Scalar anisotropic stress and non–trivial propagation of gravitational waves

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based on

I. D. Saltas, I. Sawicki, L. Amendola, M. Kunz *PRL 113, 191101 (2014)*

M. Motta, I. Sawicki, I. D. Saltas, L. Amendola, M. Kunz *PRD 88, 124035 (2013)*

L. Amendola, M. Kunz, I. D. Saltas, I. Sawicki PRD 87, 023501 (2013)

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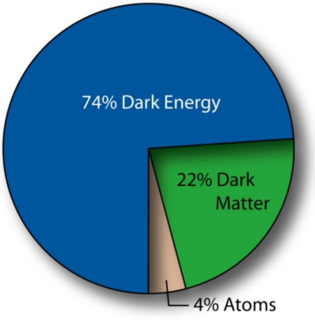
The universe is dark, isn't it?



The main topic and message of this talk:

Genuine modifications of gravity can be detected in a **model-independent** *way at the linear level, and their effect* **fundamentally connected** *with the (modified) propagation of gravitational waves.*

The universe perturbed

- The evolution of large scale structure of spacetime can be well described by small scalar fluctuations around a flat, Friedman–Lemaitre–Robertson–Walker spacetime

$$ds^2 = -\left(1 + 2\,\Psi(t, \mathbf{x}\,)\right) dt^2 + a(t)^2 \left(1 + 2\,\Phi(t, \mathbf{x}\,)\right) d\mathbf{x}^2$$

- The matter content is pressureless dark matter and baryons, with their density fractions $\delta_i \equiv \delta\rho_i/\rho_i$ related through the bias function $b(z, k)$:

$$\delta_b(z, k) = \; b(z, k)\; \delta_m(z, k)$$

- The relation between the potential Φ and the total matter density is described by

The Poisson equation: $k^2\;\Phi(t, k)\; = 4\pi a^2 G \times \delta\rho_{total} \equiv 4\pi a^2\; G_{\rm eff}\; \times \delta\rho_{\rm m}$

- The relation between the two gravitational potentials Φ and Ψ is described by

The anisotropy equation: $\Phi\; - \;\Psi\; = \sigma\left(\alpha_i(t)\right) \times \Pi(t, k)$

(Un)Observables

- **The galaxy density field** $\delta_{gal}(z,k) \equiv {}^{\delta\rho_{gal}}/_{\rho_{gal}}$ is an **observable**. It is usually related to the dark matter one through the a priori unknown bias function $b(z,k)$

$$\delta_{gal}(z,k) \;=\; b(z,k)\;\delta_m(z,k)$$

- **The galaxy velocity field** $v_{gal}(z,k)$ is also an **observable**[1] . Assuming no equivalence principle violations it equals to the dark matter one

$$v_{gal}(z,k) = v_m(z,k) \;\approx -{}^{\delta'_m}/_{k^2} \qquad [k \equiv \frac{k_{comov.}}{aH}, \;\; {}' \equiv \frac{d}{d\ln a}]$$

- **Weak lensing** is another important observable on the sky: Light reaching us from distant sources responds to the lensing potential Φ_{lens} produced by large inhomogeneities and/or modified gravity

$$\Phi_{lens}(z,k) \;\equiv \Psi + \Phi = \frac{3H_0^2(1+z)^3}{2H(z)^2}\Omega_{m,0}\cdot(1+\eta)\cdot\; G_{eff}(z,k)\cdot\delta_m(z,k) \qquad [\eta \equiv -\frac{\Phi}{\Psi}]$$

- **In a model/parametrisation independent way:** [2]

  The bias $b(z,k)$ and effective Newton's coupling G_{eff} are unknown.

  The initial condition $\delta_m(0,k)$ on some initial spatial hyper surface is unknown.

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[1] N. Kaiser, MNRAS 227 (1987) / R. Scorcimarro, H. Couchman, J. A. Frieman, Astrophys. J 517 (1999)

[2] L. Amendola, M. Kunz, I. D. Saltas, I. Sawicki, PRD 87, 023501 (2013)

A different path: Reconstructing the metric

- Linear scalar fluctuations around flat Friedman–Lemaitre–Robertson–Walker background,

$$ds^2 = -(1 + 2\Psi(t, \mathbf{x}))\, dt^2 + a(t)^2 (1 + 2\Phi(t, \mathbf{x}))\, d\mathbf{x}^2$$

- *Assume that sub horizon, galaxies move on geodesics:* Geodesic equation then provides a **measurement of the the metric potential** $\Psi(z, k)$ [3]

$$\left(a^2 \theta_{gal}\right)' = a^2 H k^2 \Psi \quad \Rightarrow \quad \frac{\theta_{gal}(z,k)}{H(z)} = (a^2 H)^{-1} \int a^2 H k^2 \Psi\, d\ln a \qquad \left[k \equiv \frac{k_{comov.}}{aH}, \quad \theta_{gal} \equiv \nabla \mathbf{u_{gal}}\right]$$

- **Complementing velocity field measurements with lensing experiments:** Light responds to the lensing potential Φ_{lens} producing a lensing effect on the sky

$$\Phi_{lens}(z, k) \equiv \Psi(z, k) + \Phi(z, k)$$

$\rightsquigarrow$ From observables θ_{gal}/H, Φ_{lens} (or L) **we can reconstruct the evolution of the metric potentials** $\Phi(z, k)$, $\Psi(z, k)$ in redshift and scale, given a known background evolution $H(z)$.

[3] M. Motta, I. Sawicki, I. D. Saltas, L. Amendola, M. Kunz, *PRD 88, 124035 (2013)*

Is it modified gravity or dark energy?

- A **model independent relation for the gravitational slip** η exists, based only on observable quantities [4]

$$\boxed{\eta(z,k) \equiv -\frac{\Phi(z,k)}{\Psi(z,k)} = \frac{3(1+z)^3}{2E^2\left(\mathcal{O}'_\theta/\mathcal{O}_\theta + E'/E + 2\right)}\frac{\Phi_{lens}}{\mathcal{O}_\theta} - 1} \quad [E(z) \equiv H(z)/H_0, \;\; \mathcal{O}_\theta \equiv -\theta_{gal}/H]$$

- **Parameter** η **is a crucial discriminator** among scalar–tensor models: $\eta = 1$ for minimal coupling to gravity (zero anisotropic stress, $\sigma = 0$), $\eta \neq 1$ otherwise (non–zero anisotropic stress $\sigma \neq 0$).

$\eta = 1$	$\eta \neq 1$
$\mathcal{L} \subset R - 2\Lambda,$	$\mathcal{L} \subset f(R)$
$\mathcal{L} \subset R + K(X,\phi),$	$\mathcal{L} \subset g(\phi)R + U(\phi)$
$\mathcal{L} \subset K(X,\phi) + G(X,\phi)\Box\phi$	$\mathcal{L} = \mathcal{L}_{Hordenski}$

$$[X \equiv -\frac{1}{2}g^{\mu\nu}\nabla_\mu\nabla_\nu\phi]$$

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[4] L. Amendola, M. Kunz, I. D. Saltas, I. Sawicki, PRD 87, 023501 (2013)
M. Motta, I. Sawicki, I. D. Saltas, L. Amendola, M. Kunz, *PRD 88, 124035 (2013)*

Connecting cosmological with astrophysical observations: The link between propagation of gravitational waves and anisotropic stress

- In GR, the only propagating field is the massless graviton h_{ij}, travelling with the speed of light $c_T = 1$

- Modified gravity models in principle affect the propagation of tensors in a non–trivial way

$$h''_{ij} + (2 + \nu)Hh'_{ij} + c_T^2 k^2 h_{ij} + a^2 \mu^2 h_{ij} = a^2 \Gamma\gamma_{ij}$$

Parameter	**Modified in ......**
Planck mass rate: $\nu \equiv H(t)^{-1} \frac{d \ln M_p^2}{dt}$	Hordenski
Speed of tensors: c_T^2	Hordenski, Einstein–Aether
Graviton's mass: μ^2	Massive bi–metric gravity
Source term: $\Gamma\gamma_{ij}$	Massive bi–metric gravity

The link between propagation of tensors and anisotropic stress

- Given the anisotropy equation and tensor evolution at the linear level

$$\Phi(z,k) - \Psi(z,k) = \sigma(\nu, \mu^2, c_T, \Gamma)\Pi(z,k)$$
$$h_{ij}'' + (2 + \nu)\mathcal{H}h_{ij}' + c_T^2 k^2 h_{ij} + a^2 \mu^2 h_{ij} = a^2 \Gamma\gamma_{ij}$$

. . . and for the most popular large classes of modified gravity models in the literature[5],

- The most general second–order, scalar–tensor Hordenski theories (one extra scalar field ϕ),
- The massive bi–metric theories (one extra spin–two field),
- The Einstein–Aether theories (one extra vector field),

. . . *the coupling σ controlling the amplitude of the linear anisotropic stress at large scales, depends on exactly those theory parameters which modify the propagator of tensor waves.*

Conjecture

This underlying relation between scalar anisotropic stress and tensor propagation is a feature of all models on general configurations

[5] I. D. Saltas, I. Sawicki, L. Amendola, M. Kunz *PRL 113, 191101 (2014)*

Tensor evolution and scalar shear in Hordenski's theory

The Hordenski theory is the most general scalar–tensor theory yielding second order equations of motion [6]:

$$\mathcal{L} = \sum_{i=2}^{4} \mathcal{L}_2 + \mathcal{L}_m$$

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X), \\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi, \\
\mathcal{L}_4 &= G_4(\phi, X)\, R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)\right], \\
\mathcal{L}_5 &= G_5(\phi, X)\, G_{\mu\nu}\,(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)\,(\nabla^\alpha\nabla_\beta\phi)\,(\nabla^\beta\nabla_\mu\phi)\right]
\end{aligned}$$

$$X \equiv -\frac{1}{2}(\nabla\phi)^2, \quad G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$

- Around a cosmological background, Hordenski theories modify the graviton's evolution through the friction (ν) and sound speed (c_T) terms

$$h''_{ij} + (2 + \nu)Hh'_{ij} + c_T^2\, k^2 h_{ij} = 0$$

- The anisotropy equation takes the form

$$\Phi(z, k) - \Psi(z, k) = \sigma(\alpha_i(t)) \times \Pi(z, k)$$

$$\sigma(\alpha_i(t)) = \nu + (c_T^2 - 1) \qquad \Pi(z, k) = \frac{\delta\phi}{\phi'} + \frac{c_T^2 - 1}{\nu + \left(1 - c_T^2\right)}\Phi$$

[6] G. W. Hordenski, Int. J. Theor. Phys. 10 (1974)

Observational implications and summary

- The **gravitational slip** can be re-constructed from observations in a model independent way. Any detection of $\eta \neq 1$ would be the smoking gun for a modification of gravity at late times

- **A fundamental link** between anisotropic stress at large scales and the propagation of tensors on a cosmological background: the theory parameters controlling the amplitude of the linear anisotropic stress at large scales, are exactly those which modify also the propagator of tensor waves. The link **opens a way to bridge observations** of large scale structure with measurements of gravitational waves at cosmological and astrophysical scales[7]

- A possible future detection of non–zero anisotropic shear $(\eta \neq 1)$ at large scales **would imply a modification of tensor propagation** at both cosmological and astrophysical scales

[7] For example, for measurements of c_T using supernovae see A.Nishizawa, T. Nakamura, arXiv:1406.5544