Phenomenology of Preferred Scale Gravity

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Introduction

One of the outstanding accomplishments of 20th century physics was the formulation of Einstein's celebrated theory of General Relativity:

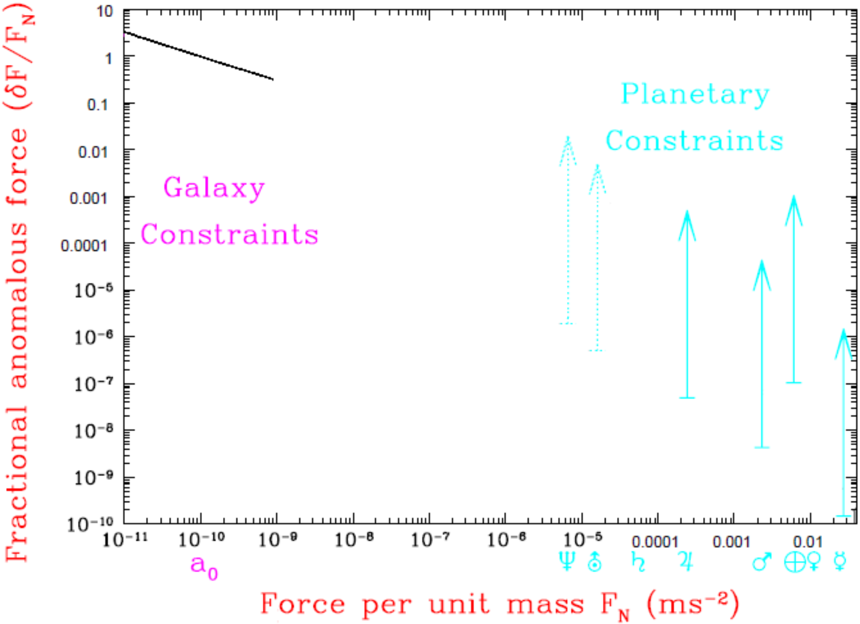
$$
\begin{aligned}
G_{\mu\nu} &= R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \\
S &= \int \left(\frac{c^4}{16\pi G} R + \mathcal{L}_m \right) \sqrt{-g}\, d^4 x
\end{aligned}
$$

This description of gravity matches a wide variety of tests -local, large scale astrophysical and cosmological. The great phenomenological success of the ΛCDM model is built upon a strong precedent that the gravity theory is a good fit to nature.

However did Einstein have the final word? Are there significant or radical departures to be made from the GR paradigm without loosing the great match to observations we cherish?

Acceleration

We may imagine that gravity matches very well the local and Solar System based experiments we may perform - but there is a lower limit to all of these, they occur at $a \gtrsim 10^{-5}$ ms^{-1}. Neatly galactic accelerations are often $a \lesssim 10^{-10}$ - giving a range of **untested** parameter space.


Fractional anomalous force (δF/F_N)
10
1
0.1
0.01
0.001
0.0001
10⁻⁵
10⁻⁶
10⁻⁷
10⁻⁸
10⁻⁹
10⁻¹⁰
Galaxy
Constraints
Planetary
Constraints
10⁻¹¹
10⁻¹⁰
10⁻⁹
10⁻⁸
10⁻⁷
10⁻⁶
10⁻⁵
0.0001
0.001
0.01
a0
♆ ♅ ♄ ♃ ♂ ⊕ ♀ ☿
Force per unit mass F_N (ms⁻²)

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Additionally we appear to find ourselves in a cosmological period of some Λ domination, what if the embedding of a scale is related to the accelerated expansion?

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Effective field theory approach for æther theories (0905.2446), adding the vector field vev to the scales in consideration - c.f. CDT/Causal Sets

Have we seen this before? - **MOND**

A Minimal Model

It would be great to modify GR using just objects derived from metric $g_{\mu\nu}$, naturally forming objects like

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Woodard and Soussa $\Rightarrow$ Action for a MONDian $g_{\mu\nu}$ metric theory, the trace of the EE in the weak field limit may be written in a conformally invariant way - classical EM is conformally invariant!

$$\begin{aligned} S_{EM} &= -\frac{1}{4}\int F_{\mu\nu}F_{\alpha\beta}g^{\mu\alpha}g^{\nu\beta}\sqrt{-g}\,d^4x \\ F_{\mu\nu} &= \nabla_\mu A_\nu - \nabla_\nu A_\mu \end{aligned}$$

To explain additional light bending - need dof external from $g_{\mu\nu}$!

Where would the scale arise?

Consider a four velocity field $\mathbf{v}^\mu$, by constructing a projector

$$h_{\mu\nu} = g_{\mu\nu} - \xi_\mu \xi_\nu$$

to define a spatial velocity vector $\mathbf{U}^\mu = h^\mu{}_\nu \mathbf{v}^\nu$. Then through $\xi^\alpha \nabla_\alpha \mathbf{U}^\beta$ we may assign a norm $a_{norm} = |\xi^\alpha \nabla_\alpha \mathbf{U}^\beta|$, $a_{pref} \sim a_{norm}$. **Local object!**

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Generically such a mechanism may be introduced through a variational principle, e.g.

$$\begin{aligned} S_{\lambda} &= \int \lambda \left(\mathcal{X}^{\alpha}\mathcal{X}^{\beta} g_{\mu\nu} - a_{pref}^{2} \right) \sqrt{-g}\, d^4x \\ \mathcal{X}^{\beta} &= \xi^{\alpha}\nabla_{\alpha}\mathbf{U}^{\beta} \end{aligned}$$

alongside the usual canonical kinetic terms for $\mathbf{U}^{\alpha}$.

A little precedent

An acceleration scale appears quite generically in relativistic theories with extra degrees of freedom, examples can include:

- Einstein-Æther $\Longleftrightarrow$ Hořava-Gravity

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\begin{aligned}
L &= \left(\frac{M_{pl}^2}{2}\left[R+\mathcal{K}\right]+\lambda\left(A^\mu A^\nu g_{\mu\nu}+1\right)\right) \\
\mathcal{K} &= \mathcal{K}_{\mu\nu}{}^{\alpha\beta}\nabla_\alpha A^\mu \nabla_\beta A^\nu \\
\mathcal{K}_{\mu\nu}{}^{\alpha\beta} &= c_1\, g_{\mu\nu} g^{\alpha\beta} + c_2\, \delta_\mu{}^\alpha \delta_\nu{}^\beta + c_3\, \delta_\mu{}^\beta \delta_\nu{}^\alpha
\end{aligned}
$$

Weak Field Limit

$$\mathcal{K} \sim |\nabla\Phi|^2$$

A Toy Model

Consider a scalar field non-minimally coupled to gravity

$$
\begin{aligned}
S &= \int \left(\frac{M_{pl}^2}{2} R + \frac{a_1}{\alpha} X - V(\phi) - F(a_1) \right) \sqrt{-g}\, d^4x \\
&+ \int \mathcal{L}_M \sqrt{-\tilde{g}}\, d^4x \\
\tilde{g}_{\mu\nu} &= A^2\, g_{\mu\nu}
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EoM's fix relations between F, a_1 and X,

$$F_{,a_1} = \frac{X}{\alpha}$$

but F, a_1 left as a free function.

Cosmological Limit

In the cosmological regime,

$$
\begin{aligned}
S &= \int \left(\frac{M_{pl}^2}{2} R + P(X,\phi) \right) \sqrt{-g}\, d^4x \\
P &= \frac{a_1}{\alpha} X - F\left(\frac{X}{\alpha}\right) - V(\phi) \\
\rho &= \frac{a_1}{\alpha} X + F\left(\frac{X}{\alpha}\right) + V(\phi) \\
-\frac{\dot{a}_1}{a_1}\dot{\phi} &= \ddot{\phi} + 3H\dot{\phi} + \frac{\alpha}{a_1} V_{,\phi}
\end{aligned}
$$

Canonical restored for $\alpha = a_1 \rightarrow$ constant
Viable inflationary theory - $n_s \simeq 1$, f_{NL} small, $c_s^2 = 1 + f(X)$
Same reheating regime as canonical inflation

Weak Field Limit

Expanding around a flat background, using a $1/c$ style expansion

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + \frac{1}{c^2} h_{\mu\nu} \\ \partial_0 &\equiv \frac{1}{c}\frac{\partial}{\partial t} \\ \phi &\equiv \phi_0(t) + \frac{1}{c^2}\phi_1(t, \mathbf{x}) \end{aligned} \tag{1}$$

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At c^{-2} order, with a quasi-static approximation $|\dot{\phi}| \ll 1$ and bosonic SET $\tilde{T}^0{}_0 = \bar{\rho}$,

$$\begin{aligned} \nabla^2 \Phi_N &= 4\pi G \bar{\rho} \\ \nabla \cdot (a_1 \nabla \phi) &= \alpha \left(V_{,\phi} + \frac{1}{4} A^4_{,\phi} \bar{\rho} \right) \end{aligned}$$

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\end{aligned}
$$

as well as physical metric perturbation

$$\Phi = f(\phi_0)\Phi_N + \phi$$

Chameleon

$$
\begin{aligned}
\mathcal{S} &= \int d^4x\sqrt{-g}\left(\frac{M^2}{2}R + X - V(\phi)\right) + \mathcal{S}_m\left(A^2(\phi)g_{\mu\nu}, \Psi_i\right) \\
\Box\phi &= V_{,\phi} + A_{,\phi}\hat{\rho} \Longrightarrow V_{eff} = V(\phi) + \hat{\rho}A(\phi)
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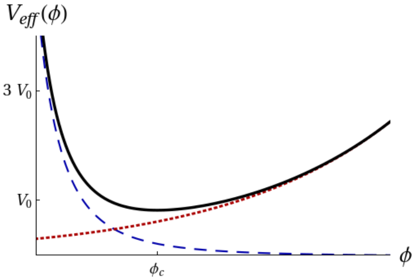
$$\begin{pmatrix} V(\phi) \sim 1/\phi^n \\ A(\phi) \sim e^{k\phi} \end{pmatrix} \Rightarrow$$

$$m^2 \equiv V_{eff,\phi\phi}(\phi_{min}) \quad \Leftarrow$$



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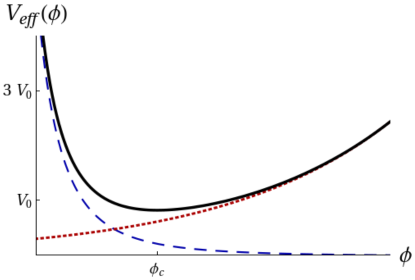

$$\nabla_\mu\nabla^\mu\phi = \frac{\alpha}{a_1}\left(V_{,\phi} + A_{,\phi}\hat{\rho}\right) - \nabla \ln a_1 \cdot \nabla\phi$$

Generalising the "Classic" Chameleon!

Beyond Æther Theories

Making a Generalised AE theory

$$
\begin{aligned}
L &= \left(\frac{M_{pl}^2}{2}\left[R+\alpha^2\mathcal{F}(\mathcal{K})\right]+\lambda\left(A^\mu A^\nu g_{\mu\nu}+1\right)\right)\\
\mathcal{K} &= \alpha^{-2}\mathcal{K}_{\mu\nu}{}^{\alpha\beta}\nabla_\alpha A^\mu \nabla_\beta A^\nu\\
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&+ \left[\left(c_5\, g^{\alpha\beta}+c_6\, A^\alpha A^\beta\right)g_{\gamma\mu}\, g_{\sigma\nu}A^\sigma\right.\\
&+ \left.\left(c_7\,\delta_\mu{}^\beta g_{\gamma\nu}+c_8\,\delta_\nu{}^\beta g_{\gamma\mu}\right)A^\alpha\right]A^\gamma
\end{aligned}
$$

The original EA theories picked $\mathcal{F}(\mathcal{K})\to\mathcal{K}$, $c_{i\geq 4}=0$

Find a generalised Hořava theory, using Orthogonal Hypersurface - (messy!)

$$A^\alpha=\frac{g^{\alpha\beta}\nabla_\beta Y}{\left(g^{\mu\nu}\nabla_\mu Y\nabla_\nu Y\right)^{1/2}}$$

A Geometric Perspective

Moving in GR with a modified spacetime, GEA theories

$$
\begin{aligned}
ds^2 &= -f(r)dt^2 + \frac{dL^2}{f(r)} \\
f(r) &= 1 + \frac{c}{3}\ln\left(\frac{r}{r_0}\right)
\end{aligned}
$$

c is a dimensionless parameter implementing the a_{pref} and r_0 is an integration constant.
Toy model:

$$
ds^2 = -\left(\frac{r}{r_0}\right)^{\frac{2c}{1-c}} dt^2 + \frac{dr^2}{(1-c)^2} + r^2 d\Omega^2
$$

[**Barriola-Wilenkin Global Monopole** - $c > 1$ unphysical]

Possible Future Constraints

Ultra-low acceleration saddle points (SP) in the Solar System, such as Earth-Sun, Jupiter-Sun, etc ...

- Tidal Stresses - A movement away from GR predicts generically different tidal stresses around SP, possible to be tests using LISA Pathfinder
- Light Delays - Additional time delays are expected in a region of modified weak-field potentials, c.f. Lunar Laser Ranging, Long Baseline Interferometry
- Strong-Field Tests - As these theories are **acceleration** dependent, it is possible to engineer tests in different regimes of curvature
- Dynamical Dark Energy / Screening Mechanism tests

Conclusions

Whilst we have the main ingredients in our gravitational theories, it could be possible to include a modification at (to be chosen) acceleration scales.

Often some more familiar theories appear to have an a_{pref} hidden in their structure.

The possibility to make a modification for both weak-field limits and also with some inflationary limit is viable - more work needs to be done to write properly consistent action.

Connection with geometry and also massive gravity could be made - these need more investigation. (Possibility of using PPN?)