Corrections to the TBM ansatz.

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- These are determined by the flavor structure of Yukawa couplings after spontaneous symmetry breaking (SSB) which is not restricted by the gauge symmetry.
- It does not explain the neutrino masses.
- In order to explain the **flavor structure** of Yukawa couplings in the SM **discrete flavor symmetries** have been extensively used .

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- **The Nearest Neighbor Interaction (NNI) form,**

$$\begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix},$$

Lepton sector

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The matrix which contain the three mixing angles, together with a CP violating phase is the lepton mixing matrix U_{PMNS},

$$U_{PMNS} = U_e^\dagger U_\nu = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\nu 1} & U_{\nu 2} & U_{\nu 3} \end{pmatrix},$$

and can be parametrized in different ways.

One of them is the standard parametrization used in the PDG:

$$U_{PMNS} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta_{CP}} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta_{CP}} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta_{CP}} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix},$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, and δ_{CP} is the CP-violating phase.

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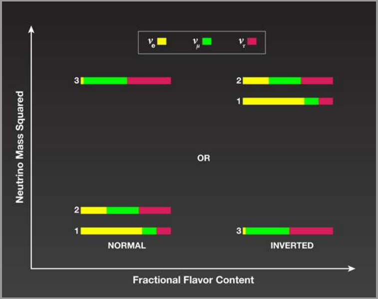

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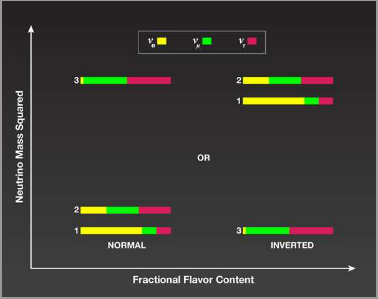

- The results on neutrino mixing angles (**T2K, Double Chooz, Daya-Bay, RENO**) have shown that $\theta_{13} \neq \mathbf{0}$.

Flavor symmetries

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- In particular, some of them predict in a natural way a **Tribimaximal (TBM) neutrino mixing matrix**[2] **($\mathbf{A}_4$, $\mathbf{S}_4$).**

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The TBM

One of the most used ansatz for the lepton mixing matrix is the TBM matrix [a]

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

If neutrinos are in the diagonal charged lepton mass basis, then

$$U_{PMNS} = U_\nu = U_{TBM}.$$

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TBM deviations

To solve this issue and continue working with the TBM, we can obtain **deviations to the θ_{13} value** coming from the charged lepton sector. (S. King, Q.-H. Cao, S. Khalil, E. Ma, H. Okada, D. Aristizabal, I. de Medeiros, E. Houet, H. Ishimori, E. Ma, C. Duarah, A. Das, N, Nimai Singh)

If we demand $U_\nu = U_{TBM}$ to be the matrix which diagonalizes M_ν and U_l the one that diagonalizes $M_l^2 \equiv M_l M_l^\dagger$,

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In this first approach we considered the particular case of a real mass matrix, then

$$M_l^2 = \begin{pmatrix} a^2+b^2+c^2 & ad+be+cf & ag+bh+ci \\ ad+be+cf & d^2+e^2+f^2 & dg+eh+fi \\ ag+bh+ci & dg+eh+fi & g^2+h^2+i^2 \end{pmatrix}.$$

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Finding the textures

Possible different textures of the non-diagonal charged lepton matrix with 3 zeros

$$M_{301} = \begin{pmatrix} 0 & 0 & c \\ d & e & 0 \\ g & h & i \end{pmatrix}; \; M_{302} = \begin{pmatrix} 0 & 0 & c \\ d & e & f \\ g & h & 0 \end{pmatrix}; \; M_{303} = \begin{pmatrix} 0 & b & c \\ d & 0 & 0 \\ g & h & i \end{pmatrix}; \; M_{304} = \begin{pmatrix} 0 & b & c \\ d & 0 & f \\ g & h & 0 \end{pmatrix};$$

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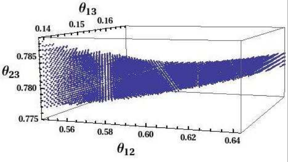
- We also searched for the solution volume of each three-zero texture
  (i.e. the set of points given by the three mixing angles that make possible to find real solutions to the entries of the M_l matrix).
- In nine of the ten cases the solution volume fills the complete experimentally allowed parameter space.

The most interesting case: $\mathbf{M_l} = \begin{pmatrix} 0 & b & c \\ d & 0 & f \\ g & h & 0 \end{pmatrix}$.

- The relevance of this case is that the angle θ_{23} is now very restricted.
- Its allowed interval is $[0.7763, 0.7876]$ for $\delta = 0$, and $[0.7750, 0.7873]$ for $\delta = \pi$.
- These intervals are near but exclude the central value of θ_{23}.

Solution volume in this case (for $\delta = \pi$)



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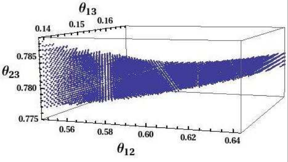

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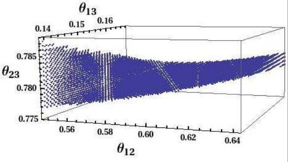

Interesting relations

We found that there are extra conditions on the entries of $Y_l = \frac{1}{\langle H \rangle} M_l$ in terms of the charged leptons masses:

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\begin{aligned}
(Y_l Y_l^T)_{11} = y_a^2 + y_b^2 + y_c^2 &= 7.7 \times 10^{-7} \\
(Y_l Y_l^T)_{22} = y_d^2 + y_e^2 + y_f^2 &= 5.2 \times 10^{-5} \\
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So, given equation $M_l^2 = U_l M_{lD}^2 U_l^\dagger$, we can expect that these conditions may be due to an internal structure of the mixing matrix U_l for the charged leptons.

A parametrization for U_l

We consider a **CKM-like parametrization** for the U_l mixing matrix in terms of the three angles θ^l_{12}, θ^l_{13} and θ^l_{23}.

Naming U' instead of U_{PMNS} one can easily find that:

$$\sin^2\theta_{13} = 1 - (U'_{11})^2 - (U'_{12})^2 ,$$

$$\sin^2\theta_{23} = \frac{(U'_{23})^2}{(U'_{11})^2 + (U'_{12})^2} ,$$

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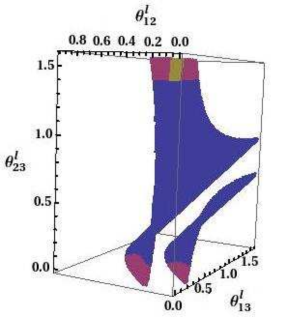
Evaluating last equations with $\theta^l_{ij} \in [0, \pi/2]$, all the points $(\theta^l_{23}, \theta^l_{13}, \theta^l_{12})$ that matched the experimental intervals are shown in the figure:



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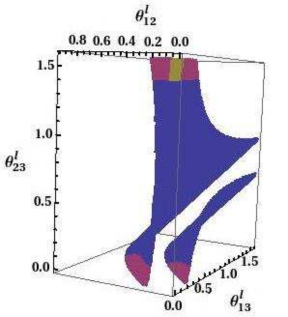

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There is very large mixing (nearly $\pi/2$) between the second and third generations of the charged leptons!

Approximation

We can use a first-order approximation for the respective rotation matrices of U_l

$$U_l \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & \epsilon & 1 \\ 0 & -1 & \epsilon \end{pmatrix} \begin{pmatrix} 1 & 0 & \theta^l_{13} \\ 0 & 1 & 0 \\ -\theta^l_{13} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \theta^l_{12} & 0 \\ -\theta^l_{12} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

where $\epsilon = \frac{\pi}{2} - \theta^l_{23}$.

Model independent relations

Then, from diagonalization condition, $M_l^2 = U_l M_{lD}^2 U_l^\dagger$, we find the following model independent relations:

$$a^2 + b^2 + c^2 \approx m_e^2 + m_\mu^2 \theta_{12}^{l2} + m_\tau^2 \theta_{13}^{l2}$$

$$d^2 + e^2 + f^2 \approx m_\tau^2 + m_e^2(-\epsilon\theta_{12}^l - \theta_{13}^l)^2 + m_\mu^2(\epsilon - \theta_{12}^l\theta_{13}^l)^2 \sim m_\tau^2$$

$$g^2 + h^2 + i^2 \approx m_\tau^2\epsilon^2 + m_e^2(\theta_{12}^l - \epsilon\theta_{13}^l)^2 + m_\mu^2(-1 - \epsilon\theta_{12}^l\theta_{13}^l)^2 \sim m_\mu^2$$

These expressions explain the observations considered before and represent the main result of this analysis.

Conclusions

- We analyzed several textures for the charged lepton mass matrix under the assumption that the neutrino mass matrix is diagonalized by the TBM matrix, and with the intention of maximizing the number of zeros in them.
- We found that there are ten three-zero textures which provide U_{PMNS} values in agreement with data and also determine the size range for their entries.

Conclusions

- Among the successful textures, the one with zeros in the diagonal shows an interesting behavior in the sense that in order to work, it requires the mixing angle θ_{23} to lie in a very restricted range
- A general analysis of the successful textures showed that there are relations between their entries and the charged lepton masses. Through a CKM-like parametrization of the U_l mixing matrix we are able to obtain the texture-independent specific relations in terms of the three rotation angles in U_l.

Thank you!

Moreover...

- We also explored the four-zero texture case. We found no solutions in this case but observe that it is possible to consider cases where one the zeros is lifted in such a way that the remaining mass matrix corresponds to some of the successful three-zero textures.

Moreover...

- It is possible to extend some of our results to the case of complex matrices with factorizable phases[3],

[3] i.e. mass matrices that can be written as $M_l = P^* \hat{M}_l P$, where $\hat{M}_l$ are matrices with real entries and P a diagonal phase matrix, $P = \mathrm{diag}(e^{\imath\phi 1}, e^{\imath\phi 2}, e^{\imath\phi 3})$.

Moreover...

- It is possible to extend some of our results to the case of complex matrices with factorizable phases[3],
- In that case we find similar results: solutions available for the three-zero textures and no solutions for the four-zero-texture, with the difference that the typical entry size of the charged lepton mass matrix is generally larger in this case.

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where $M_{lD} = \mathrm{diag}(m_e,\ m_\mu,\ m_\tau)$ and $M_{lD}^2 \equiv M_{lD} M_{lD}^{\dagger}$.

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where $M_{lD} = \mathrm{diag}(m_e,\ m_\mu,\ m_\tau)$ and $M_{lD}^2 \equiv M_{lD} M_{lD}^\dagger$.

We were interested in identifying the textures which provide solutions to this equation!.

Experimental Data

To carry out the numerical analysis we used the experimental data at 3σ from the global neutrino data analysis [4]

	Best Fit Value	3σ range
$\sin^2\theta_{12}$	0.312	$0.27 - 0.36$
$\sin^2\theta_{23}$	0.52	$0.39 - 0.64$
δ	-0.61π	$0 - 2\pi$
	(-0.41π)	

with normal (inverted) hierarchy

[4] T. Schwetz, M. Tortola, J. W. F. Valle, New J. Phys. **13**, 109401 (2011). [arXiv:1108.1376 [hep-ph]].

Experimental Data

The Daya Bay results (confirmed at 5σ)

$$\sin^2 2\theta_{13} = 0.092 \pm 0.017,$$

which can be rewritten as $\sin^2\theta_{13} = 0.0235 \pm 0.0045$.

For the charged leptons masses we use the values given in PDG

$$\begin{aligned} m_e &= 0.510998910 \pm 0.000000013 \;\; \text{MeV}\;, \\ m_\mu &= 105.658367 \pm 0.000004 \;\; \text{MeV}\;, \\ m_\tau &= 1776.82 \pm 0.16 \;\; \text{MeV}\;. \end{aligned}$$

General case: $\delta_{CP} \neq 0$

The model independent analysis of the leptonic Dirac CP violating phase has been done in:

J. A. Acosta, A. Aranda and J. Virrueta, JHEP04(2014)134 [arXiv:1402.0754 [hep-ph]],

where the mixing matrix in the neutrino sector is assumed to be the TBM and the charged lepton mixing matrix is parametrized in terms of the three angles and one phase.