S}$ About $\delta\phi = \phi - 0$ In Terms of $\widehat{\phi}$[4]. To this End We Find $\langle J\rangle$

$$J^2 = \left(\frac{d\widehat{\phi}}{d\phi}\right)^2 = \frac{f_{\rm K}}{f_{\mathcal{R}}} + \frac{3n^2c_{\mathcal{R}}^2\phi^{n-2}}{8f_{\mathcal{R}}^2} \;\Rightarrow\; \langle J\rangle = \begin{cases}\sqrt{3/2}c_{\mathcal{R}}, & \text{for } n=2,\\ 1, & \text{for } n\neq 2\end{cases} \qquad \text{For } \langle f_{\rm K}\rangle = 1\,.$$

We Observe that $\widehat{\phi} = \phi$ for $n > 2$ **At the Vacuum** of The Theory.

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- **For $n = 2$ and any $c_{\mathcal{R}}$** We obtain $\underline{\Lambda_{\rm UV} = m_{\rm P}}$ Since The Expansions Abound $\langle \phi \rangle = 0$ Give:

$$J^2 \dot{\phi}^2 = \left(1 - \sqrt{\frac{8}{3}}\widehat{\phi} + 2\widehat{\phi}^2 - \cdots\right)\dot{\widehat{\phi}}^2 \quad \text{and} \quad \widehat{V} = \frac{\lambda^2 \widehat{\phi}^2}{3c_{\mathcal{R}}^2}\left(1 - \sqrt{\frac{8}{3}}\widehat{\phi} + 2\widehat{\phi}^2 - \cdots\right).$$

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• **For $n > 2$** Similar Expansions Can Be Derived:

$$J^2\dot{\phi}^2 = \left(1 - c_{\mathcal{R}}\widehat{\phi}^{\frac{n}{2}} + \frac{3n^2}{8}c_{\mathcal{R}}^2\widehat{\phi}^{n-2} + c_{\mathcal{R}}^2\widehat{\phi}^n\cdots\right)\dot{\widehat{\phi}}^2 \;\; \text{and} \;\; \widehat{V}_{\rm CI} = \frac{\lambda^2\widehat{\phi}^n}{2}\left(1 - 2c_{\mathcal{R}}\widehat{\phi}^{\frac{n}{2}} + 3c_{\mathcal{R}}^2\widehat{\phi}^n - 4c_{\mathcal{R}}^3\widehat{\phi}^{\frac{3n}{2}} + \cdots\right).$$

Since The Term Which Yields The Smallest Denominator For $c_{\mathcal{R}} > 1$ is **$3n^2c_{\mathcal{R}}^2\widehat{\phi}^{n-2}/8$** We Find $\Lambda_{\rm UV} = m_{\rm P}/c_{\mathcal{R}}^{2/(n-2)}$.

[4] J.L.F. Barbon and J.R. Espinosa (2009); C.P. Burgess, H.M. Lee, and M. Trott (2010); A. Kehagias, A.M. Dizgah, and A. Riotto (2013)

Einstein Frame Computation

• We Analyze The Small-Field Behavior Of the Theory Expanding $\mathcal{S}$ About $\delta\phi = \phi - 0$ In Terms of $\widehat{\phi}^4$. To this End We Find $\langle J \rangle$

$$J^2 = \left(\frac{d\widehat{\phi}}{d\phi}\right)^2 = \frac{f_{\rm K}}{f_{\mathcal{R}}} + \frac{3n^2 c_{\mathcal{R}}^2 \phi^{n-2}}{8 f_{\mathcal{R}}^2} \;\Rightarrow\; \langle J \rangle = \begin{cases} \sqrt{3/2}\, c_{\mathcal{R}}, & \text{for } n = 2, \\ 1, & \text{for } n \neq 2 \end{cases} \qquad \text{For } \langle f_{\rm K} \rangle = 1\,.$$

We Observe that $\widehat{\phi} = \phi$ for $n > 2$ **At the Vacuum** of The Theory.

• **For $n = 2$ and any $c_{\mathcal{R}}$** We obtain $\underline{\Lambda_{\rm UV} = m_{\rm P}}$ Since The Expansions Abound $\langle \phi \rangle = 0$ Give:

$$J^2 \dot{\phi}^2 = \left(1 - \sqrt{\frac{8}{3}}\widehat{\phi} + 2\widehat{\phi}^2 - \cdots\right)\dot{\widehat{\phi}}^2 \quad \text{and} \quad \widehat{V} = \frac{\lambda^2 \widehat{\phi}^2}{3 c_{\mathcal{R}}^2}\left(1 - \sqrt{\frac{8}{3}}\widehat{\phi} + 2\widehat{\phi}^2 - \cdots\right).$$

• **For $n > 2$** Similar Expansions Can Be Derived:

$$J^2 \dot{\phi}^2 = \left(1 - c_{\mathcal{R}} \widehat{\phi}^{\frac{n}{2}} + \frac{3n^2}{8} c_{\mathcal{R}}^2 \widehat{\phi}^{n-2} + c_{\mathcal{R}}^2 \widehat{\phi}^n \cdots\right)\dot{\widehat{\phi}}^2 \quad \text{and} \quad \widehat{V}_{\rm CI} = \frac{\lambda^2 \widehat{\phi}^n}{2}\left(1 - 2c_{\mathcal{R}}\widehat{\phi}^{\frac{n}{2}} + 3c_{\mathcal{R}}^2 \widehat{\phi}^n - 4c_{\mathcal{R}}^3 \widehat{\phi}^{\frac{3n}{2}} + \cdots\right).$$

Since The Term Which Yields The Smallest Denominator For $c_{\mathcal{R}} > 1$ is $\mathbf{3n^2 c_{\mathcal{R}}^2 \widehat{\phi}^{n-2}/8}$ We Find $\Lambda_{\rm UV} = m_{\rm P}/c_{\mathcal{R}}^{2/(n-2)}$.

• If We Introduce **a non-Canonical Kinetic Mixing** Such That

$$\langle f_{\rm K} \rangle = c_{\rm K} \quad \text{And} \quad c_{\mathcal{R}} = r_{\mathcal{R}\rm K} c_{\rm K}^{n/4}$$

The Expansions Above Are Rewritten In Terms of the New Parameter $r_{\mathcal{R}\rm K}$

$$J^2 \dot{\phi}^2 = \left(1 - r_{\mathcal{R}\rm K} \widehat{\phi}^{\frac{n}{2}} + \frac{3n^2}{8} r_{\mathcal{R}\rm K}^2 \widehat{\phi}^{n-2} + r_{\mathcal{R}\rm K}^2 \widehat{\phi}^n \cdots\right)\dot{\widehat{\phi}}^2 \quad \text{and} \quad \widehat{V}_{\rm CI} = \frac{\lambda^2 \widehat{\phi}^n}{2 c_{\rm K}^{n/2}}\left(1 - 2r_{\mathcal{R}\rm K}\widehat{\phi}^{\frac{n}{2}} + 3r_{\mathcal{R}\rm K}^2 \widehat{\phi}^n - 4r_{\mathcal{R}\rm K}^3 \widehat{\phi}^{\frac{3n}{2}} + \cdots\right),$$

Consequently, **No Problem** With The Perturbative Unitarity Emerges for $r_{\mathcal{R}\rm K} \leq 1$, Even If $c_{\mathcal{R}}$ and $c_{\rm K}$ Are Large.

[4] J.L.F. Barbon and J.R. Espinosa (2009); C.P. Burgess, H.M. Lee, and M. Trott (2010); A. Kehagias, A.M. Dizgah, and A. Riotto (2013)

Jordan Frame Computation

- If we Expand $g_{\mu\nu}$ About The Flat Spacetime Metric $\eta_{\mu\nu}$ and ϕ About Its V.E.V As Follows

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu} \quad \text{and} \quad \phi = \langle\phi\rangle + \delta\phi \quad \text{Where} \quad \langle\phi\rangle \simeq 0.$$

The Lagrangian Corresponding To The Two First Terms In The Right-Hand Side Of $\mathcal{S}$ Takes The Form ($\langle\phi\rangle = 0$):

$$\delta\mathcal{L} = -\frac{\langle f_{\mathcal{R}}\rangle}{4} F_{\rm EH}\left(h^{\mu\nu}\right) + \frac{\langle f_{\rm K}\rangle}{2}\partial_\mu\delta\phi\partial^\mu\delta\phi + \left(\langle f_{\mathcal{R},\phi}\rangle\delta\phi + \frac{1}{2}\langle f_{\mathcal{R},\phi\phi}\rangle\delta\phi^2 + \frac{1}{6}\langle f_{\mathcal{R},\phi\phi\phi}\rangle\delta\phi^3 + \cdots\right) F_{\mathcal{R}}$$

$$= -\frac{1}{8} F_{\rm EH}\left(\bar{h}^{\mu\nu}\right) + \frac{1}{2}\partial_\mu\overline{\delta\phi}\partial^\mu\overline{\delta\phi} + \Lambda_{\rm UV}^{-1}\overline{\delta\phi}^{n/2}\,\Box\bar{h}\,, \quad \left(\text{The Only non-Vanishing Term is } (n/2)!\,c_{\mathcal{R}}\delta\phi^{n/2}\right)$$

Where The Functions $F_{\rm EH}$ and $F_{\mathcal{R}}$ Read: $F_{\rm EH} = h^{\mu\nu}\Box h_{\mu\nu} - h\Box h + 2\partial_\rho h^{\mu\rho}\partial^\nu h_{\mu\nu} - 2\partial_\nu h^{\mu\nu}\partial_\mu h$ and $F_{\mathcal{R}} = \Box h - \partial_\mu\partial_\nu h^{\mu\nu}$

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- The **JF Canonically Normalized Fields** $\bar{h}_{\mu\nu}$ and $\overline{\delta\phi}$ Are Defined By The Relations:

$$\overline{\delta\phi} = \sqrt{\frac{\langle\bar{f}_{\mathcal{R}}\rangle}{\langle f_{\mathcal{R}}\rangle}}\delta\phi \quad \text{and} \quad \bar{h}_{\mu\nu} = \sqrt{\langle f_{\mathcal{R}}\rangle}\,h_{\mu\nu} + \frac{\langle f_{\mathcal{R},\phi}\rangle}{\sqrt{\langle f_{\mathcal{R}}\rangle}}\eta_{\mu\nu}\delta\phi \quad \text{with} \quad \bar{f}_{\mathcal{R}} = f_{\rm K}f_{\mathcal{R}} + \frac{3}{2}f_{\mathcal{R},\phi}^2, \quad \text{With} \quad \langle f_{\mathcal{R}}\rangle = 1 \quad \text{and} \quad \langle f_{\mathcal{R},\phi}\rangle = 0 \quad \text{for } n > 2.$$

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$$\mathcal{A} \sim \left(\frac{E}{\Lambda_{\rm UV}}\right)^2 \quad \text{With} \quad \Lambda_{\rm UV} \simeq \frac{1}{c_{\mathcal{R}}}\frac{\langle \bar f_{\mathcal{R}}\rangle^{n/4}}{\langle f_{\mathcal{R}}\rangle^{(n-2)/4}} = \frac{\langle f_{\rm K}\rangle^{n/4}}{c_{\mathcal{R}}} = r_{\mathcal{R}\rm K}^{-1} \quad \text{if } \langle f_{\rm K}\rangle = c_{\rm K} = \left(c_{\mathcal{R}}/r_{\mathcal{R}\rm K}\right)^{4/n}$$

Therefore, We Verify That Perturbative Unitarity Is Retained Up to Planck Scale, if $r_{\mathcal{R}\rm K} \leq 1$. For These Reasons

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> We Propose to Analyze Models of **Kinetically Modified non-MCI** With $f_{\rm K} = c_{\rm K}f_{\mathcal{R}}^m$ Where $c_{\rm K} = (c_{\mathcal{R}}/r_{\mathcal{R}\rm K})^{4/n}$

Realization of non-MCI Within SUGRA (Using a Gauge Singlet Inflaton)

- The General **EF** Action For The Scalar Fields Φ^α Plus Gravity In Four Dimensional $\mathcal{N}=1$ SUGRA is:

$$\mathcal{S}=\int d^4x\sqrt{-\widehat{\mathfrak{g}}}\left(-\frac{1}{2}\widehat{\mathcal{R}}+K_{\alpha\bar\beta}\widehat{g}^{\mu\nu}\partial_\mu\Phi^\alpha\partial_\nu\Phi^{*\bar\beta}-\widehat{V}\right)\quad \text{Where}\quad \widehat{V}=\widehat{V}_{\rm F}=e^K\left(K^{\alpha\bar\beta}(D_\alpha W)(D^*_{\bar\beta}W^*)-3|W|^2\right),$$

$$K\ \text{ is The Kähler Potential With }\ K_{\alpha\bar\beta}=\frac{\partial^2 K}{\partial\Phi^\alpha\partial\Phi^{*\bar\beta}}>0\ \text{ and }\ K^{\alpha\bar\beta}K_{\alpha\bar\gamma}=\delta^{\bar\beta}_{\bar\gamma};\quad D_\alpha W=W_{,\Phi^\alpha}+K_{,\Phi^\alpha}W.$$

Therefore, Implementing non-MCI Within SUGRA Requires The Appropriate Selection of W and K

[5] M.B. Einhorn and D.R.T. Jones (2010); S. Ferrara et al. (2010, 2011); H.M. Lee (2010); C.P. and N. Toumbas (2011).

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- If We Set [5] $K=-3\ln\left(-\Omega/3\right)$ Where **Ω is The Frame Function** and Perform a Conformal Transformation, **$\mathcal{S}$ In JF** Reads

$$\mathcal{S}=\int d^4x\sqrt{-\mathfrak{g}}\left(\frac{\Omega}{3}\frac{\mathcal{R}}{2}+\Omega_{\alpha\bar\beta}\partial_\mu\Phi^\alpha\partial^\mu\Phi^{*\bar\beta}-\Omega\mathcal{A}_\mu\mathcal{A}^\mu-V\right),\ \ \text{With}\ \mathcal{A}_\mu=-i\left(\partial_\mu\Phi^\alpha\Omega_\alpha-\partial_\mu\Phi^{*\bar\alpha}\Omega_{\bar\alpha}\right)/2\Omega$$

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Selecting Conveniently the Superpotential and Frame Function

- We Use 2 Superfields $\Phi^1 = \Phi$ (**Inflaton**) and $\Phi^2 = S$ (**"Stabilized" Field**) And Adopt the Following Superpotential:

$$W = \lambda S\,\Phi^{n/2}$$

From Which in the The SUSY Limit We Get $V_{\rm CI} = \lambda^2\,|\Phi|^n + \lambda^2|S|^2$

Charge Assignments

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$$\Phi \to \Phi^*, \ \Phi \to \Phi + c \quad \text{and} \quad S \to e^{i\alpha} S\,, \quad \text{in the Limits} \quad c_{\mathcal{R}} \to 0 \ \& \ \lambda \to 0$$

Framework Of Inflationary Analysis

- Expanding Φ and S in Real And Imaginary Parts as Follows:

  $\Phi = \phi e^{i\theta}/\sqrt{2}$ and $S = (s + i\bar{s})/\sqrt{2}$ We Obtain $\widehat{V}_{\rm CI} = \lambda^2\phi^n/(1 + c_{\mathcal{R}}\phi^{n/2})^2$ (**No Dependence on m and $c_{\rm K}$** Arises)

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$$\left.\frac{\partial V}{\partial\widehat{\chi}^{\alpha}}\right|_{s=\bar{s}=\theta=0} = 0 \quad \text{and} \quad \widehat{m}^2_{\chi^\alpha} > 0 \quad \text{Where} \quad \widehat{m}^2_{\chi^\alpha} = \text{Egv}\left[\widehat{M}^2_{\alpha\beta}\right] \quad \text{With} \quad \widehat{M}^2_{\alpha\beta} = \left.\frac{\partial^2 V}{\partial\widehat{\chi}^{\alpha}\partial\widehat{\chi}^{\beta}}\right|_{\theta=s=\bar{s}=0} \quad \text{and} \quad \chi^\alpha = \theta, s, \bar{s}.$$

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The Mass Spectrum Along The Inflationary Trajectory

Fields	Eingestates	Masses Squared
1 Real Scalar	$\widehat{\theta}$	$\widehat{m}^2_\theta \simeq 4\widehat{V}_{\rm CI}/3 \simeq 4H^2_{\rm CI}$
2 Real Scalars	$\widehat{s},\ \widehat{\bar{s}}$	$\widehat{m}^2_s \simeq 2(6k_S f_{\mathcal{R}} - 1)\widehat{V}_{\rm CI}/3$
2 Weyl Spinors	$\widehat{\psi}_\pm = (\widehat{\psi}_\Phi \pm \widehat{\psi}_S)/\sqrt{2}$	$\widehat{m}^2_{\psi\pm} \simeq n^2\widehat{V}_{\rm CI}/2c_{\rm K}\phi^2 f_{\mathcal{R}}^{1+m}$

- We Observe the Following:
  - $\forall\,\alpha,\ \widehat{m}^2_{\chi^\alpha} > 0$. Especially $\widehat{m}^2_s > 0 \;\Leftrightarrow\; k_S > (0.5 - 1.5)$;
  - $\forall\,\alpha,\ \widehat{m}^2_{\chi^\alpha} > \widehat{H}^2_{\rm CI}$ and So Any Inflationary Perturbations Of The Fields Other Than The Inflaton Are Safely Eliminated;
  - The One-Loop **Radiative Corrections** (RCs) à la Coleman-Weinberg to $\widehat{V}_{\rm CI}$ Have The Usual Form:

$$\Delta\widehat{V}_{\rm CI} = \frac{1}{64\pi^2}\left(\widehat{m}^4_\theta \ln\frac{\widehat{m}^2_\theta}{\Lambda^2} + 2\widehat{m}^4_s \ln\frac{\widehat{m}^2_s}{\Lambda^2} - 4\widehat{m}^4_{\psi_\pm} \ln\frac{\widehat{m}^2_{\psi\pm}}{\Lambda^2}\right)$$

Where $\Lambda \simeq (1-5)\cdot 10^{14}$ is A Renormalization Group Mass Scale Determined By Requiring $\Delta\widehat{V}_{\rm CI}(\phi_\star) = 0$ or $\Delta\widehat{V}_{\rm CI}(\phi_{\rm f}) = 0$. Under These Circumstances, $\Delta\widehat{V}_{\rm CI}$ Has **No Significant Effect** On The Results.

Approximating the Inflationary Dynamics

- The Slow-Roll Parameters Are Determined Using the Standard Formulae in the EF:
$$\widehat{\epsilon} = n^2/(2\phi^2 c_{\rm K} f_{\mathcal{R}}^{1+m}) \quad \text{and} \quad \widehat{\eta} = \Big(2\,(1-1/n) - \Big(4 + n(1+m)c_{\mathcal{R}}\phi^{\frac{n}{2}}/2n\Big)\Big)\widehat{\epsilon}\,.$$

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$$\widehat{N}_\star \simeq \frac{c_{\rm K}\phi_\star^2}{2n}\,{}_2F_1\left(-m, \frac{4}{n}; 1+\frac{4}{n}; -c_{\mathcal{R}}\phi_\star^{n/2}\right) \;\Rightarrow\; \phi_\star \simeq \begin{cases} \sqrt{\frac{2n\widehat{N}_\star}{c_{\rm K}}} & \text{for } m=0, \text{ Since } {}_2F_1\left(0, \frac{4}{n}; 1+\frac{4}{n}; -c_{\mathcal{R}}\phi_\star^{n/2}\right) = 1; \\ \sqrt{\frac{f_{m\star}-1}{r_{\mathcal{R}\rm K}c_{\rm K}}} & \text{for } n=4, \text{ Since } {}_2F_1\left(-m, 1; 2; -c_{\mathcal{R}}\phi_\star^{n/2}\right) = \frac{f_{\mathcal{R}}^{1+m}-1}{(1+m)c_{\mathcal{R}}\phi_\star^2}. \end{cases}$$

Here ${}_2F_1$ is the Gauss Hypergeometric Function and $f_{m\star} = \left(1 + 8(m+1)r_{\mathcal{R}\rm K}\widehat{N}_\star\right)^{1/(1+m)}$.

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- For Every m and n, There is a Lower Bound on $c_{\rm K}$, Above Which $\phi_\star < 1$.
- The Power Spectrum Normalization Implies A **Dependence of λ on $c_{\rm K}$** for Every $r_{\mathcal{R}\rm K}$

$$\lambda = \sqrt{3A_{\rm s}}\pi\cdot\begin{cases} \left(c_{\rm K}/n\widehat{N}_\star\right)^{\frac{n}{4}}\left(2nf_{n\star}/\widehat{N}_\star\right)^{\frac{1}{2}} & \text{for } m=0, \\ 16c_{\rm K}r_{\mathcal{R}\rm K}^{3/2}/(f_{m\star}-1)^{\frac{3}{2}}\,f_{m\star}^{\frac{1+m}{2}} & \text{for } n=4, \end{cases} \quad \text{where } f_{n\star} = f_{\mathcal{R}}(\phi_\star) = 1 + r_{\mathcal{R}\rm K}(2n\widehat{N}_\star)^{n/4}.$$

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$$\widehat{N}_\star \simeq \frac{c_{\rm K}\phi_\star^2}{2n}\, {}_2F_1\left(-m, \frac{4}{n}; 1+\frac{4}{n}; -c_{\mathcal{R}}\phi_\star^{n/2}\right) \Rightarrow \phi_\star \simeq \begin{cases} \sqrt{\dfrac{2n\widehat{N}_\star}{c_{\rm K}}} & \text{for } m=0, \text{ Since } {}_2F_1\left(0, \frac{4}{n}; 1+\frac{4}{n}; -c_{\mathcal{R}}\phi_\star^{n/2}\right) = 1; \\ \sqrt{\dfrac{f_{m\star}-1}{r_{\mathcal{R}\rm K}c_{\rm K}}} & \text{for } n=4, \text{ Since } {}_2F_1\left(-m, 1; 2; -c_{\mathcal{R}}\phi_\star^{n/2}\right) = \dfrac{f_{\mathcal{R}}^{1+m}-1}{(1+m)c_{\mathcal{R}}\phi_\star^2}. \end{cases}$$

Here ${}_2F_1$ is the Gauss Hypergeometric Function and $f_{m\star} = \left(1 + 8(m+1)r_{\mathcal{R}\rm K}\widehat{N}_\star\right)^{1/(1+m)}$.

- For Every m and n, There is a Lower Bound on $c_{\rm K}$, Above Which $\phi_\star < 1$.
- The Power Spectrum Normalization Implies A **Dependence of λ on $c_{\rm K}$** for Every $r_{\mathcal{R}\rm K}$

$$\lambda = \sqrt{3A_{\rm s}}\pi \cdot \begin{cases} \left(c_{\rm K}/n\widehat{N}_\star\right)^{\frac{n}{4}} \left(2nf_{n\star}/\widehat{N}_\star\right)^{\frac{1}{2}} & \text{for } m=0, \\ 16c_{\rm K}r_{\mathcal{R}\rm K}^{3/2}/(f_{m\star}-1)^{\frac{3}{2}} f_{m\star}^{\frac{1+m}{2}} & \text{for } n=4, \end{cases} \quad \text{where } f_{n\star} = f_{\mathcal{R}}(\phi_\star) = 1 + r_{\mathcal{R}\rm K}(2n\widehat{N}_\star)^{n/4}.$$

- A Clear Efficient **Dependence of The Observables** On $r_{\mathcal{R}\rm K}$ Arises, I.e.,

$$n_{\rm s} = 1 - (f_{m\star}^{1+m}-1)\frac{m-1+(m+2)f_{m\star}}{(f_{m\star}-1)f_{m\star}^{1+m}(1+m)\widehat{N}_\star},\ r = \frac{16(f_{m\star}^{1+m}-1)}{(f_{m\star}-1)f_{m\star}^{1+m}(1+m)\widehat{N}_\star},$$

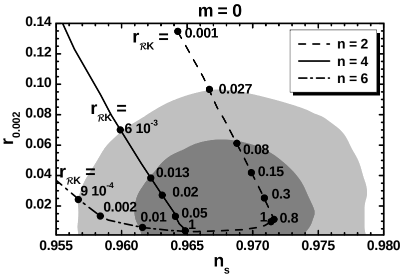
$$\alpha_{\rm s} = \frac{f_{m\star}^{-2(1+m)}}{(1+m)\widehat{N}_\star}\frac{(f_{m\star}^{1+m}-1)^2}{(f_{m\star}-1)^2}\left(2f_{m\star}(1+f_{m\star}) + 3(f_{m\star}-1)f_{m\star}m + (f_{m\star}-1)^2m^2 - 1\right)$$

- E.g., Expanding the Relevant Formulas for $1/\widehat{N}_\star \ll 1$ We Find **for $n=4$ and $m=1$**:

$$n_{\rm s} \simeq 1 - 3/2\widehat{N}_\star - 3/8(\widehat{N}_\star^3 r_{\mathcal{R}\rm K})^{1/2},\ r \simeq 1/2\widehat{N}_\star^2 r_{\mathcal{R}\rm K} + 2/(\widehat{N}_\star^3 r_{\mathcal{R}\rm K})^{1/2},\ \alpha_{\rm s} \simeq -3/2\widehat{N}_\star^2 - 9/16(\widehat{N}_\star^5 r_{\mathcal{R}\rm K})^{1/2}$$

Testing Against Observations

- Imposing the *Planck* Constraints for $\widehat{N}_\star = 55$, $k_S = 0.5 - 1$ and $k_\Phi = 1$ we Obtain the Following Allowed Curves:



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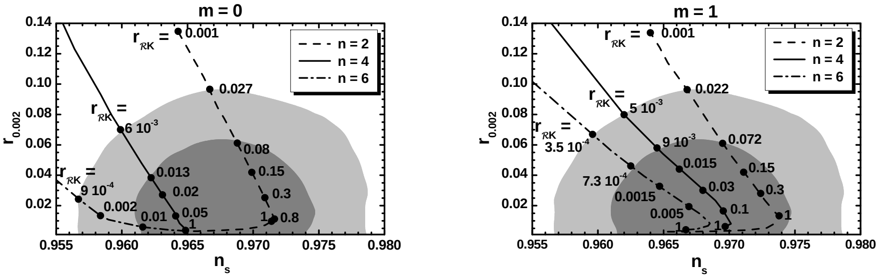

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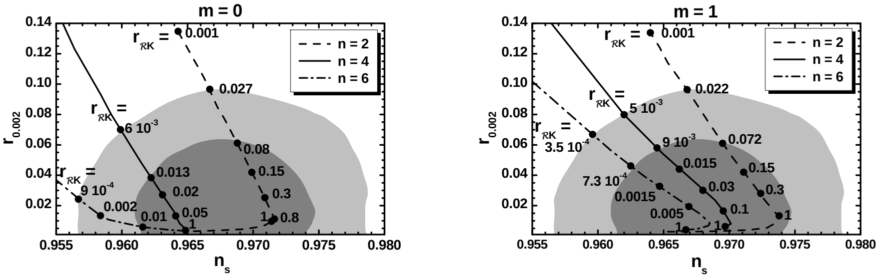

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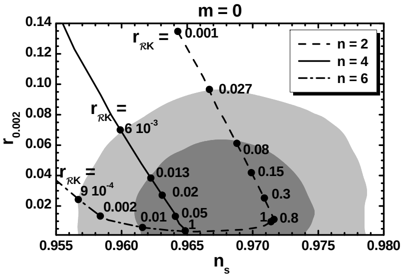

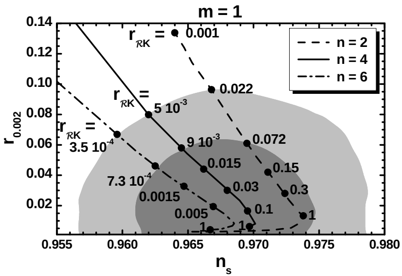

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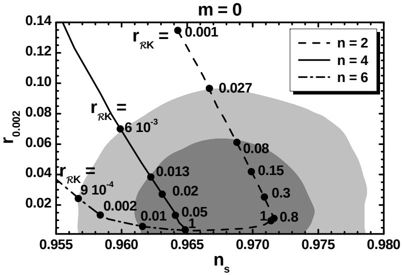

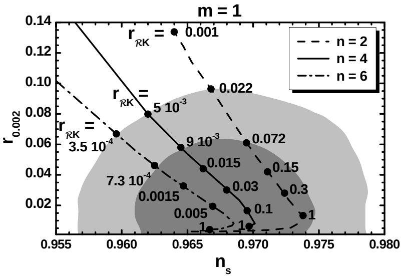

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  - The Requirement $r_{\mathcal{R}K} \leq 1$ Provides **a Lower Bound on r**, Which Ranges From 0.0032 (for $m = 0$ and $n = 6$) to 0.015 (for $m = 4$ and $n = 4$).

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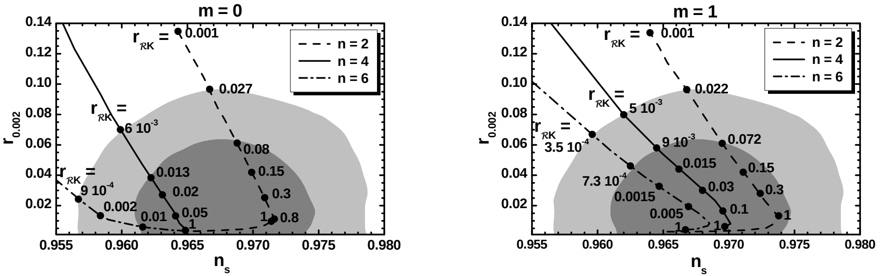

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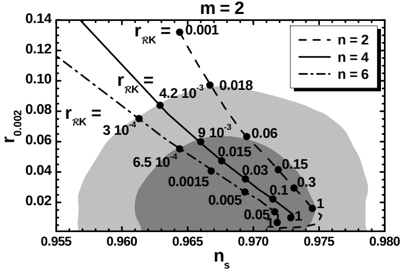
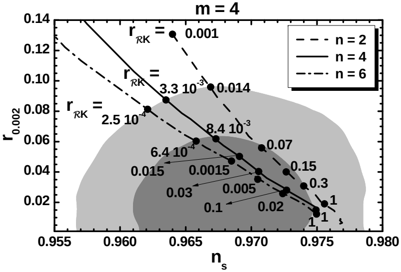
  - The $n = 2$ Line Approaches **an Attractor Value** for $c_\mathcal{R} \gg 1$ any m.

- For $m = 2$ & $m = 4$ The Curves Move More And More To The Right And Tend To Go Further From The 1-σ Ranges:





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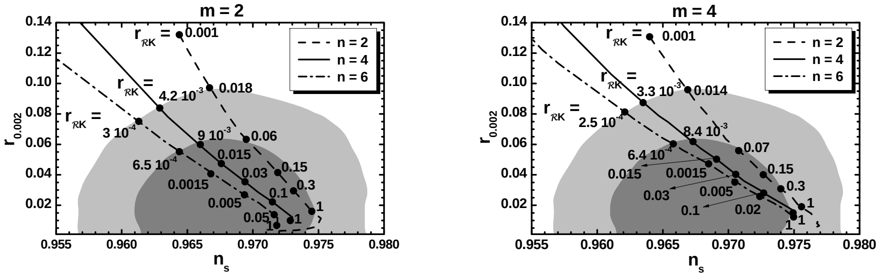

CONCLUSIONS

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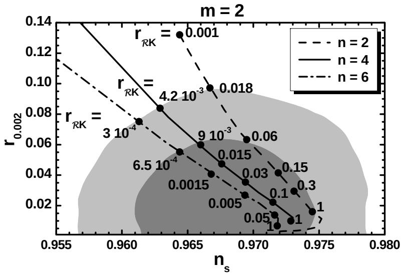

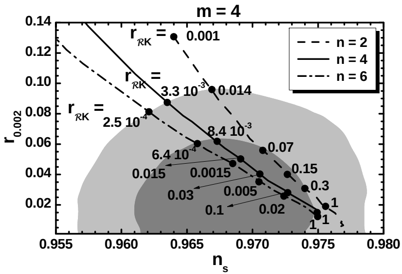

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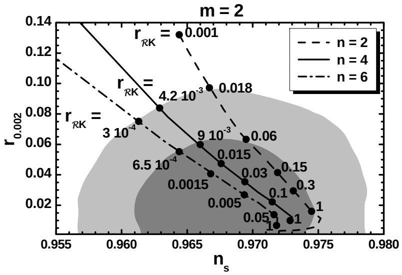

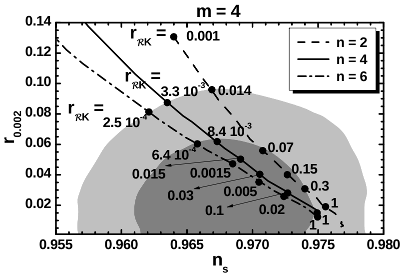

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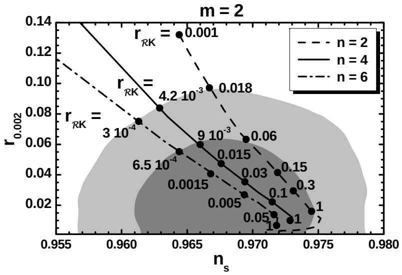

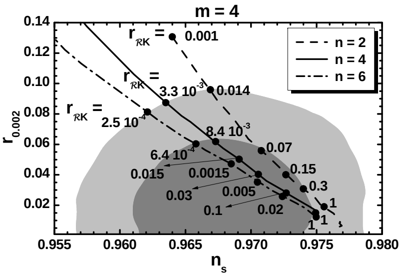

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