Formation and evolution of quantum condenastes as orgin of inflation and dark energy

Houri Ziaeepour.

Institut UTINAM (CNRS)
OSU-THETA
Universite de Franche Comté
France

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hep-ph/0412314
arXiv:1003.2996
arXiv:1205.3304
arXiv:1411.0620
arXiv:1502.04308

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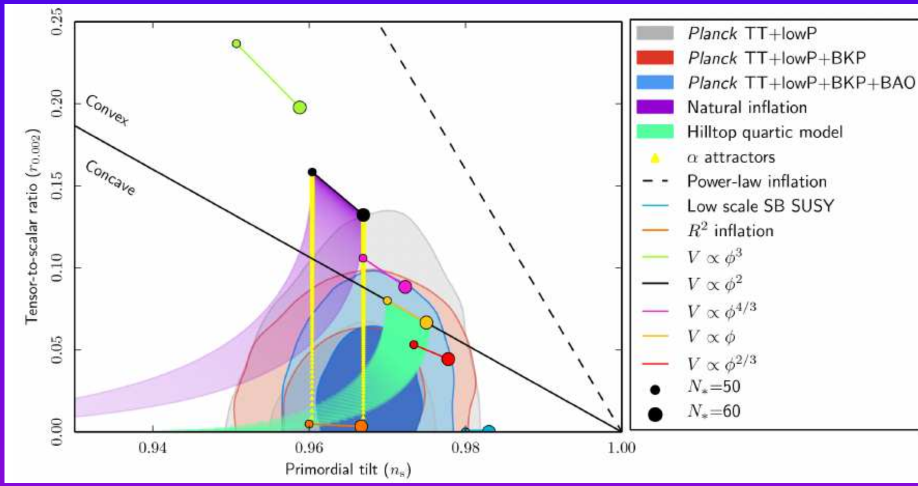
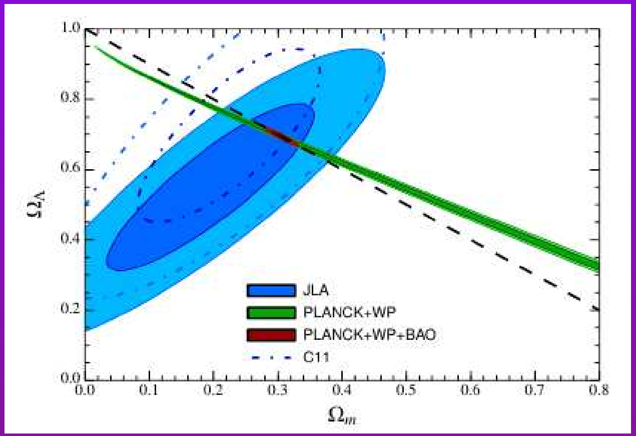
Epochs of accelerating expansion



⋆ During two epochs Universe has had an accelerating expansion:

- ⋆ Inflation in the early Universe.
- ⋆ Since ∼ 6 Myr up to present

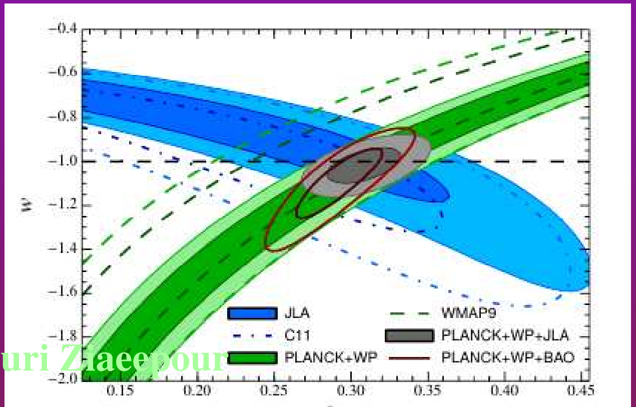


⋆ We call its origin **Dark Energy**

⋆ At present a **Cosmological Constant** is consistent with all observations

⋆ Many models are proposed for both inflation and dark energy

⋆ In many of these models dark energy / inflation can be formulated as scalar field.



[Planck & BICEP2 Collab's. 15, SDSS II Collab. 14]

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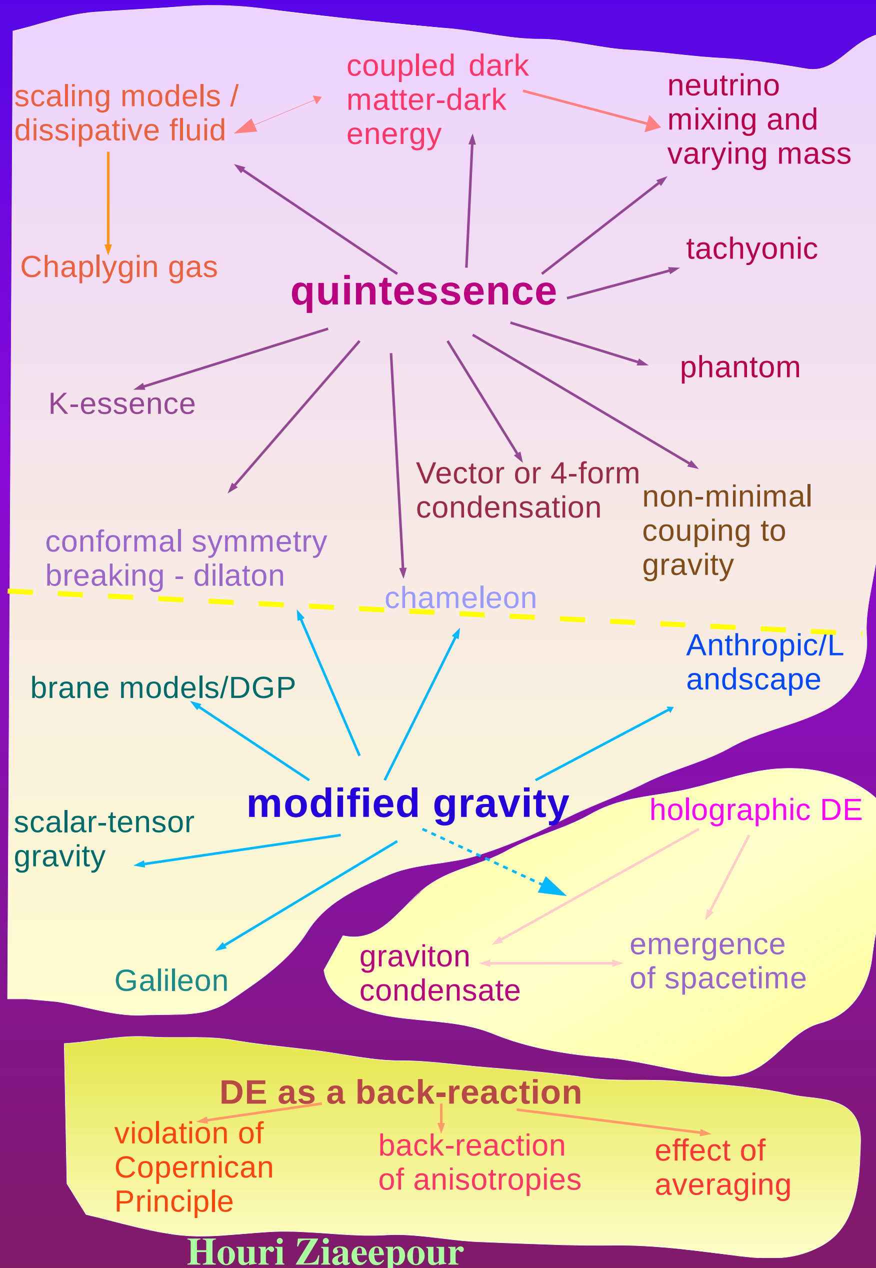


[HZ arXiv:1411.0620]

Universe is ruled by Quantum Mechanics !

- ⋆ Quantum aspects of inflation are studied more extendedly than for dark energy.
- ⋆ Studies are performed in the frame work of non-equilibrium quantum field theory.
  - ⋆ Correlation functions - Green's functions, power spectrum and non-Gaussianity [Vilenkin 83, Starobinsky & Yokoyama 94, Weinberg 06, Seery 07, Kühne & Schwarz 08,]
  - ⋆ Effective potential of inflation and particle production, **directly observable** [Boyanovsky & de Vega 04, Yokoyama 04, Collins & Holman 05]
  - ⋆ Preheating and reheating [Yokoyama 95, 04, Garbrecht & Rigopoulos 11, Gautier & Serreau 13, . . .]
- ⋆ Mostly studied for $\mathbf{O}(\mathbf{N})$ models;
- ⋆ Usually performed in de Sitter.

Unsolved problems

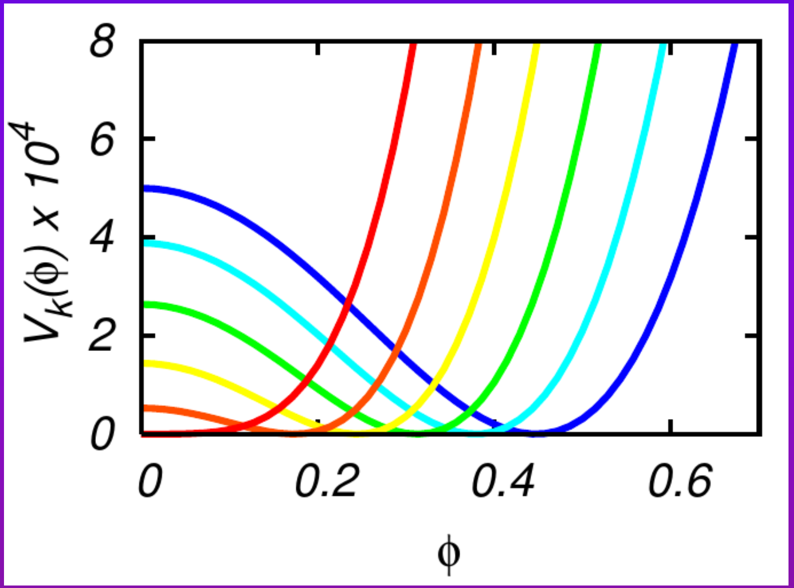
⋆ Does the condensate survive the expansion ?

⋆ **Yes, and symmetry breaks** [Boyanovsky 12]

⋆ **No, and symmetry restored at the end** [Kaya 12, Lazzari & Prokorec 13, Serreau 13]

⋆ IR singularities, specially for massless fields [Starobinsky 79, Giddings & Sloth 10, Marlof & Morrison 10, Hollands 10, 11, Boyanovsky & Holman 11, Higuchi *et al.*11]

⋆ Consistency with Planck depends on parameters and details of the model.



Serreau 13

⋆ Relation between IR behaviour and survival of the condensate ? Not studied

Relation with dark energy

- Many of these questions are relevant also for dark energy.
  - **Survival of condensate is crucial for dark energy.**
- It is more difficult to study the recent expansion:
  - **Conformal symmetry** of de Sitter simplifies solution of equations, etc.
  - Study of inflation in de Sitter is an idealization and does not include backreaction on the metric.
- Evolution of inflation and dark energy models must be performed in a general FLRW geometry.

Condensate and coherent states

- **Condensate state $|\Psi\rangle$:** $\langle\Phi\rangle \equiv \langle\Psi|\Phi|\Psi\rangle = \varphi(\mathbf{x})$
- There is no general expression for condensate states.
- **Coherent states:**
  - Eigen state of annihilation operator $\mathbf{a}_\alpha$:

$$\mathbf{a}_\alpha|\Psi\rangle = \mathbf{C}_\alpha|\Psi\rangle, \quad [\mathbf{a}_\alpha, \mathbf{a}'^{\dagger}_\alpha] = \delta_{\alpha\alpha'}, \quad \langle\Psi|\Phi|\Psi\rangle = \mathbf{C}_\alpha$$

- **Glauber coherent state: Closest to classical QM quantum state [Zhang *et al.*90]:**

$$|\mathbf{C}_\alpha\rangle \equiv \mathbf{e}^{-|\mathbf{C}_\alpha|^2}\mathbf{e}^{\mathbf{C}_\alpha\mathbf{a}^\dagger_\alpha} = \mathbf{e}^{-|\mathbf{C}_\alpha|^2}\sum_{\mathbf{i}=0}\mathbf{C}^{\mathbf{i}}_\alpha/\mathbf{i}!(\mathbf{a}^\dagger_\alpha)^{\mathbf{i}} \quad \mathbf{a}|\mathbf{C}_\alpha\rangle = \mathbf{C}_\alpha|\mathbf{C}_\alpha\rangle$$

- It is superposition of infinite particles with finite energy.
- **Generalization:** Multiple condensates:

$$|\Psi_{\mathbf{GC}}\rangle \equiv \sum_{\mathbf{k}}\mathbf{A}_{\mathbf{k}}\mathbf{e}^{\mathbf{C}_{\mathbf{k}}\mathbf{a}^\dagger_{\mathbf{k}}}|0\rangle = \sum_{\mathbf{k}}\mathbf{A}_{\mathbf{k}}\sum_{\mathbf{i}=0}^{\mathbf{N}\to\infty}\frac{\mathbf{C}^{\mathbf{i}}_{\mathbf{k}}}{\mathbf{i}!}(\mathbf{a}^\dagger_{\mathbf{k}})^{\mathbf{i}}|0\rangle$$

- A frame independent vacuum can be defined based on this state. **HZ arXiv:1205.3304**

A toy model

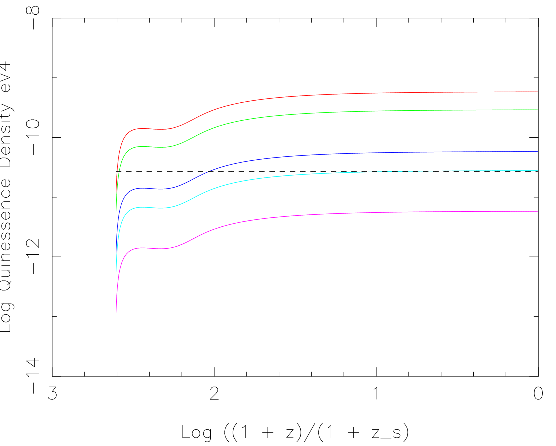
- **Lagrangian:**

$$
\begin{aligned}
\mathcal{L}_{\Phi} &= \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\mu\Phi - \frac{1}{2}m_\Phi^2\Phi^2 - \frac{\lambda}{n}\Phi^n\right] \\
\mathcal{L}_{X} &= \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu X\partial_\mu X - \frac{1}{2}m_X^2 X^2\right] \\
\mathcal{L}_{A} &= \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu A\partial_\mu A - \frac{1}{2}m_A^2 A^2 - \frac{\lambda'}{n'}A^{n'}\right] \\
\mathcal{L}_{int} &= \int d^4x\sqrt{-g}\,g\Phi XA,
\end{aligned}
$$

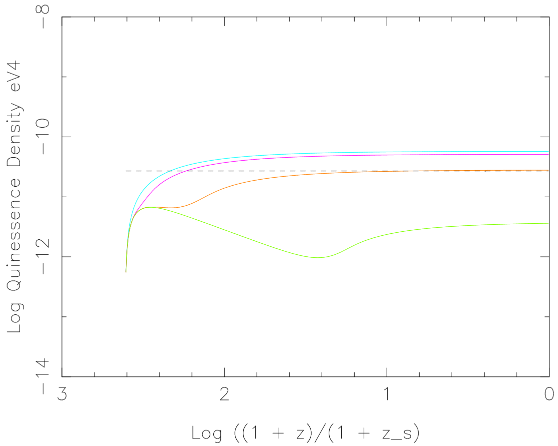
- The fields present 3 physically important mass scales:
  - **X**: A heavy particle, e.g. inflaton or Planck-scale field, a super heavy decaying/interacting dark matter.
  - **A**: Medium mass particle - can be a collective notation, e.g. Standard Model particles.
  - **Φ**: An axion-like light field - Quintessence, dilaton, etc.

Motivations

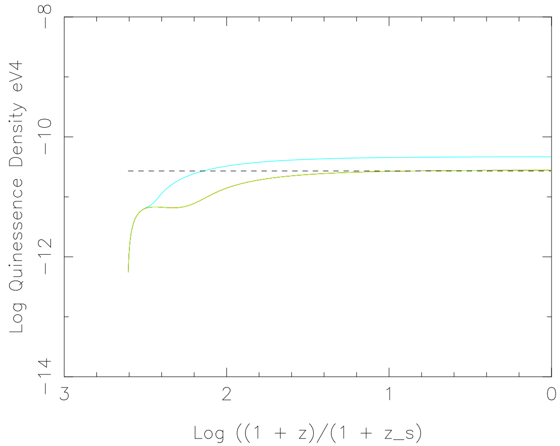
- Quintessence field produced by the decay of dark matter;
- Inherent correlation between dark matter and dark energy;
- Solution for coincidence problem of dark energy;
- In the context of inflation **X** can be an effective field from Planck scale physics.



$\Gamma_0 = \Gamma_q/\Gamma = 10^{-16}$, $5\Gamma_0$, $10\Gamma_0$, $50\Gamma_0$, $100\Gamma_0$. $m_q = 10^{-6}$ eV, $\lambda = 10^{-20}$.



$m_q = 10^{-3}$ eV, $m_q = 10^{-5}$ eV, $m_q = 10^{-6}$ eV, $m_q = 10^{-8}$ eV, $\lambda = 10^{-20}$;



$\lambda = 10^{-10}$, $\lambda = 10^{-15}$, $\lambda = 10^{-20}$, and $\lambda = 10^{-25}$, $m_q = 10^{-6}$ eV.

[HZ astro-ph/0308515, astro-ph/0312606, astro-ph/0406079]

Decomposition

- ⋆ We decompose $\Phi(x)$ to classical (condensate) and quantum components: **[HZ hep-ph/0603125, arXiv:1003.2996]**

$$\Phi(x) = \varphi(x)I + \phi(x) \qquad \langle\Phi\rangle \equiv \langle\Psi|\Phi|\Psi\rangle = \varphi(x) \qquad \langle\phi\rangle \equiv \langle\Psi|\phi|\Psi\rangle = 0$$

- ⋆ We assume that only $\Phi(x)$ forms a condensate $\Longrightarrow \langle X\rangle = \langle A\rangle = 0$
- ⋆ Field equation for the condensate for model (a):

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi) + m_\Phi^2\varphi + \frac{\lambda}{n}\sum_{i=0}^{n-1}(i+1)\binom{n}{i+1}\varphi^i\langle\phi^{n-i-1}\rangle - g\langle XA\rangle = 0$$

- ⋆ **Solution of this equation is the main goal.**
- ⋆ **We must use non-equilibrium QFT methods such as Schwinger-Keldysh in-in method to calculate expectation values which depend on the condensate field.**

2PI formalism

- ⋆ **Evolution of exact propagators:**

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu + M_i^2(x))\, G_i^F(x,y) =$$

$$-\int_\infty^{x^0} d^4z\sqrt{-g(z)}\,\Pi_{ij}^\rho(x,z)\,G_{ij}^F(z,y) + \int_\infty^{y^0} d^4z\sqrt{-g(z)}\,\Pi_{ij}^F(x,z)\,G_{ij}^\rho(z,y)$$

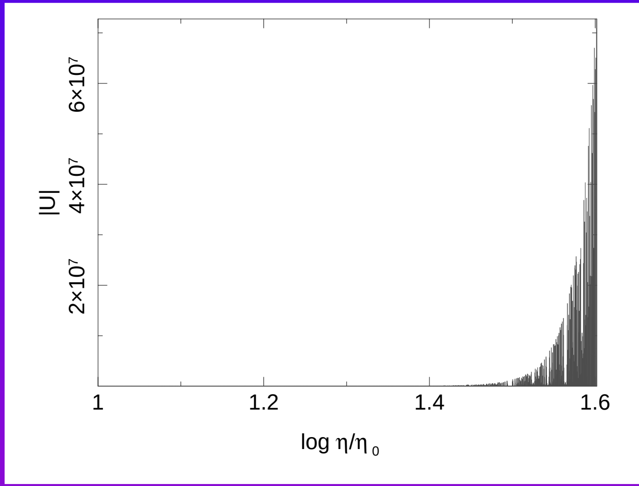
$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu + M_i^2(x))\, G_i^\rho(x,y) = -\int_{y^0}^{x^0} d^4z\sqrt{-g(z)}\,\Pi_{ij}^\rho(x,z)\,G_{ij}^\rho(z,y)$$

$$M_\Phi^2(x) = m_\Phi^2 + \frac{-i\lambda}{(n-2)!}\sum_{j=0}^{[n/2]-1} C_{2j}^{n-2}C_2^{2j}\varphi^{n-2ij}(x)(G_\Phi^F(x,x))^j, \quad M_{X,A}^2 = m_{X,A}^2$$

$$\Pi(\varphi, G) \equiv 2i\frac{\partial\Gamma_2[\varphi, G]}{\partial G}$$

- ⋆ **Approximations:** Free propagators, lowest order 2PI corrections for condensate.
- ⋆ They allow analytical solutions when cosmology is evolved separately.

Simplified Analysis for Radiation and Matter Domination E



⋆ **Matter domination:** The condensate grows exponentially.

⋆ **Matter domination epoch:** Solutions without self-interaction or linearized decrease asymptotically: $\varphi_{\mathbf{k}} \propto \mathbf{t_0/t}$ meaning the condensate does not survive.

⋆ **Full nonlinear equations include terms similar to an effective potential with negative power.**

⋆**Necessary condition for tracking solutions** [Weterisch 89, Steinhardt *et al.*99, Brax *et al.*00, 01]:

$$\mathbf{\Gamma \equiv V"V/V'2 > 1}$$

$$\mathbf{V(\Phi) \propto \Phi^n \Longrightarrow \Gamma = n(n-1)/n^2 > 1,\ n < 0}$$

⋆ **Quantum correction terms of order i have negative component and roughly constant coefficient if** [HZ arXiv:1003.2996]:

$$\mathbf{17 - 6n + 2i \geqslant 0, \quad i < n - 1}$$

Cosmological evolution in 2PI

- ⋆ We use semi-classical Einstein equation **[Ramsey & Hu 97]**:

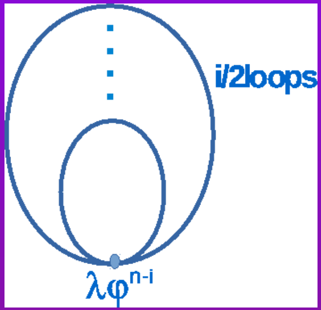
$$\mathbf{R}_{\mu\nu} - \frac{\mathbf{1}}{\mathbf{2}}\mathbf{g}_{\mu\nu}\mathbf{R} = \mathbf{8}\pi\mathbf{G}\mathbf{T}^{\mathbf{eff}}_{\mu\nu}$$

- ⋆ The effective energy-momentum tensor $\mathbf{T}^{\mathbf{eff}}_{\mu\nu}$ is calculated from effective action:

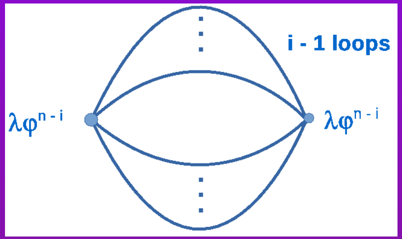
$$\boldsymbol{\Gamma}_{\mathbf{eff}} = \mathbf{S}_{\mathbf{cl}}(\varphi) + \mathbf{i}/\mathbf{2}(\mathrm{tr}\mathbf{G}^{-\mathbf{1}} + \mathrm{tr}\mathbf{G}_{\mathbf{0}}^{-\mathbf{1}}\ \mathbf{G}) + \boldsymbol{\Gamma}_{\mathbf{2}}(\varphi, G)$$

- ⋆ Lowest order diagrams contributing **[HZ arXiv:1502.04308, HZ *et al.*(in preparation)]**

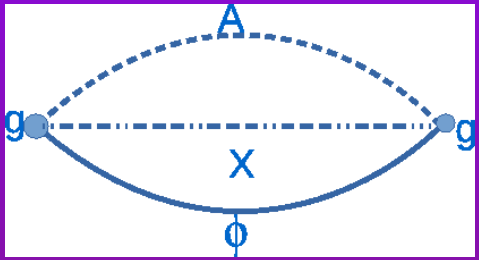
$$\Gamma_2 = \sum_{i=2}^{n} N_1$$



$$+ \sum_{i=2}^{n} N_2$$



$$+$$



$$N_1 = C_i^n i!! \qquad N_2 = i!(C_i^n)^2$$

$$\mathbf{T}^{\mathbf{00}} = \mathbf{T}^{\mathbf{00}}_{\mathbf{cl}}(\varphi) + \mathbf{M}^{\mathbf{2}}\mathbf{i}\mathbf{G}$$

- ⋆ Apriori a condensate is not necessary for accelerating expansion, but Planck results may need it.

Initial conditions

- Solutions of field equations depend on two integration constants $\mathbf{C_k},\ \mathbf{D_k}$ which define the **vacuum** of the model.
- Quantization imposes:

$$|\mathbf{C_k}|^2 + |\mathbf{D_k}|^2 = 1$$ **They cannot be both null for any value of k.**

- Considering $\mathbf{T^{00}_{cl}(t_0) = 0}$, **wave function renormalization** condition:

$$\frac{2}{(2\pi)^3\sqrt{-g(t_0)}}\int d^3k\,\omega_k^2\ |u_k(t_0)|^2 = \frac{3H^2(t_0)}{8\pi G_N}$$

- $\mathbf{H \neq 0 \Longrightarrow}$ **Universe could never be static.**
- In semi-classical gravity, $\mathbf{H}$ is identified with Hubble function.
- In inflation models $\mathbf{H(t_0) \sim 10^{(-3)-(-6)} M_P}$ $\mathbf{3H^2(t_0)/8\pi G_N \gg \rho_{de}(t_{present})}$ [Schwarz *et al.*11].
- **The initial value - related to Planck scale physics - cannot be the Dark Energy.**

Condensation in Early Universe

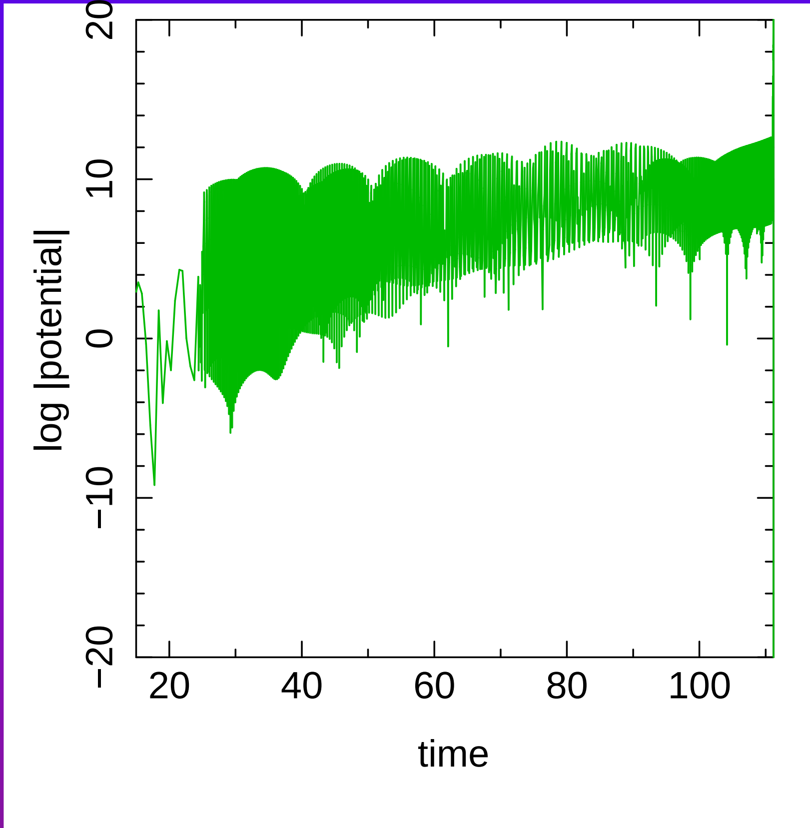
⋆ **Preliminary results are consistent with formation of a condensate with small variations.**

⋆ **Interaction between condensate and non-condensed (particles) is crucial for backreaction process, control of geometry, and decoherence.**

⋆ **Full consideration of backreactions in presence of very high number densities is non-perturbative:**

⋆ Renormalization group technique [Kaya 12, Serreau 13]

⋆ A method similar to Color Glass Condensate used in small-x regime of QCD [McLerran & Venugopalan 94] and in extra-dimension curved spacetimes **[HZ hep-ph/0407046, hep-ph/0412314]**



Outline

- The present expansion of the Universe may be due to quantum superposition and coherence at cosmological distances.
- In this framework the origin of inflation and dark energy may be the same, but this is not a necessity.
- Better theoretical and simulations are necessary to investigate relation between dark energy and inflation.
- According to a definition of what we call *modified gravity*, it is possible to observationally verify whether scalar field responsible for present accelerating expansion belongs to gravity or matter sector **[HZ arXiv:1112.6025]**.
- Along with CMB and LSS findings in other domains of astro-particle physics such as UHECRs, dark matter direct and accelerator phyisics may be necessary to pin down the nature of dark energy. **[HZ arXiv:0709.0115]**