Possible doomsday behaviours for the Universe: classical versus quantum

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Outline

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Introduction-1-: A brief sketch of the universe

- The universe is homogeneous and isotropic on large scales.
- The matter content of the universe:
  - Standard matter
  - Dark matter
  - Something that accelerates the universe (broadly speaking dark energy)
- What is dark energy (DE)? No idea. But we know it implies acceleration in a homogeneous and isotropic universe

Introduction-2-

- The effective equation of state of dark energy is roughly -1
- There could be room for dark energy with $w_0 < -1 \Longrightarrow$ phantom energy
- In phantom energy models
  - Null energy condition is not satisfied
  - Energy density is a growing function of the scale factor (in an expanding Universe like ours)
  - May be a big rip singularity in the future

Starobinsky 00, Caldwell 02, Caldwell, Kamionkowski and Weinberg 03

Outline

Cosmological singularities related to dark energy

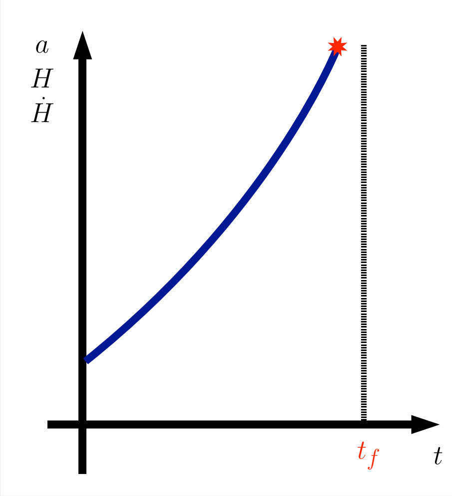
- Classification of the cosmological singularities related to dark energy
  - Big rip singularity
  - Sudden singularity, big brake singularity, big démarrage singularity
  - Big freeze singularity
  - Type IV singularity
  - Little rip event
  - Little sibling of the big rip singularity

Kamenshchik 13 (review mainly on type II sing.)

Odintsov has a lot of works on this topic

Big rip singularity-1-

- For this singularity the null energy condition is violated. The scale factor diverges in a finite time. It is accompanied with a divergence of the Hubble rate and the cosmic derivative of the Hubble rate.



Starobinsky 00, Caldwell 02, Caldwell, Kamionkowski and Weinberg 03

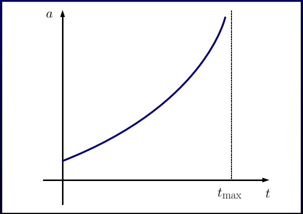
Big rip singularity-2-

- Equation of state $p = w\rho$, $w = \mathrm{const.}$ and $w < -1$
- Energy density $\rho = \tilde{A}a^{-3(w+1)}$
- Scale factor for a flat FLRW ($C = (\kappa_4^2/3)\tilde{A}$)

$$a(t) = \left[a_0^{3(w+1)/2} + \frac{3(w+1)}{2}C^{1/2}(t - t_0)\right]^{2/(3(w+1))}$$

- Big rip in the future

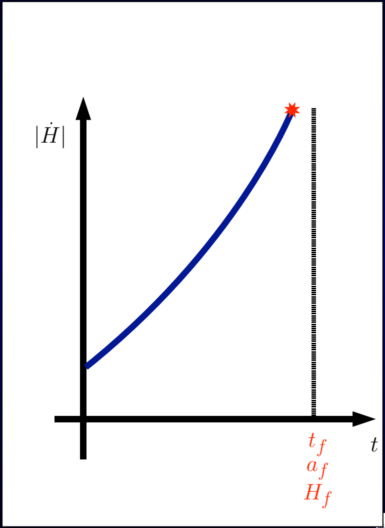
$$t_{\max} = t_0 - \frac{2}{3(w+1)C^{1/2}}a_0^{3(w+1)/2}$$



Sudden singularity

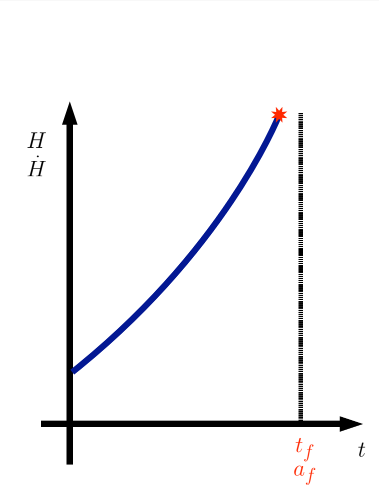
- This singularity occurs at a finite value of the scale factor and the Hubble rate. It is accompanied with a divergence of the cosmic derivative of the Hubble rate.

Barrow '04



Big freeze singularity

- This extremal events happens also at a finite scale factor. The Hubble rate and its cosmic derivative blow up at that scale factor.

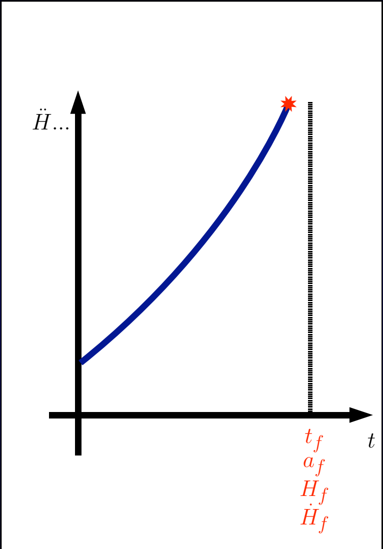


Nojiri, Odintsov and Tsujikawa 05'
BL, González-Díaz and Martín-Moruno 06, 07

Type IV singularity

- None of the Hubble rate or $\dot{H}$ blow up in this case. However, second and higher derivatives blow up at a finite value of the scale factor.

Nojiri, Odintsov and Tsujikawa 05'



What type of matter can drive those singularities? Example: Generalised Chaplygin gas

- A sudden, big freeze and type IV singularity can emerges on the realm of a Chaplygin gas
- Chaplygin gas: $\alpha = 1$ and $A > 0$. Kamenshchik et al 01, Bilic et al 01
- Generalised Chaplygin gas: $P = -A/\rho^{\alpha}$, $0 < \alpha < 1$ and $A > 0$. Bento et al '02
- Motivated initially not only as a dark energy component but also as a dark component playing the role of dark matter and dark energy

GCG and dark energy related singulaties-1-:

The asymptotic behaviour of a universe filled with each type of a "plain" GCG; i.e. it doesn't violate the null, strong and weak energy conditions

A,B	$1+\alpha$	a	ρ	Past	Future
$A<0$	positive	$0 \leq a < a_{\max}$	$0 \leq \rho < \infty$	dust-like	(1) no singularity/infinite future
					(2) type IV singularity
					(3) sudden singularity
$B>0$	negative	$a_{\min} \leq a < \infty$	$0 \leq \rho < \infty$	big freeze singularity	dust-like
$A>0$	$(2n)^{-1}>0$	$0 \leq a < a_{\max}$	$0 \leq \rho < \infty$	dust-like	no singularity/infinite future
$B<0$	$(2n)^{-1}<0$	$a_{\min} \leq a < \infty$	$0 \leq \rho < \infty$	big freeze singularity	dust-like

- (1) and (3) correspond to $-1<\alpha\leq -1/2$ and $0<\alpha$, respectively. (2) corresponds to $-1/2<\alpha<0$, where α cannot be expressed as $\alpha=1/(2p)-1/2$, with p a positive integer. If $-1/2<\alpha<0$ and α can be expressed as $\alpha=1/(2p)-1/2$, with p a positive integer, there is no past singularity and the universe is born at a finite past.

$$\rho=(A+B/a^{3(1+\alpha)})^{1/1+\alpha}$$

BL, González-Díaz, Martín-Moruno '06, '07

GCG and dark energy related singulaties-2-:

The asymptotic behaviour of a universe filled with each type of a phantom GCG

A,B	$1+\alpha$	a	ρ	Past	Future
$A>0$	positive	$a_{\rm min} \le a < \infty$	$0 \le \rho \le A^{1/(1+\alpha)}$	(1) ∞ past	asymptotically dS
				(2) Type IV singularity	
				(3) Sudden singularity	
$B<0$	negative	$0 \le a < a_{\rm max}$	$A^{1/(1+\alpha)} \le \rho < \infty$	asymptotically dS/∞ past	big freeze singularity
$A<0$	$(2n)^{-1}>0$	$a_{\rm min} \le a < \infty$	$0 \le \rho \le \lvert A\rvert^{1/(1+\alpha)}$	∞ past	asymptotically dS
$B>0$	$(2n)^{-1}<0$	$0 \le a < a_{\rm max}$	$\lvert A\rvert^{1/(1+\alpha)} \le \rho < \infty$	asymptotically dS/∞ past	big freeze singularity

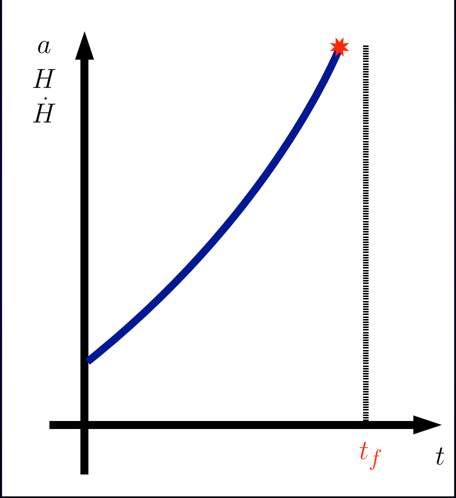
- (1) and (3) correspond to $-1<\alpha\le -1/2$ and $0<\alpha$, respectively. (2) corresponds to $-1/2<\alpha<0$, where α cannot be expressed as $\alpha = 1/(2p)-1/2$, with p a positive integer. If $-1/2<\alpha<0$ and α can be expressed as $\alpha = 1/(2p)-1/2$, with p a positive integer, there is no past singularity and the universe is born at a finite past.

$$\rho = (A + B/a^{3(1+\alpha)})^{1/1+\alpha}$$

BL, González-Díaz, Martín-Moruno '06, '07

Little rip singularity-1-

- For this singularity the null energy condition is violated. The scale factor diverges in an infinite time ($t_f \to \infty$). It is accompanied with a divergence of the Hubble rate and the cosmic derivative of the Hubble rate.



Little rip singularity-2-

- The name of little rip was introduced by Frampton, Ludwick and Scherrer '11
- This kind of singularity corresponds to a big rip sent towards an infinite cosmic time
- Examples:
  - This kind of singularity can happens in a FLRW universe filled with a perfect fluid $p = -\rho - A\rho^{1/2}$
    (Nojiri, Odintsov and Tsujikawa 05', Stefançic 05')
  - Also presents in some dilatonic brane-world models (BL 05').
  - First example was found by Ruzmaikina and Ruzmaiki back in 1970 corresponding to a past little rip

Little sibling of the big rip singularity

- This event is much smoother than the big rip singularity. When the little sibling of the big rip is reached, the Hubble rate and the scale factor blow up but the cosmic time derivative of the Hubble rate does not. This abrupt event takes place at an infinite cosmic time where the scalar curvature explodes.
- It turns out that eventhough the event seems to be harmless as it takes place in the infinite future, the bound stuctrure in the universe would be unavoidably destroyed in a finite cosmic time from now.

BL, Errahmani, Martín-Moruno, Ouali, Tavakoli (2014)

Outline

Motivation-1-

- Einstein general relativity (GR) is an extremely successful theory
- However, it is expected to break down at some point at very high energies
- GR cannot explain the current acceleration of the universe unless a dark energy component is considered
- These are some motivations for looking for possible extension of GR

Motivation-2-

- GR: (i) From a geometrical point of view, the theory is fullly determined by the metric. (ii) From a dynamical point of view, the theory is determined by Einstein equation or Hilbert-Einstein action.
- Natural questions:
  1. Is this the most general geometry?
  2. Is this the most general theory of gravity?
- Answers
  1. Answer to (1): In principle, the connection and the metric can be independent. This leads to Palatini type of theories. In addition, the connection can be non symmetric (very important to incorporate spin in the theory), this excentially leads to a non-vanishing torsion.
  2. Answer to (2): Certainly not. Of course, any generalisations of GR must incorporate the good features and achievements of GR.
- We will present two examples of alternatives theories of gravity (one with torsion and one without torsion) that have been recently proposed as a mean to avoid the Big Bang singularity and see if the same happens with dark energy related singularities.

EiBI theory-1-

- There have been many proposals for alternative theories of GR as old as the theory itself
- One of the oldest proposal was due to Eddington
- In Eddington proposal, the connection rather than the metric plays the fundamental role of the theory
- It is equivalent to GR in vacuum
- BUT does not incorporate matter
- An Eddington-inspired-Born-Infeld theory has been proposed by Bañados and Ferreira

Bañados and Ferreira (2010)

EiBI theory-2-

$$\mathcal{S}_{\rm EiBI}(g,\Gamma,\Psi)=\frac{2}{\kappa}\int d^4x\left[\sqrt{|g_{\mu\nu}+\kappa R_{\mu\nu}(\Gamma)|}-\lambda\sqrt{|g|}\right]+\mathcal{S}_{\rm m}(g,\Gamma,\Psi)$$

- We consider the action under the Palatini formalism, i.e., the connection $\Gamma^{\alpha}_{\mu\nu}$ is *not* the Levi-Civita connection of the metric $g_{\mu\nu}$
- This Lagrangian has two well defined limits: (i) when $|\kappa R|$ is very large, we recover Eddington's theory and (ii) when $|\kappa R|$ is small, we obtain the Hilbert-Einstein action with an effective cosmological constant $\Lambda=(\lambda-1)/\kappa$
- A solution of the above action can be characterized by two different Ricci tensors: $R_{\mu\nu}(\Gamma)$ as presented on the action and $R_{\mu\nu}(g)$ constructed from the metric g
- There are in addition three ways of defining the scalar curvature. These are: $g^{\mu\nu}R_{\mu\nu}(g)$, $g^{\mu\nu}R_{\mu\nu}(\Gamma)$ and $R(\Gamma)$. The third one is derived from the contraction between $R_{\mu\nu}(\Gamma)$ and the metric compatible with the connection Γ
- Therefore whenever one refers to singularity avoidance, one must specify the specific scalar curvature(s)

EiBI theory-3-

- Gravitational action:
$$\mathcal{S}_{\rm EiBI}(g,\Gamma,\Psi)=\frac{2}{\kappa}\int d^4x\left[\sqrt{|g_{\mu\nu}+\kappa R_{\mu\nu}(\Gamma)|}-\lambda\sqrt{|g|}\right]+\mathcal{S}_{\rm m}(g,\Gamma,\Psi)$$
- The parameter κ has been constrained using observationally for example from BBN (Casanellas et al 2012, Avelino 2012).
- The model can avoid the Big Bang singularity, for example, in a radiation dominated universe (Bañados and Ferreira 2012).
- Has been proposed as an alternative scenario to the inflationary paradigm (Avelino 2012)
- The theory can be unstable from a perturbative point of view (See for example Escamilla-Rivera et al 2012).
- It was shown that if the null energy condition is fullfilled then the *apparent* null energy condition is also fullfilled (Deslate and Steinhoff 2012).
- Can this theory avoid the big rip singularity? This is the main question, we will address

EiBI theory-4-

- The physical metric: flat FLRW metric with scale factor $a(t)$
- The auxiliary metric: $q_{\mu\nu} = -U(t)dt^2 + a^2(t)V(t)d\mathbf{X}^2$
- Friedmann eq: $H^2 = H^2(\kappa, \rho_t, p_t, \frac{dp_t}{d\rho_t})$
- The auxiliary metric: $U = U(\rho_t, p_t)$ and $V = V(\rho_t, p_t)$.
- The conservation of the energy momentum tensor holds.

EiBI theory-5-: radiation dominated universe

- A radiation dominated universe faces in the past a bounce ($\kappa < 0$) or a loitering effect ($\kappa > 0$); i.e. it avoids the big bang singularity. The reason behind this is that the energy density is bounded as a consequence of the modified Friedmann equation. Notice nevertheless the behaviour of the curvature is defined from the connection.
- The curvature behaviour (a_l, a_b minimum scale factors for κ positive and negative, respectively)

Curvature	Loitering ($\kappa > 0$)	Bounce ($\kappa < 0$)
$R_{00}(g)$	0	$4/\kappa$
$R_{ij}(g)$	0	$-4/(3\kappa)\, a_b^2$
$g^{\mu\nu}R_{\mu\nu}(g)$	0	$-8/\kappa$
$R_{00}(\Gamma)$	$1/\kappa$	∞
$R_{ij}(\Gamma)$	$-a_l^2\delta_{ij}/\kappa$	$-(1/\kappa)\, a_b^2\delta_{ij}$
$h^{\mu\nu}R_{\mu\nu}(\Gamma)$	$-\infty$	$+\infty$
$g^{\mu\nu}R_{\mu\nu}(\Gamma)$	$-4/\kappa$	$-\infty$

- Can the big rip be avoided as well?

The EiBI scenario filled with CDM and PE

- We consider the EiBI model filled with CDM and a dark energy component with a constant equation of state $w \sim -1$.
- In GR a matter component such that $w < -1$ (and constant) implies a big rip singularity. Can the EiBI scenario avoid this singularity as happens with the big bang (with respect to the metric $g_{\mu\nu}$) singularity?

Curvature	Big Freeze ($\kappa < 0$)	Big Rip $\kappa > 0$
$R_{00}(g)$	$-\infty$	$-\infty$
$R_{ij}(g)$	$+\infty$	$+\infty$
$g^{\mu\nu}R_{\mu\nu}(g)$	$+\infty$	$+\infty$
$R_{00}(\Gamma)$	finite	$-\infty$
$R_{ij}(\Gamma)$	finite	$+\infty$
$h^{\mu\nu}R_{\mu\nu}(\Gamma)$	finite	$4/\kappa$
$g^{\mu\nu}R_{\mu\nu}(\Gamma)$	finite	$+\infty$

- Notice that EiBI reduce to GR at late-time for a dust filled universe this is no longer the case for a universe filled with phantom matter. Nevertheless the big rip singularity is not cured BL, Chen and Chen 14

What about the other singularities?

Singularity in GR	EiBI physical metric	EiBI auxiliary metric
Big Rip	Big Rip	expanding de-Sitter
past Sudden ($\alpha > 0$)	past Type IV ($0 < \alpha \leq 2$)	contracting de-Sitter
	past Sudden ($\alpha > 2$)	
future Big Freeze ($\alpha < -1$)	future Big Freeze ($-3 < \alpha < -1$)	expanding de-Sitter
	future Type IV ($\alpha = -3$)	
	future Sudden ($\alpha < -3$)	
past Type IV ($-1 < \alpha < 0$) ($\alpha \neq -n/(n+1)$)	past Sudden ($-2/3 < \alpha < -1/3$)	past Type IV
	(1)past Type IV	
	(2)finite past without singularity	finite past without singularity
	past loitering effect ($a_b > a_{\mathrm{min}}$)	Big Bang
finite past without singularity ($\alpha = -n/(n+1)$) ($-1 < \alpha < 0$)	finite past without singularity	finite past without singularity
	past loitering effect ($a_b > a_{\mathrm{min}}$)	Big Bang
Little Rip	Little Rip	expanding de-Sitter

BL, Chen and Chen 15

The geodesic analyses of a Newtonian object in the EiBI setup-1-

- A spherical Newtonian object with mass M and a test particle rotating around the object with a physical radius r
- Both of them are embedded in a spherically symmetric FLRW background
- We will analyse the fate of the bound structure near the singularities corresponding to the physical metric and the auxiliary metric
- The evolution equation of the physical radius: $\ddot{r} = \frac{\ddot{a}}{a} r - \frac{GM}{r^2} + \frac{L^3}{r_3}$
  conservation of angular momentum: $r^2 \dot{\phi} = L$
- Near the Big Rip, Little Rip, Big Freeze and the Sudden singularities: $\ddot{r} \approx \frac{\ddot{a}}{a} r$
- $r_1 = a(t)$, and $r_2 = r_1 \int \frac{dt}{{r_1}^2}$
- $r(t) = A_1 r_1(t) + A_2 r_2(t)$

Faraoni, Jacques 2007

The geodesic analyses of a Newtonian object in the EiBI setup-2-

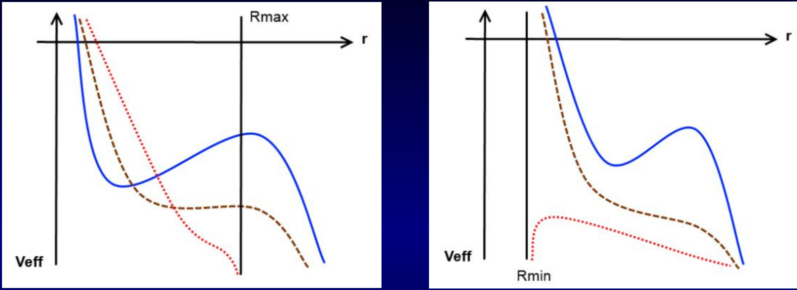


Figure: We show the behaviour of the effective potential *Veff* ($\dot{r}^2 = -2V_{\text{eff}}$) for future singularities (left figure) and past singularities (right figure). R_{max} is finite for a sudden and big freeze singularities while infinite for a big rip and little rip singularity. Likewise R_{min} is finite for a past sudden singularity. On the left figure: the blue solid curve shows the current bound structure, the brown dashed one the intermediate future behaviour and the red dotted one the final state. On the right figure the colors appear in an inverted chronological order, first red dotted, then brown dashed and finally blue solid, as the singularity takes place in the past.

The geodesic analyses of a Newtonian object in the EiBI setup-3-: Type IV singularity and the geodesic defined by the auxiliary metric

- Near a type IV singularity, all the terms in the evolution equation are finite
- A bound system remains bounded
- As for the geodesic equations defined by the auxiliary metric, the singularities are substituted by a de-Sitter or a type IV
- the auxiliary metric and the physical connection have a much smoother behaviour close to the singularities

A cosmographic approach of the EiBI scenario-1-

- The cosmographic approach is the Taylor expansion of the scale factor $a(t)$ with respect to the cosmic time t around the present time t_0

$$a(t) \equiv 1 + \sum_{i=1}^{\infty} \frac{1}{i!} \frac{d^i a}{dt^i}|_{t=t_0} (t - t_0)^i$$

- It is convenient to define the following cosmographic parameters: $H(t) = \frac{1}{a}\frac{da}{dt}$, $q(t) = -\frac{1}{a}\frac{d^2a}{dt^2}\frac{1}{H^2}$, $j(t) = \frac{1}{a}\frac{d^3a}{dt^3}\frac{1}{H^3}$, $s(t) = \frac{1}{a}\frac{d^4a}{dt^4}\frac{1}{H^4}$, $l(t) = \frac{1}{a}\frac{d^5a}{dt^5}\frac{1}{H^5}$, which are commonly called the Hubble, deceleration, jerk, snap and lerk parameters.
- If we can measure/constrain those parameters at present we could get the expansion of the Universe.
- These parameters can be used to constrain cosmological models in an easy way.
- Unfortuntely, it is not so easy to contrain these parameters (from the snap onward).

See for example: Capozziello, Cardone and Salzano (2008)

A cosmographic approach of the EiBI scenario-2-

- We follow this approach to have a clue of what happens in the scenario we discussed before.
- We follow two types of approach. Here we will present one of them
- There are six parameters in our model: κ, α, $a_{\max}$ (or $a_{\min}$), Ω_m, Ω_{de}, and Ω_r. We set the values $\Omega_m = 0.31$, $\Omega_r = 8.48 \times 10^{-5}$, and Ω_κ from 10^{-7} to 10^{-4}, and numerically solve the resulting α, Ω_{de}, $a_{\max}$ using the cosmographic parameters H, q, j ($\Omega_\kappa \equiv 3\kappa {H_0}^2$).
- We used the cosmographic parameters obtained by Gruber and Luongo (2014).

A cosmographic approach of the EiBI scenario-3-

- This approach is applied to the fits deviating from the ΛCDM model and those consistent with cases in which $\alpha < -1$ (future singularities)
- Only fits (2) and (3) in Gruber and Luongo (2014) are consistent (other fits are incompatible with phantom energy)
  - fit (2): $H_0 = 70.25$, $q_0 = -0.683$, $j_0 = 2.044$

Ω_κ	α	Ω_{de}	$a_{\max}$	$H_0(t_{\max} - t_0)$
0 (GR)	-1.94103	0.684915	2.05115	0.621529
10^{-7}	-1.94103	0.684915	2.05115	0.622249
10^{-6}	-1.94103	0.684916	2.05115	0.6235
10^{-5}	-1.94102	0.684919	2.05113	0.62681
10^{-4}	-1.94095	0.684948	2.05102	0.635257

- fit (2) with $\Omega_m = 0.315$ prefers the parameter space $-3 < \alpha < -1$, implying a big freeze singularity in the future

- This approach is applied to the fits deviating from the ΛCDM model and those consistent with cases in which $\alpha < -1$ (future singularities)
- Only fits (2) and (3) in Gruber and Luongo (2014) are consistent (other fits are incompatible with phantom energy)
  - fit (3): $H_0 = 70.09$, $q_0 = -0.658$, $j_0 = 2.412$

Ω_κ	α	Ω_{de}	$a_{\max}$	$H_0(t_{\max} - t_0)$
0 (GR)	-3.19514	0.684915	1.39279	0.317249
10^{-7}	-3.19514	0.684915	1.39279	0.318152
10^{-6}	-3.19514	0.684916	1.39279	0.31972
10^{-5}	-3.19516	0.68492	1.39276	0.323837
10^{-4}	-3.19532	0.68496	1.39248	0.334114

- fit (3) with $\Omega_m = 0.315$ prefers the parameter space $\alpha < -3$, implying a sudden singularity in the future

BI determinantal gravity-1-

- Born-Infeld determinantal gravity has been recently proposed as a way to smooth the Big Bang singularity
- The theory is constructed within the Weitzenböck space-time and it ensures a second order equation of motion of vielbein field.
- Regular cosmological solutions were obtained for some regions of the parameter space.
- In fact, the possible divergence of the Hubble rate at high energies is substituted by a de Sitter phase or a bounce in a FLRW.

Fiorini (2013)

BI determinantal gravity-2-

- Gravitational action:

$$\mathcal{S}_{\rm BId} = \frac{\lambda}{2}\int d^D x\Big[\sqrt{|g_{\mu\nu}+2\lambda^{-1}F_{\mu\nu}|}-\sqrt{|g_{\mu\nu}|}\Big],$$

- $F_{\mu\nu}$ (as well as $S_{\lambda\mu}{}^{\rho}$) is constructed from the Weitzenböck torsion which can be defined through the Weitzenböck torsion: $\Gamma^{\rho}{}_{\mu\nu} = e^{\rho}{}_{a}\partial_{\nu}e^{a}{}_{\mu}, \quad F_{\mu\nu} = \alpha S_{\mu}{}^{\lambda\rho}T_{\nu\lambda\rho} + \beta S_{\lambda\mu}{}^{\rho}T^{\lambda}{}_{\nu\rho} + \gamma g_{\mu\nu}T$
- In the Weitzenböck representation, the dynamical field is the vielbein e^{a} rather than the metric $g_{\mu\nu}$. The metric relates with the vielbein through $g_{\mu\nu} = \eta_{ab}e^{a}{}_{\mu}e^{b}{}_{\nu}$.
- The low energy limit of this theory ($|\lambda| \to \infty$) recovers GR as long as $Tr(F_{\mu}{}^{\nu}) = T$; i.e. $\alpha+\beta+D\gamma = 1$. Note that the relation between the standard Riemannian version of GR and the teleparallel version of GR is ensured by the equation $T = -R + 2e^{-1}\partial_{\nu}(eT_{\sigma}{}^{\sigma\nu})$, where R is the Ricci scalar constructed within the standard Riemannian representation, e is the determinant of the vielbein field and T is the Weitzenböck invariant $T \equiv S_{\rho}{}^{\mu\nu}T^{\rho}{}_{\mu\nu}$.

BI determinantal cosmology-1-

- Friedmann equation:

$$\frac{\sqrt{1-BH^2}}{\sqrt{1-AH^2}}[1+2BH^2-3ABH^4]-1=\frac{2\rho}{\lambda},$$

where $A=6(\beta+2\gamma)/\lambda$ and $B=2(2\alpha+\beta+6\gamma)/\lambda$. The low energy limit is recoverd as long as $A+3B=\frac{12}{\lambda}$. In fact, on that regime $H^2=\frac{\rho}{3}+O^2(\rho)$.

- The Raychaudhuri equation:

$$\dot{H}=-\frac{3}{2}(\rho+p)\frac{dH^2}{d\rho}$$

where $\frac{dH^2}{d\rho}=\frac{4\sqrt{(1-AH^2)^3(1-BH^2)}}{\lambda K(H^2)}$, and $K(H^2)=\frac{12}{\lambda}-14ABH^2-6B^2H^2+9A^2BH^4-12A^2B^2H^6$.

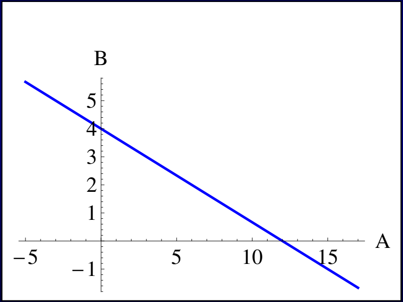
- Not at all straightforward to analyse even for a spatially flat FLRW.

BI determinantal cosmology-2-

- Constraint on the theory: $1 - AH^2 > 0 \rightarrow \frac{1}{H^2} > A$, $1 - BH^2 > 0 \rightarrow \frac{1}{H^2} > B$.
- The previous constraint implies the signature of $g_{\mu\nu} + 2\lambda^{-1}F_{\mu\nu}$ does not change when recovering GR (i.e. $|\lambda| \rightarrow \infty$).
- The Big Bang singularity is avoided for any matter content fullfilling the dominant energy condition for $B = 0$. The universe would be asymptotically de Sitter in the past (Fiorini 2013).
- Here again, we ask the question: is this still the case for dark energy related singularities and even for the Big Bang singularity in other regions of the parameter space?

BI determinantal cosmology versus cosmological singularities ($\lambda > 0$)

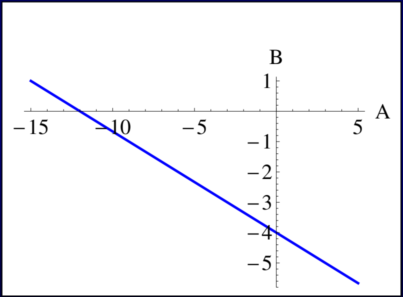
- Wherever A or B are positive there are no strong singularities (H does not blows up)
- This is the case for $\lambda > 0$: remember $\frac{1}{H^2} > A$ and $\frac{1}{H^2} > B$ + see Fig.
- We can however have weaker singularities: Ex. if $A > 0$ there is a sudden singularity at $\rho = (\sqrt{2} - 1)\lambda/2$



BL, Chen and Chen 14

BI determinantal cosmology versus cosmological singularities ($\lambda < 0$)

- The constant A and B can be simultaneously negative this might allow the existence of strong singularities. Indeed this is the case.
- If $B = 0$, any matter content whose pressure is finite when $\rho \to |\lambda|/2$ for a finite scale factor, there is a big freeze from purely geometrical effects.
- If $A = 0$, at the high energy regime $H \propto \rho^{1/3}$, therefore for a constant equation of state different from -1 a Big Rip or a Big Bang can happen.



Outline

On the quantum fate of singularities in a dark-energy dominated universe-1-

- On the previous part of the talk, we tackled the issue of cosmological singularities in the context of modified theories of gravity which in certain regimes could be thought as effective description of a more fundamental theory which could incorporate some aspect of the quantum realm in it.
- There is no final quantum gravity theory so far that would lead to THE theory of quantum cosmology
- There are, however, several approaches in this direction (Talk by Calcagni).
- Here we will follow the most conservative one which corresponds to the Wheeler deWitt approach.

On the quantum fate of singularities in a dark-energy dominated universe-2-

- We will use this approach which is based in the quantum geometrodynamical framework, where we have only one degree of freedom corresponding to the scale factor or two degrees of freedom corresponding to the scale factor and the the matter content.
- Within the first approach we will analyse the Big Rip singularity while the second approach will be used to analyse the Sudden, Big Freeze and type IV singularities.
- We have seen at the beginning of the talk that a mGCG may induce almost any kind of DE singularity but not a Big Rip. In order to carry the quantisation in this case, we need another classical model. For simplicity we will choose the Holographic Ricci Dark energy model.

The holographic dark energy scenario

- Bekenstein proposal: The entropy of a system of volume L^3 is bounded by a quantity proportional to its area L^2 (Bekenstein 73, 81)
- It was proposed as well a bound on the energy density of such a system (Cohen et al 99)
- This idea was applied to cosmology given rise to the holographic dark energy model (Li 04, Hsu 04)

$$\rho_{\rm H} = \frac{3c^2}{\kappa_4^2 L^2} \qquad (1)$$

- There are several choices for the scale: the Hubble scale, the event horizon, the particle horizon, a scale related to the curvature ...
- Here, we will consider an holographic Ricci dark energy model (Gao, Wu, Chen, Shen '09).

The holographic Ricci dark energy model

- After using the Friedmann equation for a universe filled with rad. and matter, the energy density of HRDE reads

$$\rho_R = \frac{3H_0^2}{8\pi G}\left[\left(\frac{\beta}{2-\beta}\right)\Omega_{m0}\left(\frac{a}{a_0}\right)^{-3} + \Omega_{p0}\left(\frac{a}{a_0}\right)^{-2\left(2-\frac{1}{\beta}\right)}\right]$$

  where Ω_{p0} is an integration constant that quantifies the effective amount of DE in the HRDE.

  1. If $1 < \beta$, the cosmic acceleration is negative.
  2. If $1/2 < \beta < 1$, the Universe enters in an accelerating state when the HRDE dominates. The Universe is asymptotically flat in the future.
  3. If $\beta = 1/2$, the model is asymptotically de Sitter.
  4. If $0 < \beta < 1/2$, the Universe not only enters in an accelerated state, but also super accelerates ($\dot{H} > 0$) in the future hitting a Big Rip. The universe hits a singularity at a finite cosmic time.

- Obsevertional constraints on the HRDE favours the case $0 < \beta < 1/2$ (Xu and Wang 2010 and Suwa, Kobayashi, Oshima 2014).

Example on how to obtain the Wheeler deWitt Eq.-1-

- We start assuming a FLRW universe, then the gravitational Lagrangian reads

$$L = N\left[\frac{3\pi}{4G}\left(-\frac{a\dot{a}^2}{N^2} + ka - \Lambda(a)\frac{a^3}{3}\right)\right].$$

where $\Lambda(a)$ encodes the matter content of the universe

- We next obtain the Hamiltonian

$$\mathcal{H} = N\left[-\frac{G}{3\pi}\frac{p_a^2}{a} + \frac{\pi}{4G}\Lambda\left(a\right)a^3\right]$$

where p_a is the canonical momentum of a.

- So far everything is classical

Example on how to obtain the Wheeler deWitt-2-

- In the quantum framework, the term p_a^2/a generates the operator

$$\frac{p_a^2}{a} = -\hbar^2 \left[a^{-\frac{1}{2}}\partial_a\right]\left[a^{-\frac{1}{2}}\partial_a\right]$$

- Then, the quantum Hamiltonian operator can be written as

$$\hat{\mathcal{H}} = N\left\{\frac{3G\hbar^2}{4\pi a_0^3}\partial_x^2 + \frac{3\pi H_0^2 a_0^3}{4G}\left[\Omega_{r0}x^{-\frac{2}{3}} + \left(\frac{2}{2-\beta}\right)\Omega_{m0} + \Omega_{p0}x^{-\frac{2}{3}\left(1-\frac{2}{\beta}\right)}\right]\right\}$$

where $x = \left(\frac{a}{a_0}\right)^{\frac{3}{2}}$.

- Finally, the WDW equation comes from the variation of the Hamiltonian with respect to the lapse function N which produces the Hamiltonian constraint

$$\hat{\mathcal{H}}\Psi(x) = 0.$$

Quantisation of the holographic Ricci dark energy model

- We devided the expanssion of the universe in 3 different regions.
  - A purely initial radiation domintes epoch
  - An epoch where the main components are radiation and matter
  - A final epoch where matter and DE on the form of the HRDE are present
- We solved the Wheeler DeWitt equation on the different regions and connected them smoothly
- It is possible to impose the DeWitt argument for small and very large factors which can be seen as a quantum avoidance of the big bang in the past and big rip in the future
- We next tackle the other DE related singularities using the WdW formalism within the BO approximation.

Albarran, BL 2015 (To appear soon)

More on the quantum fate of singularities in a dark-energy dominated universe

Within the framework of quantum geometrodynamics and a Born Oppenheimer approximation

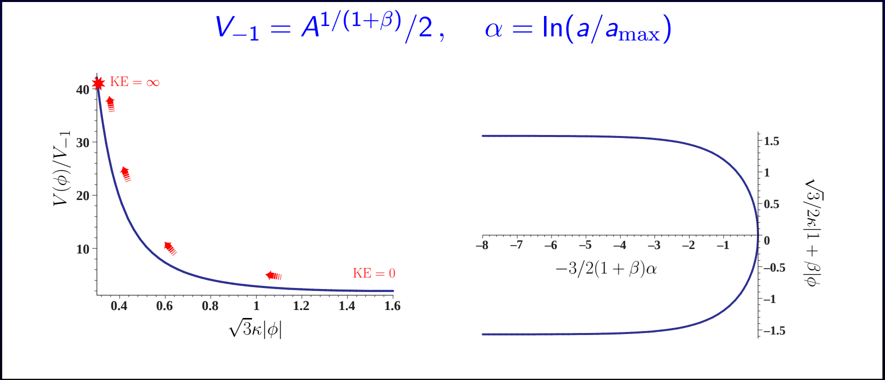
- It was shown by Dabrowski, Kiefer and Sandhöfer 06' that the big rip can be removed.
- It was shown by Kamenshchik, Kiefer and Sandhöfer 07' the avoidance of a big brake singularity.
- It was shown by BL, Kiefer, Sandhöfer and Vargas Moniz 09' the avoidance of a big démarrage singularity and a big freeze.
- Type IV singularity are partially removed (BL, Krämer and Kiefer 2014).

The BF singularity driven by a phantom scalar field

- Phantom scalar field ϕ: $\rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi)$, $\quad p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi)$

$$\ddot{\phi} + 3H\dot{\phi} - V'(\phi) = 0, \quad V(\phi) \simeq V_{-1}\left(\frac{\sqrt{3}}{2}\kappa|1+\beta||\phi|\right)^{-\frac{2\beta}{1+\beta}}$$

- Identify ϕ and GCG; i.e. $\rho_\phi = \rho$, $p_\phi = P$ (with $P = -A/\rho^\beta$, $0 < A$ and $\beta < -1$)



BL, Kiefer, Sandhöfer, Vargas Moniz, 09

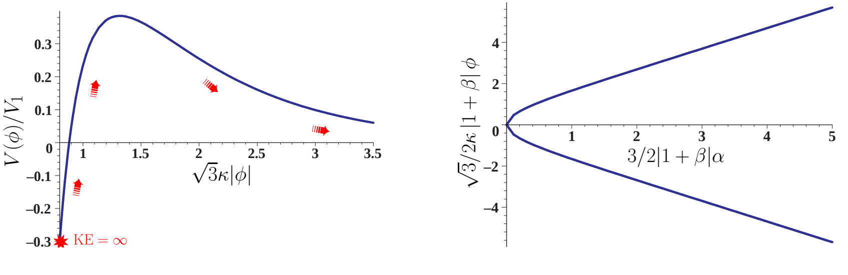
The BF singularity driven by a standard scalar field

- Standard scalar field ϕ: $\rho_\phi = +\frac{1}{2}\dot{\phi}^2 + V(\phi)$, $\quad p_\phi = +\frac{1}{2}\dot{\phi}^2 - V(\phi)$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \quad V(\phi) \simeq -V_1\left(\frac{\sqrt{3}}{2}\kappa|1+\beta||\phi|\right)^{-\frac{2\beta}{1+\beta}}$$

- Identify ϕ and GCG; i.e. $\rho_\phi = \rho$, $p_\phi = P$ (with $P = -A/\rho^\beta$, $A < 0$ and $\beta < -1$)

$$V_1 = |A|^{1/(1+\beta)}/2\,, \qquad \alpha = \ln(a/a_{\mathrm{min}})$$



The Wheeler-DeWitt equation

- Quantisation of the classical scenario in the quantum geometrodynamical framework
- The Wheeler-DeWitt equation in quantum cosmology is the analogous to Schrödinger equation in quantum mechanics.
- The Wheeler-DeWitt equation for the space variables (a,ϕ)

$$\frac{\hbar^2}{2}\left(\frac{\kappa^2}{6}\frac{\partial^2}{\partial\alpha^2} - \ell\frac{\partial^2}{\partial\phi^2}\right)\Psi\left(\alpha,\phi\right) + a_0^6 e^{6\alpha} V(\phi)\Psi\left(\alpha,\phi\right) = 0, \;\; \alpha := \ln\left(\frac{a}{a_0}\right)$$

  - Standard scalar field $\ell = 1$, $a_0 = a_{\rm min}$
  - Phantom scalar field $\ell = -1$, $a_0 = a_{\rm max}$

- Notice that in the quantum case ϕ is no longer a function of a
- General remark: the Wheeler-DeWitt equation does not depend on time (!)

Decomposing the Wheeler-DeWitt equation

- We use the ansatz

$$\Psi(\alpha,\phi)=\varphi_k(\alpha,\phi)C_k(\alpha)$$

- We require the matter part of the Wheeler-DeWitt equation to satisfy

$$-\ell\frac{\hbar^2}{2}\frac{\partial^2\varphi_k}{\partial\phi^2}+a_0^6e^{6\alpha}V(\phi)\varphi_k=E_k(\alpha)\varphi_k$$

- Such a Born–Oppenheimer-type of ansatz was first used in quantum cosmology in Kiefer 88
- Schrödinger type of equation and in the vicinity the singularity reads

$$\varphi_k''+\left[\ell k^2+\tilde{V}_\alpha|\phi|^{-\frac{2\beta}{1+\beta}}\right]\varphi_k=0$$

where $k^2:=\frac{2E_k}{\hbar^2}$, $\tilde{V}_\alpha:=\frac{2V_\alpha}{\hbar^2}$

$$V_\alpha:=a_0^6e^{6\alpha}V_\ell\left[\frac{\sqrt{3}\kappa}{2}|1+\beta|\right]^{-\frac{2\beta}{1+\beta}}$$

Singular potentials-1-

- The matter part of the wave function satisfies

$$\varphi_k'' + \left[\ell k^2 + \tilde{V}_\alpha |\phi|^{-\frac{2\beta}{1+\beta}}\right]\varphi_k = 0$$

- This equation is formally the same as the radial part of the stationary Schrödinger equation for an *attractive* potential of inverse power $V \sim r^{-\frac{2\beta}{1+\beta}}$, where $|\phi|$ plays the role of the radial coordinate r, and the angular momentum vanishes.
- The potential corresponds to a singular potential; i.e. a potential that approaches (plus or minus) infinity faster than r^{-2} for $r \to 0$. For an attractive r^{-2}-potential there exists a transitional case: if the coupling is more negative than a critical value, the potential is singular, otherwise regular.

Singular potentials-2-

- Analytical solutions for polynomial singular potentials are known for the inverse square, inverse fourth-power, and inverse sixth-power potentials.
  - The inverse square potential is realized for $\beta \ll -1$, where β is chosen such that $|1+\beta||\phi|$ is still small
  - The inverse fourth-power potential corresponds to $\beta = -2$
  - The inverse sixth-power potential corresponds to $\beta = -\frac{3}{2}$
- We focus on the case $\beta \ll -1$. We thus deal with the case of the inverse-square potential $\frac{\tilde{V}_\alpha}{|\phi|^2}$ with

$$\tilde{V}_\alpha = \frac{2a_0^6 e^{6\alpha} V_\ell}{\hbar^2}\left[\frac{\sqrt{3}\kappa|\beta|}{2}\right]^{-2} > 0$$

- This case is sufficiently generic to accommodate also the features of other singular potentials.

More on the matter part of the wave function

- For the least singular potential, which is realized for $\beta \ll -1$, we have to solve the equation

$$\varphi_k'' + \left[\ell k^2 + \frac{\tilde{V}_\alpha}{|\phi|^2}\right]\varphi_k = 0, \quad \tilde{V}_\alpha = \frac{2a_0^6 e^{6\alpha} V_\ell}{\hbar^2}\left[\frac{\sqrt{3}\kappa|\beta|}{2}\right]^{-2} > 0$$

- The phantom and scalar matter have to obey the same quantum equation, where the realm of positive energy for the ordinary scalar field $k^2 > 0$ corresponds to the realm of negative energy for the case of the phantom field, $k^2 < 0$
- The general solution is

$$\varphi_k(\alpha, |\phi|) = \sqrt{|\phi|}\left[c_1 \mathrm{J}_\nu(\sqrt{\ell}k|\phi|) + c_2 \mathrm{Y}_\nu(\sqrt{\ell}k|\phi|)\right], \ \nu := \sqrt{\frac{1}{4} - \tilde{V}_\alpha}$$

- There are four cases to distinguish: k can be real or imaginary, depending on whether the energy entering k^2 is positive or negative. Furthermore, ν can be real or imaginary, depending on the parameters β, A, and the value of α

The gravitational part of the wave function

- The gravitational part of the wave function fulfils

$$\frac{\kappa^2}{6}\left(2\dot{C}_k\dot{\varphi}_k + C_k\ddot{\varphi}_k\right) + \left(\frac{\kappa^2}{6}\ddot{C}_k + k^2 C_k\right)\varphi_k = 0$$

- The Born–Oppenheimer approximation: $\dot{C}_k\dot{\varphi}_k$ and $C_k\ddot{\varphi}_k$ can be neglected.
  - C_k varies much more rapidly with α than φ_k
  - Neglect the back reaction of the matter part on the gravitational part
  - The change in the matter part does not influence the gravitational part
- The matter part simply contributes its energy through k^2

$$\left(\frac{\kappa^2}{6}\ddot{C}_k + k^2 C_k\right)\varphi_k = 0 \Longrightarrow C_k(\alpha) = b_1 e^{\mathrm{i}\frac{\sqrt{6}k}{\kappa}\alpha} + b_2 e^{-\mathrm{i}\frac{\sqrt{6}k}{\kappa}\alpha}$$

- Same solution for a phantom or a scalar field.

The wave function at the singularity

- It can be shown that the matter part of the wave function always vanishes at $\phi = 0$. Notice that we have not used any boundary condition
- What does it mean that the matter wave function vanishes at the singularity? Singularity avoidance but not yet we have to make sure that the gravitational part of the wave function is bounded at the singularity
- The gravitational part of the wave function remains finite at the respective singularities and we can safely speak of singularity avoidance.
- Finally, as the total wave function vanishes, we can interpret this as a singularity avoidance.

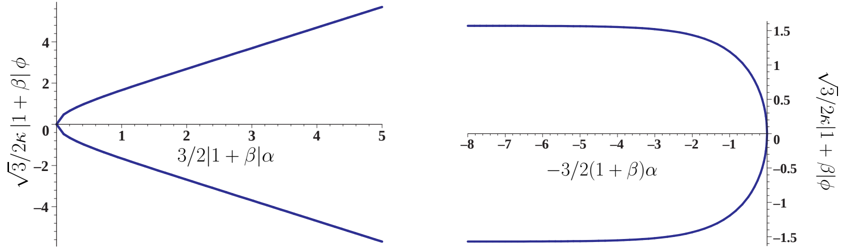
A Wise Boundary condition for the wave function-1

- Nobody knows what the correct boundary condition for the quantum universe are.
- There have been several proposals, most of them using the boundary condition with the ambition to lead to singularity avoidance
- We impose the BC: The wave function decreases in the classically forbidden region.
- Why? Because then it is possible to construct wave packets that follow classical trajectories with turning point in configuration space.
- Namely, one has to require that the wave packet decays in the classically forbidden region. This allows the interference of wave packets following the two branches of the classical solution behind the classical turning point.
- In general, out of solutions to the Wheeler–DeWitt equation which grow in the classically forbidden region, no wave packet can be constructed that follows the classical path

Kiefer 88

A Wise Boundary condition for the wave function-2

- How do we impose this boundary conditions?



Standard scalar field(p) Phantom scalar field(f)

Imposing the BC: standard scalar field-1-

- The region $a < a_{\min}$ is a forbidden region $\Longrightarrow$ we impose the boundary condition that the wave function decay there.
- Then, $\Psi \to 0$ as $\alpha \to -\infty$. The total wave function has to vanish well inside the forbidden region. This happens whenever the matter (or gravitational part) vanishes while the other part is bounded.
- The physical solutions read

$$\Psi_k(\alpha, \phi) = c_1 \sqrt{|\phi|} \mathrm{J}_\nu(k|\phi|) \left[b_1 e^{\mathrm{i}\frac{\sqrt{6}k}{\kappa}\alpha} + b_2 e^{-\mathrm{i}\frac{\sqrt{6}k}{\kappa}\alpha} \right], \quad k^2 > 0$$

$$\Psi_k(\alpha, \phi) = b_2 e^{\frac{\sqrt{6}k}{\kappa}\alpha} \left[c_1 \mathrm{J}_\nu(\mathrm{i}\bar{k}|\phi|) + c_2 \mathrm{Y}_\nu(\mathrm{i}\bar{k}|\phi|) \right], \quad k^2 < 0$$

Imposing the BC: phantom scalar field

- The region $a > a_{\max}$ is a forbidden region $\Longrightarrow$ we impose the boundary condition that the wave function decay there.
- Then, $\Psi \to 0$ as $\alpha \to \infty$. The total wave function has to vanish well inside the forbidden region. This happens whenever the matter (or gravitational part) vanishes while the other part is bounded.
- The physical solutions are

$$
\begin{aligned}
\Psi_k(\alpha,\phi) &= \left(b_1 e^{\mathrm{i}\frac{\sqrt{6}k}{\kappa}\alpha} + b_2 e^{-\mathrm{i}\frac{\sqrt{6}k}{\kappa}\alpha}\right)\sqrt{|\phi|}\mathrm{K}_{i\nu}(k|\phi|), \quad k^2 > 0 \\
\Psi_k(\alpha,\phi) &= d_2 \exp\left(-\frac{\sqrt{6}}{\kappa}\tilde{k}\alpha\right)\sqrt{|\phi|}\mathrm{H}^{(2)}_{\mathrm{i}\nu}(\tilde{k}|\phi|), \qquad k^2 < 0.
\end{aligned}
$$

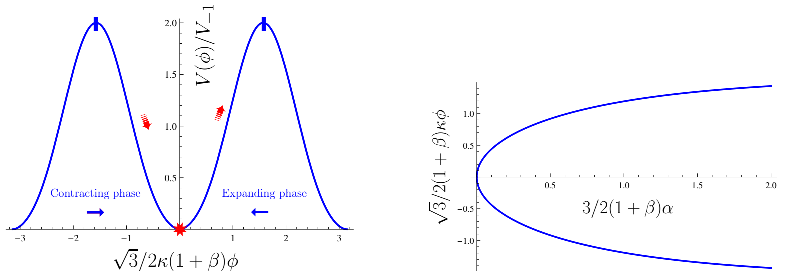
Type IV singularity driven by a phantom scalar field

- Phantom scalar field ϕ: $\rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \quad p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi)$

$$\ddot{\phi} + 3H\dot{\phi} - V'(\phi) = 0, \quad V(\phi) \simeq V_{-1}\left(\frac{\sqrt{3}}{2}\kappa|1+\beta||\phi|\right)^{-\frac{2\beta}{1+\beta}}$$

- Identify ϕ and GCG; i.e. $\rho_\phi = \rho$, $p_\phi = P$ $(0 < A,\ -1/2 < \beta < 0,$ $\beta \neq 1/(2p) - 1/2)$

$$V_{-1} = A^{1/(1+\beta)}/2\,, \qquad \alpha = \ln(a/a_{\min})$$



BL, Kiefer, Krämer 14

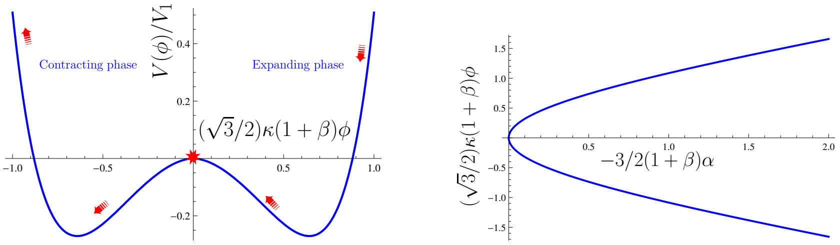
Type IV singularity driven by a standard scalar field

- Standard scalar field ϕ: $\rho_\phi = +\frac{1}{2}\dot{\phi}^2 + V(\phi)\ , \quad p_\phi = +\frac{1}{2}\dot{\phi}^2 - V(\phi)$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \quad V(\phi) \simeq -V_1 \left(\frac{\sqrt{3}}{2}\kappa|1+\beta||\phi|\right)^{\frac{2\beta}{1+\beta}}$$

- Identify ϕ and GCG; i.e. $\rho_\phi = \rho$, $p_\phi = P$ ($A < 0$, $-1/2 < \beta < 0$, $\beta \neq 1/(2p) - 1/2$)

$$V_1 = |A|^{1/(1+\beta)}/2\,, \qquad \alpha = \ln(a/a_{\max})$$



The quantum analysis of type IV singularity

- We follow a Born-Oppenheimer (BO) approximation
- We can solve exactly the matter part in same cases $\beta \to -1/2$ and $l = \pm 1$: it involves Heun functions.
- The gravitational part can be as well be solved within a WKB approximation
- Singularity avoidance for type IV singularities occurs only in special cases. In general, the singularity is not avoided; i.e. only a subset of the solutions of the Wheeler DeWitt equation vanishes at the singularity.

Outline

Conclusions

- In this talk, we have reviewed the cosmological singularities that have appeared on the literature over the last few years, motivated (initially) from the possible presence of an exotic dark energy component
- Then we have shown how these singularities could be appeased or removed either through some modified theories of gravity or within a quantum approach
- We have chosen some BI theories as an example of modified theory of gravity (with or without torsion)
- The Quantum approach has been carried out in the quantum geometrodynamics setup and within the Wheeler DeWitt formalism

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