Post-inflationary preheating with weak coupling

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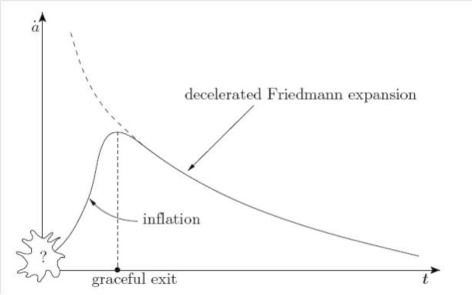
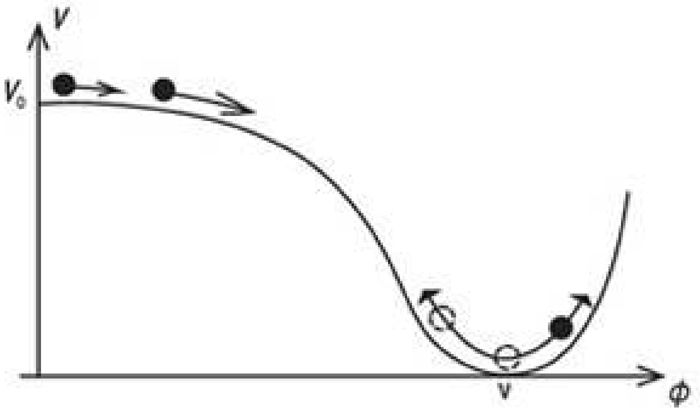
March 19, 2015

Inflation solves the problems of Big Bang theory



1. horizon problem
2. flatness problem
3. initial perturbations problem
4. entropy problem

Reheating



Universe is usually preheated by particle creation in the background of an oscillating inflaton

Mechanism of particle production

Minkowski space

Strong coupling
Parametric resonance

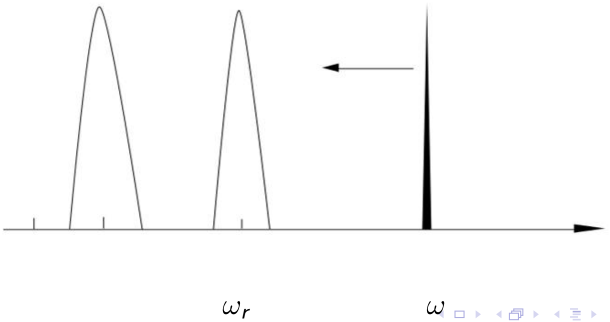
Weak coupling
Parametric resonance

Expanding universe

Strong coupling
Parametric resonance

Weak coupling
Born approximation

???



Parametric resonance in Minkowski space

For the interaction $\mathcal{L}_{int} = -\sigma\phi\varphi^2$ with weak coupling $\sigma\phi_0 \ll M^2$, $\phi(t) = \phi_0(t)\cos Mt$ in the first resonance band $\omega_{res} = \frac{M}{2}$ occupation number of the created particles is

$$N_k = \frac{1}{1 + \Delta^2/\sigma^2\phi_0^2}\sinh^2\lambda t, \tag{1}$$

$$\lambda = \frac{1}{M}\sqrt{\sigma^2\phi_0^2 - \Delta^2}, \quad \Delta = \omega_k^2 - \omega_{res}^2, \quad \omega_k = \sqrt{m_\varphi^2 + k^2} \approx k \tag{2}$$

Total particle number grows asymptotically exponentially with time

Parametric resonance in the expanding universe

Modifications in the case of expanding universe

$$\phi_0 \propto a^{-3/2}, \quad \omega_k = \sqrt{m_\varphi^2 + \frac{k^2}{a^2}} \approx \frac{k}{a} \tag{3}$$

Parametric resonance still occurs if

$$\left|\frac{\dot{\phi}_0}{\phi_0}\right| = \frac{3}{2}H \ll M, \quad \left|\frac{\dot{\lambda}}{\lambda}\right| \ll \lambda \tag{4}$$

$$N_k \simeq \sinh^2 \int \lambda(t) dt \tag{5}$$

If the adiabatic conditions are violated one usually employs the Born approximation

$$\Gamma_\varphi = \frac{\sigma^2}{8\pi M}, \qquad \Gamma_\psi = \frac{\Upsilon^2 M}{8\pi} \tag{6}$$

How to justify this?

Production of bosons

$$\Box\varphi + (m_\varphi^2 + 2\sigma\phi)\varphi = 0, \quad \phi(t) = \phi_0(t)\cos Mt \qquad (7)$$

Violation of the adiabatic conditions means $|\sigma| \leq \sqrt{\frac{3}{8}}\frac{M^2}{M_P}$

It is always true in our case as $\sigma\phi_0 \ll M^2$, $\phi_0 \ll M_P$

Using Bogolubov coefficients method and stationary-phase approximation one gets the result:

Solution for coefficient β

$$\beta_k = \frac{\sigma\phi_0(t_k)}{M^{3/2}}\sqrt{\frac{2\pi}{H(t_k)}}, \quad \omega_k(t_k) = M/2$$

Production of bosons

Energy density of created particles

$$\rho_\varphi(t) = \frac{1}{a^4(t)} \int \frac{d^3k}{(2\pi)^3} \theta(k - k_{min}) \theta(Ma(t) - 2k) k |\beta_k|^2 \tag{8}$$

$$\dot{\rho}_\varphi = -4H\rho_\varphi + \Gamma_\varphi \rho_\varphi$$

$$\Gamma_\varphi \rho_\varphi = \left. \frac{k^3 |\beta_k|^2 M\dot{a}(t)}{4\pi^2 a^4(t)} \right|_{k = Ma(t)/2}$$

$$\Gamma_\varphi = \frac{\sigma^2}{8\pi M}$$

As it was in Born approximation

Production of fermions

$$\mathcal{L}_{int} = \Upsilon\phi\bar{\psi}\psi, \quad [i\gamma^{\mu}(x)\mathcal{D}_{\mu} - m(t)]\psi(x) = 0 \tag{9}$$

$$m(t) = m_{\psi} - \Upsilon\phi(t), \quad \frac{\Upsilon\phi_0}{M} \ll 1 \tag{10}$$

Analogous calculations gives the result

Solution for coefficient β

$$\beta_k = -\frac{1}{2}\Upsilon\phi_0(t_k)\sqrt{\frac{2\pi}{MH(t_k)}}$$

and for the decay rate

$$\Gamma_{\psi} = \frac{\Upsilon^2 M}{8\pi}$$

Our theory is applicable to the Starobinsky model

$$S_g = -\frac{M_P^2}{2}\int d^4x\sqrt{-g}\left(R - \frac{R^2}{6\mu^2}\right), \quad \mu = 1.3\times 10^{-5} M_P \qquad (11)$$

R^2-modified theory of gravity is equivalent to the Einstein theory of gravity with a new special scalar field ϕ. $\Box\phi + \mu^2\phi = 0$

Bosons

$$\Box\varphi + \left[m_\varphi^2 - \frac{\mu^2\phi}{\sqrt{6}M_P}\right]\varphi = 0, \qquad (12)$$

$$\sigma = -\frac{\mu^2}{2\sqrt{6}M_P} \qquad (13)$$

$$\Gamma_\varphi = \frac{\sigma^2}{8\pi M} = \frac{\mu^3}{192\pi M_P^2} \qquad (14)$$

Fermions

$$\left[i\gamma_\mu(x)\mathcal{D}_\mu - m\right]\psi x = 0 \qquad (15)$$

$$\Upsilon = \sqrt{\frac{2}{3}}\frac{m_\psi}{M_P} \qquad (16)$$

$$\Gamma_\psi = \frac{\Upsilon^2 M}{8\pi} = \frac{\mu m_\psi^2}{12\pi M_P^2} \qquad (17)$$

Conclusions

1. Due to the specific features of expanding universe parametric resonance does not develop in the case of weak coupling
2. Universe expansion restores validity of Born formulas for the decay rates

$$\Gamma_{\varphi} = \frac{\sigma^2}{8\pi M}, \quad \Gamma_{\psi} = \frac{\Upsilon^2 M}{8\pi} \tag{18}$$

do not depend on the details of the universe expansion

decay rate	mean occupation number		rate of filling new modes
$\Gamma \sim$	$N_k \sim 1/H(t)$	$\times$	$\sim H(t)$

decay rates eventually do not depend on $H(t)$