The Simplest Extension to the Starobinsky Model

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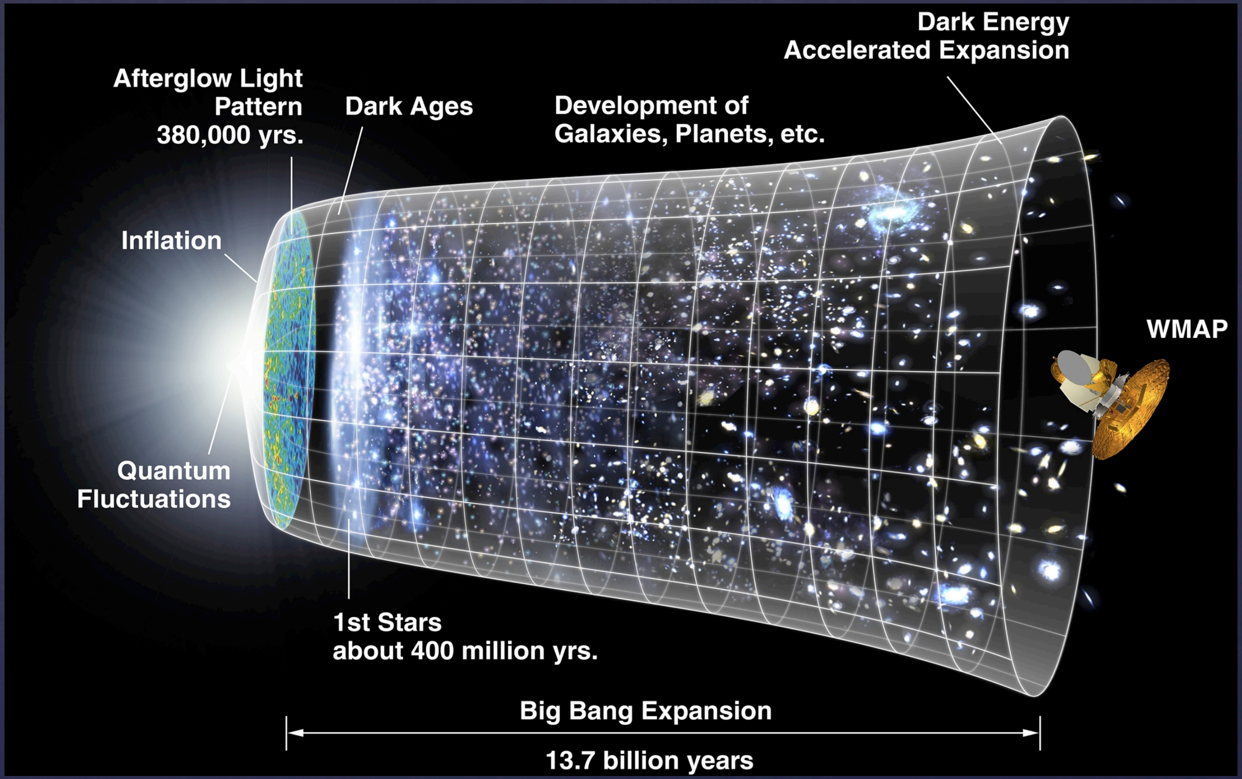
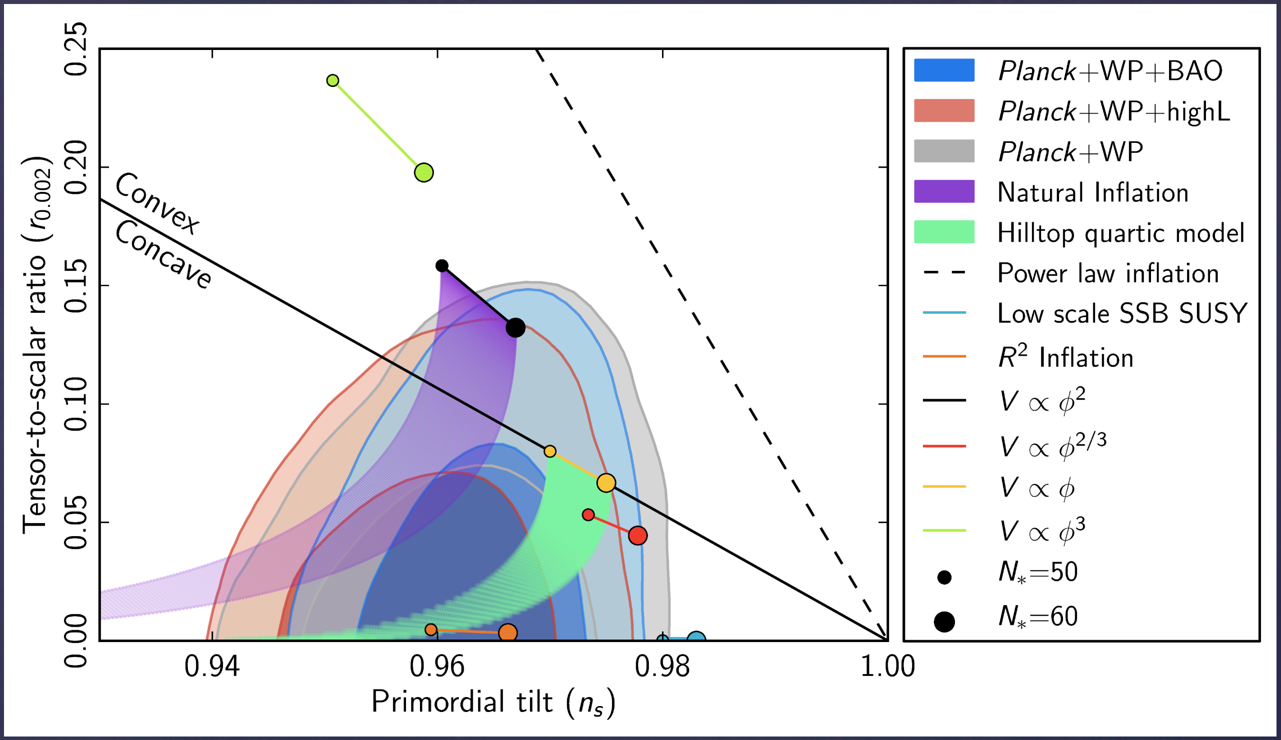
Inflation

- Time of rapid expansion
- Slow-roll regime



Inflationary Models



- Earliest models contain quantum corrections to the Einstein-Hilbert action
- Great agreement with Planck predictions

Simplest Model 1

- Einstein-Hilbert Action

$$S_{EH} = \int d^4x \sqrt{-g}\, \frac{R}{2\kappa}$$

- Starobinsky Action

$$S_S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa} + \frac{\mu R^2}{2\kappa} \right]$$

- Starobinsky & Matter Field

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa} + \frac{\mu}{2} R^2 \right] + \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} m_\chi^2 \chi^2 \right]$$

Frame change

- Perform conformal transformation from Jordan to Einstein frame

$$S_E = \int d^4x \sqrt{-\tilde{g}} \left[\frac{\tilde{R}}{2\kappa} - \frac{\tilde{g}^{\mu\nu}}{2} (\tilde{\partial}_\mu \psi)(\tilde{\partial}_\nu \psi) - \frac{1}{2} \tilde{g}^{\mu\nu} e^{-2\alpha\psi} (\tilde{\partial}_\mu \chi)(\tilde{\partial}_\nu \chi) - V \right]$$

- Potential $$V = \frac{(1 - e^{-2\alpha\psi})^2}{8\kappa^2\mu} + \frac{1}{2} m_\chi^2 e^{-4\alpha\psi} \chi^2$$

- The mass of the scaleron is defined as: $$m_\psi^2 = \frac{1}{6\kappa\mu}$$

Perturbation Theory 1

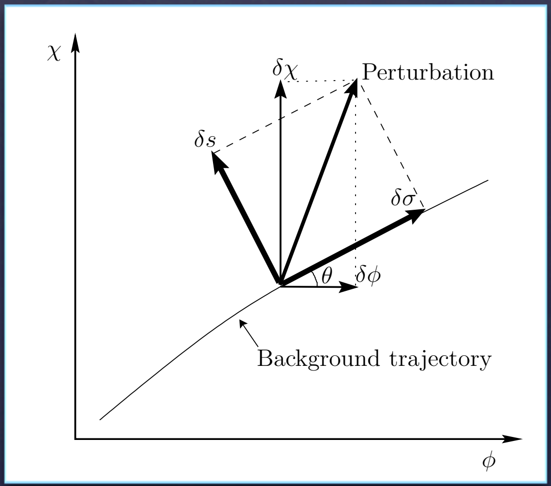
- One field models have a straight-forward behaviour
- Two field model with non-standard kinetic terms
  - there is potential for isocurvature perturbations seeding curvature perturbations after horizon crossing
- Follow the formalism developed by C.van de Bruck and M.Robinson **arXiv:1404.7806**:
  - generalizes the transfer matrix method to second-order in slow-roll in non-canonical cases.
- Instead of integrating the full perturbation equations we evaluate the perturbations at horizon crossing and at the end of inflation
- Second order are in great agreement with first order

$$S_E = \int d^4x \sqrt{-\tilde{g}} \left[\frac{\tilde{R}}{2\kappa} - \frac{\tilde{g}^{\mu\nu}}{2} (\tilde{\partial}_\mu \psi)(\tilde{\partial}_\nu \psi) - \frac{1}{2} \tilde{g}^{\mu\nu} e^{-2\alpha\psi} (\tilde{\partial}_\mu \chi)(\tilde{\partial}_\nu \chi) - V \right]$$

Perturbation Theory 2

- Equations of motion:

$$\ddot{\psi} + 3H\dot{\psi} + V_\psi = b_\psi e^{2b}\dot{\chi}^2,$$
$$\ddot{\chi} + (3H + 2b_\psi\dot{\psi})\dot{\chi} + e^{-2b}V_\chi = 0$$



arXiv:astro-ph/009131

- Instantaneous rotation in field space
- With the directions defined as:

$$d\sigma = \cos\theta d\psi + \sin\theta e^b d\chi$$
$$ds = e^b\cos\theta d\chi - \sin\theta d\psi$$

$$\cos\theta = \frac{\dot{\psi}}{\sqrt{\dot{\psi}^2 + e^{2b}\dot{\chi}^2}}$$
$$\sin\theta = \frac{e^b\dot{\chi}}{\sqrt{\dot{\psi}^2 + e^{2b}\dot{\chi}^2}}$$

Numerical Results 1

We look at three observables:

- The spectral index: $\mathbf{n}_s$
- The Amplitude at the pivot scale 0.05 Mpc^{-1}: $\mathbf{A_s}$
- The tensor to scalar ratio: **r**

And compare them to the Plank 2015 values:

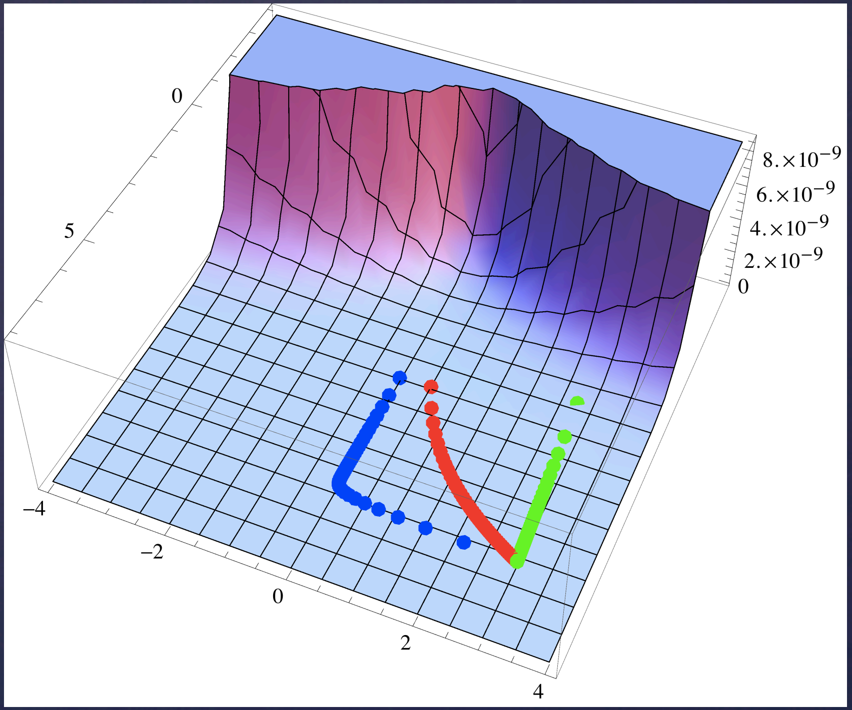
- $\mathbf{n}_s$ = [0.9530, 0.9676]
- **$A_s\, 10^9$ = [2.07, 2,39]**
- **r = <0.1**

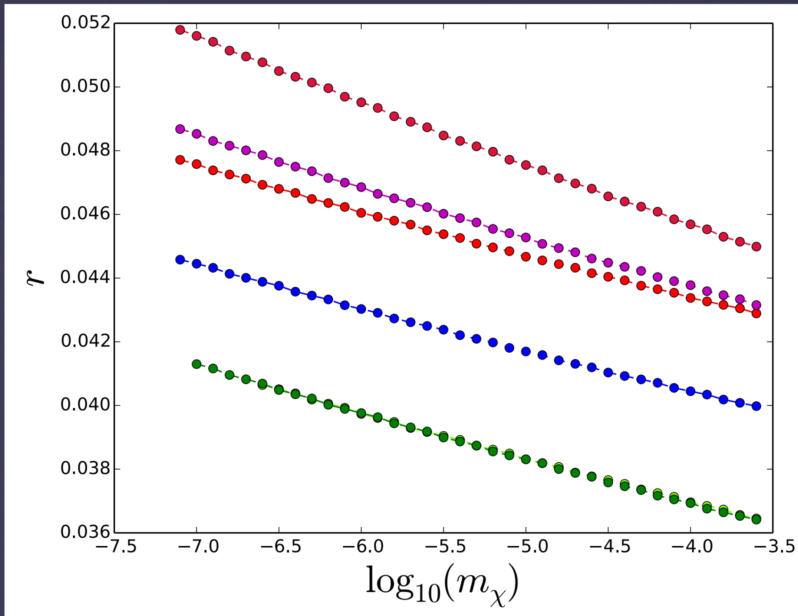
Numerical Results 2

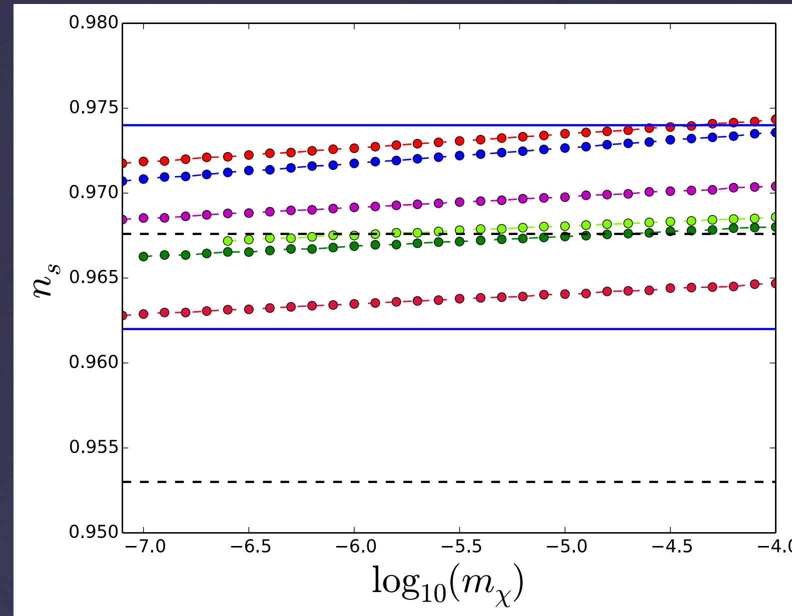
- Changing the masses of the fields, changes the potential landscape

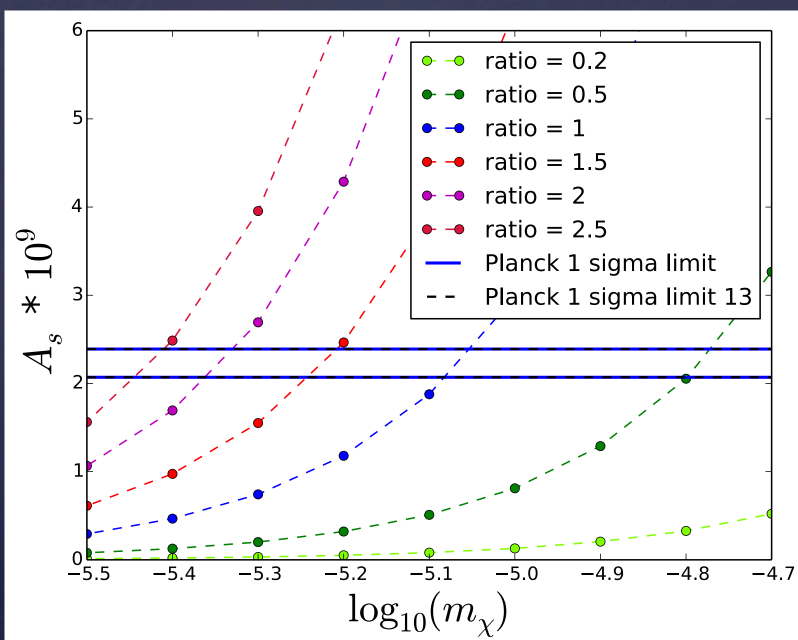
Different initial conditions:

- Keep one field initial value fixed and vary the other one
- We find that the variation in $\mathbf{n_s}$ and **r** is **<1%**
- The variation in $\mathbf{A_s}$ can be as high as **10%**









- Different mass ratios
- Mass ratio defined :

$$ratio = m_\psi / m_\chi$$

Numerical Results 3

Conclusions

- This model is stable under variations in initial conditions
- The model allows for a variety of mass ratios between the two fields
- When calculating the power spectra there is great agreement between first and second order approximation in slow-roll
- Good model for embedding in a fundamental theory
- Worth exploring the limits of the model at times subsequent inflation – asymmetrical potential

- There have been attempts to obtain Starobinsky inflation from more fundamental theories
- Supergravity motivations use chiral multiplets or combinations to drive inflation
- One such attempt is done in arXiv:1405.0271v3

$$K = -3\ln(T+\bar{T}) + \frac{|\phi|^2}{(T+\bar{T})^\omega} \qquad W = \sqrt{\frac{3}{4}}\frac{m}{a}\phi(T-a)$$

$$V = e^K(K^{i\bar{j}}D_iW\bar{D}_{\bar{j}}\bar{W} - 3|W|^2) \qquad V = \frac{3m^2}{4a^2}|T-a|^2(T+T^*)^{w-3}$$

$$\begin{aligned}\mathcal{L} =& \frac{1}{2}\partial_\mu\partial^\mu\rho + \frac{1}{2}e^{2\sqrt{\frac{2}{3}}\rho}\partial_\mu\partial^\mu\sigma \\ &- \frac{3}{2^{(5-\omega)}a^{(3-\omega)}}m^2 e^{\sqrt{\frac{2}{3}}(3-\omega)\rho}(1-e^{-\sqrt{\frac{2}{3}}\rho})^2 \\ &- \frac{1}{2^{(4-\omega)}a^{(3-\omega)}}m^2 e^{\sqrt{\frac{2}{3}}(3-\omega)\rho}\sigma^2\end{aligned}$$

- The case when both fields have the same mass, w = 1 and a^2= 1\12 reduces our Lagrangian