Cosmology and quantum gravities: Where are we?

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March 30th, 2015

Outline

1. Quantum gravity?

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2. Cosmological problems

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Goal

Systematic introduction and comparison of the status of the most prominent theories of quantum and emergent gravity in relation to cosmology.

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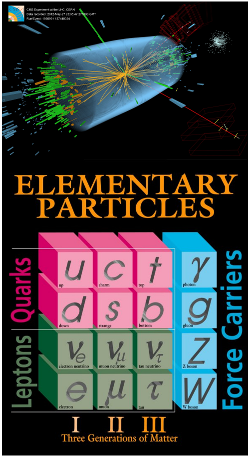
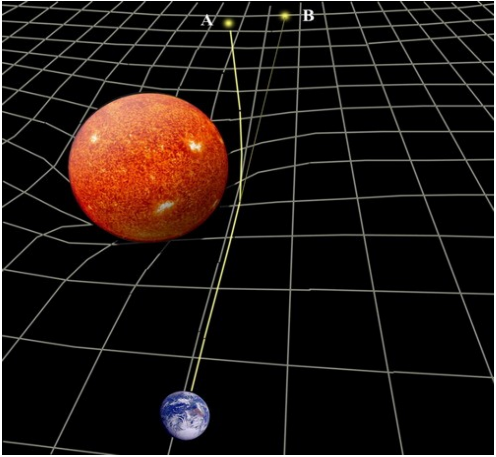
02/27– Particles and gravity





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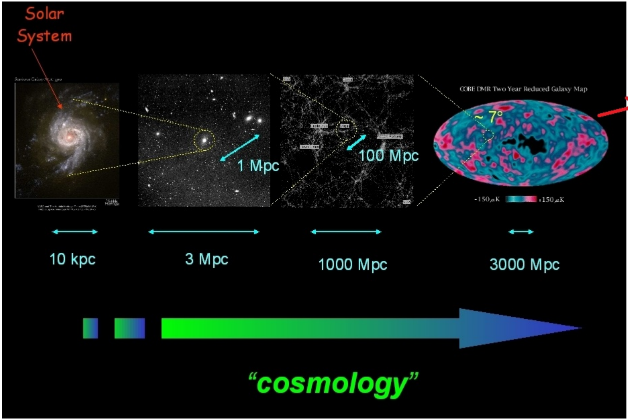
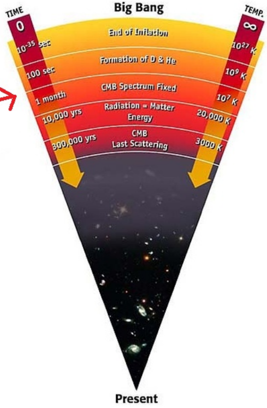
- Non-predictive gravity if quantized perturbatively [Goroff & Sagnotti 1985,1986].
- Models of theories of everything and quantum gravity are very formal and with little contact with observations.
- Cosmological problems must be addressed.

04/27– Cosmology and quantum gravity





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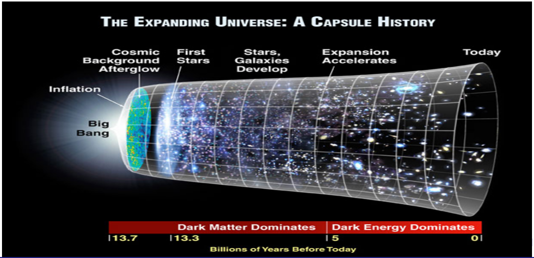
2. Cosmological problems
3. Quantum and emergent gravities
4. Final remarks

05/27– Big bang problem

- Singularities typical of classical gravity (black holes, big bang).
- Borde–Guth–Vilenkin theorem (2003): *Let* $(\mathcal{M}, g)$ *be a spacetime with a congruence* u^μ *continuously defined along any past-directed timelike or null geodesic* v^μ (the observer). *Let* u^μ *obey the averaged expansion condition* $\mathscr{H}_{\rm av} > 0$ *for almost any* v^μ. *Then* $(\mathcal{M}, g)$ *is geodesically past-incomplete* (finite proper/affine length of geodesics).



06/27– Inflation

- Graceful exit.
- Trans-Planckian problem.
- Model building.

07/27– Cosmological constant problems

- Old problem: zero-point energy (dim. reg. [Koksma & Prokopec 2011]) $\rho_{\rm vac} \sim 10^{-68} m_{\rm Pl}^4 \sim 10^{56} \rho_\Lambda$ wrong magnitude. E.g., $\rho_{\rm eq} \approx 2.4 \times 10^{-113} m_{\rm Pl}^4$ is calculable.

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- Shift symmetry: $\mathcal{L}_{\text{m}} \to \mathcal{L}_{\text{m}} + \rho_0 \quad \Rightarrow \quad T_\mu^{\ \nu} \to T_\mu^{\ \nu} + \rho_0 \delta_\mu^\nu$. E.o.m.s $\nabla_\nu T_\mu^{\ \nu} = 0$ invariant, Einstein eqs. are not: Why?

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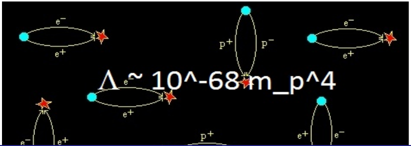
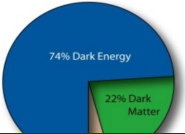
- 4π puzzle: For the observed value of ρ_Λ, the duration of the matter-radiation era (# modes reentered) is $4\pi \pm 10^{-3}$ e-folds [Padmanabhan 2012].





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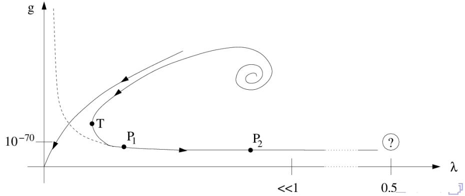
08/27– Asymptotic safety: setting

Weinberg, Reuter, Bonanno, Lauscher, Litim, Saueressig, . . .

- All dimensionless couplings approach a UV NGFP $\lim_{k\to\infty}\bar{\lambda}_i(k)=\bar{\lambda}_i^*\neq 0$ (existence checked a posteriori).
- Gravity: effective action

$$\Gamma_k=\frac{1}{16\pi G_k}\int \mathrm{d}^D x\sqrt{-g}\,(R-2\Lambda_k)\,,\qquad \frac{\delta\Gamma_k}{\delta g_{\mu\nu}}[\langle g_{\mu\nu}\rangle_k]=0$$

- Λ and average metric are scale-dependent: $\langle g_{\mu\nu}\rangle_k=k^{-2}\langle g_{\mu\nu}\rangle_{k_0}$, $\Lambda_k=k^2\Lambda_{k_0}$ as $k\to\infty$.



09/27– Asymptotic safety: cosmology

Cutoff identification $k = k(t) \propto H(t) \quad \Rightarrow \quad \Lambda, G \to \Lambda(t), G(t)$.
RG-improved dynamics:

$$H^2 = \frac{8\pi G(t)}{3}\rho + \frac{\Lambda(t)}{3}, \qquad \dot{\rho} + 3H(\rho + P) = -\frac{\dot{\Lambda} + 8\pi\rho\dot{G}}{8\pi G}$$

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• Near the NGFP: $\tilde G(p^2) \simeq -1/p^4$ for $p^2 \gg m_{\rm Pl}^2$,
$\langle h(t,\mathbf{x})h(t,0)\rangle \sim \ln|\mathbf{x}|^2$, $\langle \delta R(t,\mathbf{x})\delta R(t,0)\rangle \sim |\mathbf{x}|^{-4}$ for $\delta R \sim \partial^2 h$.
Scale-invariant power spectrum.

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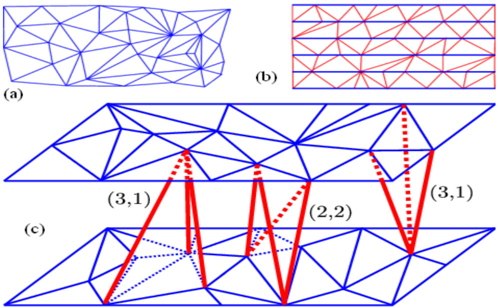
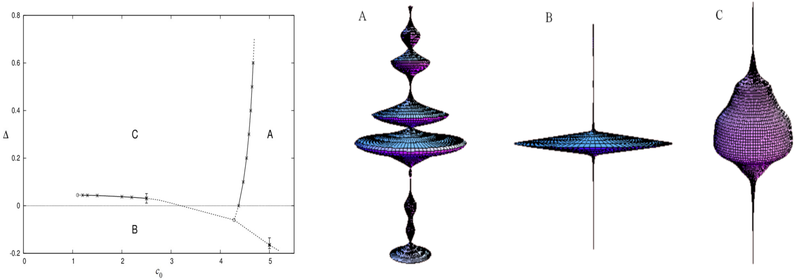
10/27– Causal dynamical triangulations: setting

Ambjørn, Loll, Jurkiewicz, . . .

$$Z = \int [\mathcal{D}g]\, e^{iS[g]} \quad \rightarrow \quad \sum_T \frac{1}{\mathrm{Aut}(T)}\, e^{-S_E^{\mathrm{Regge}}(T)} \,.$$



11/27– Causal dynamical triangulations: cosmology



A: branched-polymer phase, disconnected "lumps" of space, non-Riemannian geometry.
B: crumpled phase, vanishing temporal extension and almost no spatial extension (many simplices clustered around very few vertices).
C: semi-classical de Sitter universe (several checks).

12/27– Non-local gravity: setting

Krasnikov, Tomboulis, Mazumdar, Modesto, G.C., . . .

Minimal requirements: (i) continuous spacetime with Lorentz invariance; (ii) classical local (super)gravity good approximation at low energy; (iii) perturbative super-renormalizability or finiteness; (iv) unitary and ghost free; (v) typical classical solutions singularity-free.

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Example [G.C. & Modesto 2014]:

$$S_g = \frac{1}{2\kappa^2}\int \mathrm{d}^D x\sqrt{-g}\left[R - 2\Lambda - G_{\mu\nu}\,\frac{\mathrm{e}^{-f(\Box/M^2)}-1}{\Box}\,R^{\mu\nu}\right].$$

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Reproduces the linearized effective action of string field theory when $f = \Box/M^2$. Exponential operators have good properties (Cauchy problem well defined, etc.).

13/27– Non-local gravity: cosmology

G.C., Modesto, Nicolini 2014

Typical classical bouncing profiles in $D = 4$:

$$a(t) = a_* \cosh\left(\sqrt{\frac{\omega}{2}}\,t\right),$$

$$a(t) = a_* \exp\left(\frac{H_1}{2}\,t^2\right).$$

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Difficult dynamics, e.o.m.s still under study [G.C., Modesto & Nardelli in progress].

14/27– Canonical quantum cosmology

DeWitt, Hawking, Vilenkin, Ashtekar, Bojowald, ...

Hamiltonian formalism (unconstrained):

$$S=\int \mathrm{d}t\, L[q,\dot q] \;\to\; H[q,p]=p\dot q - L[q,\dot q] \;\to\; \hat H[\hat q,\hat p=\mathrm{i}\hbar\partial_q]|\psi\rangle = E|\psi\rangle$$

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Gravity$+$matter dynamics (constrained): Wheeler–DeWitt equation $\hat{\mathcal{H}}(g,\phi)\Psi[g,\phi]=0$.

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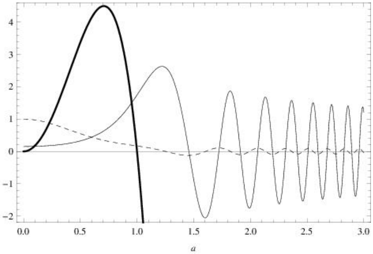
Symmetry reduction. FLRW: $g_{\mu\nu} = (-1, a^2(t), a^2(t), a^2(t))$, $p_{(a)} = -6a\dot a$, $\Pi_\phi = a^3\dot\phi$:

$$\mathcal{H} = \frac{1}{2a^3}\left[-\frac{a^2 p_{(a)}^2}{6\kappa^2} + \Pi_\phi^2\right] + \cdots = 0 \to \hat{\mathcal{H}} = \frac{1}{2a^3}\left[\frac{\kappa^2}{6}\frac{\partial^2}{(\partial \ln a)^2} - \frac{\partial^2}{\partial\phi^2}\right] + \ldots$$

15/27– WDW quantum cosmology



PDF (nucleation probability) of the initial state of the Universe: ratio of the squared wave-function at the classical turning point $a = H^{-1}$ and at $a = 0$, $P(\phi) \sim |\Psi[a = H^{-1}, \phi]/\Psi[a = 0, \phi_{\rm i}]|^2 \sim |\Psi[a = 0, \phi_{\rm i}]|^{-2} \propto \exp[\pm 4/(H^2\kappa^2)]$.

16/27– WDW quantum cosmology and Λ

Probabilistic interpretation [Baum 1983; Hawking 1984; Wu 2008]:

$$P_{\rm V}(\Lambda)=\exp\left(-\frac{3m_{\rm Pl}^2}{2\pi\Lambda}\right)\,,\qquad P_{\rm HH}(\Lambda)=\exp\left(\frac{12}{\kappa^2\Lambda}\right)=\exp\left(\frac{3m_{\rm Pl}^2}{2\pi\Lambda}\right)\,.$$

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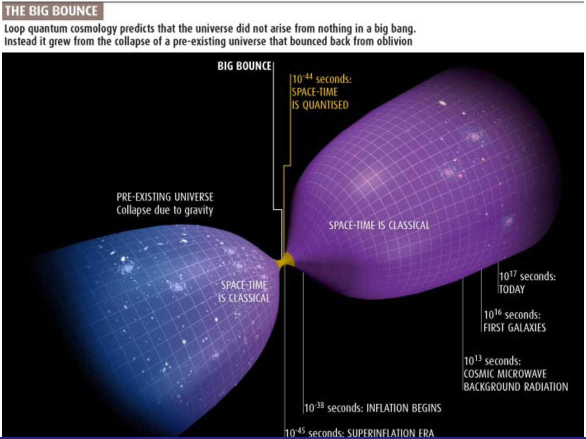
Problem: a Λ-dependent normalization of Ψ may erase the effect. Undecided issue in canonical theory (linear in Ψ).

17/27– Loop quantum cosmology

Other canonical variables, $p = a^2 \to \hat{p}$, $c \sim \dot{a} \to \hat{h} = \widehat{e^{i\mu(p)c}}$.
Quantum bounce ($a = 0$ never):



17/27– Loop quantum cosmology

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1. Bounded spectrum of inverse-volume operator:
   $\widehat{|v|^{l-1}}|v\rangle = \frac{1}{2l}\left(|v+1|^l - |v-1|^l\right)|v\rangle$.
2. State $|v=0\rangle$ disappears from dynamics:
   $c_{v+2}\Psi_{v+4} - (c_{v+2} + c_{v-2})\Psi_v + c_{v-2}\Psi_{v-4} + \langle v|\hat{\mathcal{H}}_\phi|v\rangle\Psi_v = 0$.
3. Volume expectation value (massless field):
   $\langle|\hat{v}|\rangle = \mathcal{V}_* \cosh(\kappa_0\phi)$.
4. Effective dynamics: $\sin^2(\bar{\mu}c) = \frac{\rho}{\rho_*} \leftrightarrow H^2 = \frac{\kappa^2}{3}\rho\left(\alpha - \frac{\rho}{\rho_*}\right)$,
   $\alpha = 1 + \delta_{\rm Pl} = 1 + Ca^{-\sigma}$.

18/27– Quantum gravity and superconductivity

S. Alexander & G.C. PLB 672 (2009) 386; Found. Phys. 38 (2008) 1148

- LQG with Λ in vacuum, Chern–Simons state annihilates the constraints.
- Different gravity vacua connected via large gauge transformations.
- Gravity in a degenerate sector described with fermionic variables, behaves as a **Fermi liquid** (BCS): $\Lambda = \Lambda_0 \exp(-j_5^z) = \Lambda_0 \exp(-\bar{\psi}\gamma^5\gamma^z\psi)$, exponentially suppressed if $\langle \mathbf{j_5} \rangle \sim \mathbf{O}(\mathbf{10^2})$.
- Correspondence made rigorous via a deformed CFT ($SU(2)_{k=2}$, WZW model).

19/27– Group field theory: setting

Freidel, Oriti, Rovelli, . . .

$$S_{\mathrm{GFT}} = \int_G \mathrm{d}^4 g \left[\int_G \mathrm{d}^4 g' \, \varphi^*(g) \mathcal{K}(g, g') \, \varphi(g') + V \right].$$

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- Fock quantization: $[\hat\varphi(g),\,\hat\varphi^\dagger(g')]=\mathbb{1}_G(g,g')$, vacuum $|\emptyset\rangle$ "no-spacetime" configuration, one-particle state $|g\rangle:=\hat\varphi^\dagger(g)|\emptyset\rangle$ 4-valent spin-network vertex or dual tetrahedron, ...

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• ∞ many particles, continuity! All GFT quanta in the same state, homogeneity! Condensate (coherent state):

$$|\xi\rangle := A\,\mathrm{e}^{\hat\xi}|\emptyset\rangle\,, \qquad \hat\xi := \int \mathrm{d}^4 g\,\xi(g)\,\hat\varphi^\dagger(g)\,, \qquad \hat\varphi|\xi\rangle = \xi|\xi\rangle$$

20/27– Group field theory: cosmology

Gielen, Oriti & Sindoni 2014; G.C. Phys. Rev. D 90 (2014) 064047

Gross–Pitaevskii equation:

$$0 = \langle\xi|\mathbb{1}\hat{\mathcal{C}}|\xi\rangle = \int \mathrm{d}^4 g'\, \mathcal{K}(g, g')\xi(g') + \left.\frac{\delta V}{\delta\varphi^*(g)}\right|_{\varphi=\xi}.$$

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$G=SU(2)$, isotropy:

$$2\chi(1-\chi)\xi''(\chi)+(3-4\chi)\xi'(\chi)+m\,\xi(\chi)=0\,,\qquad \chi=\sin^2\left(\frac{\bar{\mu}c}{2}\right)$$

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$$0=\langle\xi|\mathbb{1}\hat{\mathcal{C}}|\xi\rangle=\int \mathrm{d}^4g'\,\mathcal{K}(g,g')\xi(g')+\left.\frac{\delta V}{\delta\varphi^*(g)}\right|_{\varphi=\xi}.$$

$G=SU(2)$, isotropy:

$$2\chi(1-\chi)\xi''(\chi)+(3-4\chi)\xi'(\chi)+m\,\xi(\chi)=0\,,\qquad \chi=\sin^2\left(\frac{\bar\mu c}{2}\right)$$

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$$p_\chi\sim a/(\bar\mu^2H)\,,\qquad p_\phi\sim a^3\dot\phi\,,\qquad (a\bar\mu)^{-2}\propto-\mathcal{E}^2\dot\phi^2\,.$$

$\mathcal{E}^2<0$, l.h.s. is H^2 if $a\propto\mathrm{e}^{Ht}$ (de Sitter) in the LQC improved quantization scheme $n=1/2$.

20/27– Group field theory: cosmology

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LQC dynamics? $4\chi(1-\chi)=\sin^2(\bar{\mu}c)$, l.h.s. of LQC Friedmann eq., r.h.s. depends on form of p_χ. Beyond WKB...

21/27– Causal sets: setting

Bombelli, Dowker, Sorkin, . . .

- “Order and number”: discrete structure of partially ordered points ($x \preceq y$) reproducing the causal structure of continuous Lorentzian geometries.

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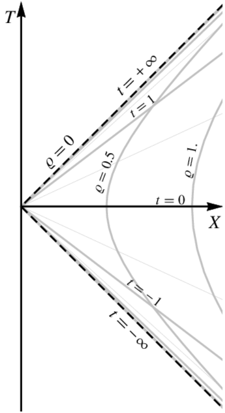
(vii) Effective time-varying $\alpha(t)$? Dynamics?

23/27– Emergent gravity: setting

Padmanabhan

Local Rindler observer (constant proper acceleration)



23/27– Emergent gravity: setting

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Total heat within $\mathcal{V}$:

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Dynamics (for all Rindler observers):

$$\frac{\delta\mathcal{Q}}{\delta n^\mu} = 0 \qquad \Rightarrow \qquad \left(G_{\mu\nu}+\Lambda g_{\mu\nu}-\kappa^2 T_{\mu\nu}\right) n^\mu n^\nu = 0$$

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$$\mathcal{N}_{\rm c}\stackrel{?}{=}N_{\partial\mathcal{V}_{\rm Pl}}\,,\qquad \Lambda\propto {\rm e}^{-N_{\partial\mathcal{V}}/4}?$$

Outline

1. Quantum gravity?
2. Cosmological problems
3. Quantum and emergent gravities
4. Final remarks

25/27– Comparison: How far from realistic cosmology?

- *Asymptotic safety*: types of $f(R)$ actions naturally produced. Λ problem reformulated.
- *Multi-scale spacetimes*: Λ problem reformulated.
- *WDW QC*: probabilistic interpretation for Λ problem.
- Causal dynamical triangulations: de Sitter universe emerges from full quantum gravity.
- Group field theory: cosmology from full theory, LQC dynamics possibly obtained.
- Non-local gravity: big bang removed.
- Loop quantum gravity: big bang removed, Λ as a condensate.
- Causal sets: prediction for Λ. Big bang perhaps removed.
- Emergent gravity: towards a resolution of the Λ problem.

26/27– More can be found in . . .

Classical and Quantum Cosmology (Graduate Texts in Physics, Springer, to appear).

Discussion

どうもありがとうございました！

Thank you!

¡Muchas gracias!

Grazie!

Danke schön!