# Flavor symmetry and mass relations

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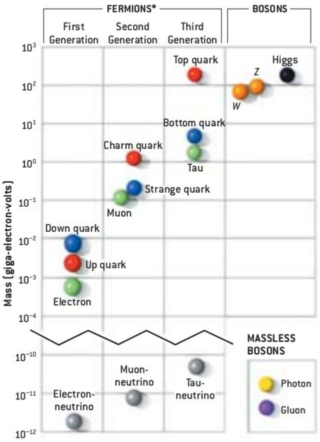
[0] C.B., S. Morisi, E. Peinado, J. Valle. Based on Phys. Lett. B **742**, 99 (2015)

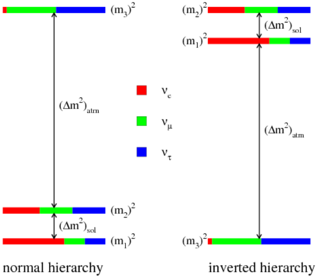
# Matter content in nature:



# The flavor problem:

- neutrinos have mass ($m_\nu \neq 0$) and is pretty small:

$$\Delta m^2_{21} = 7.59^{+0.20}_{-0.18} \times 10^{-5}\text{eV}^2 \text{ and } |\Delta m^2_{31}| = 2.5^{+0.09}_{-0.16} \times 10^{-3}\text{eV}^2.$$



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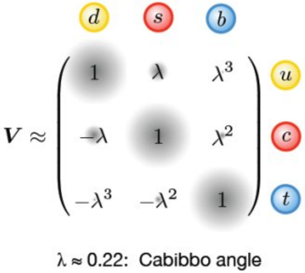
- why are the **lepton mixing angles** larger than the **quark mixing angles**?,



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- why are the **lepton mixing angles larger** than the **quark mixing angles**?,
- what is the **neutrino nature** (Majorana Vs Dirac)?,
- why 3 families? ...

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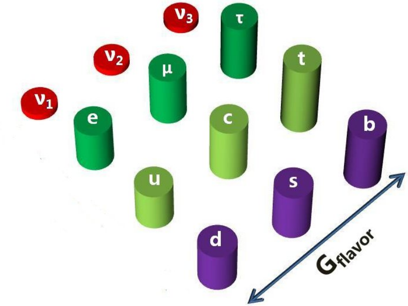
Fermion masses and mixing angles can be explained assuming relations among the families, **flavor symmetries**,




where the symmetry group $\mathbf{G}_{\mathbf{flavor}}$ could be:

- **Abelian, Non-Abelian, Continuous, Discrete**.

# Imprints of flavor models

- Mixing Sum Rules
- Mass Sum Rules

} Neutral Sector

- Mass relations

} Charged Sector

# Imprints of flavor models

- **mixing sum rules**[1]. For instance,

$$s \approx r\cos\delta \tag{1}$$

where $s$ and $r$ represent deviations of the TBM [2] in the following parametrization,

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}(1+s), \quad \sin\theta_{23} = \frac{1}{\sqrt{2}}(1+a)$$
$$\text{and} \quad \sin\theta_{13} = \frac{r}{\sqrt{2}}$$

[1] King JHEP0508:105,2005. For details see King et al. New J.Phys. 16 (2014) 045018 and references therein.

[2] See Alma Rojas' talk.

# Imprints of flavor models

- **neutrino mass sume rules** [3].

$\Longrightarrow$ lower bounds for the effective mass $|m_{ee}|$ in $\nu 0\beta\beta$

For example,

$$3m_2 + 3m_3 = m_1 \quad \text{(NH)} \tag{2}$$

[3] Altarelli and Meloni, J.Phys.G36:085005,2009. Barry and Rodejohann, Phys.Rev.D81:093002,2010. Dorame et al. , Nucl.Phys. B861 (2012) 259-270. King, Merle and Stuart, JHEP 1312 (2013) 005.

Neutrino mass sum rule, [4]

$$3m_2 + 3m_3 = m_1 \tag{3}$$

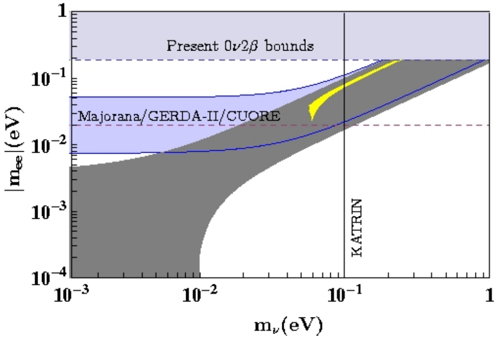



Figure : Effective $0\nu\beta\beta$ mass parameter $\langle|m_{ee}|\rangle$ as a function of the lightest neutrino mass. The yellow band corresponds to the model which predicts the mass sum rule $3m_2 + 3m_3 = m_1$ for the case of NH.

[4] Figure taken from Dorame et al. , Nucl.Phys. B861 (2012) 259-270.

# Imprints of flavor models

- **mass relation in a GUT-less framework**

$$\frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_s m_d}} \left( = \frac{m_t}{\sqrt{m_u m_c}} \right)^5. \qquad (4)$$

[5] This part predicted by Wilczek and Zee in a model with SU(2) as flavor symmetry. That relation is ruled out by the top quark mass.

# Mass relation in the charge sector [6]

- **mass relation in a GUT-less framework**

$$\frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_s m_d}}$$

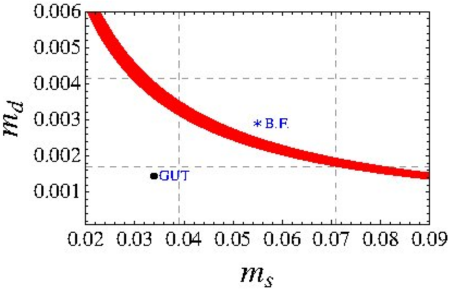



Figure : The shaded band gives our prediction for the down-strange quark masses at the $M_z$ scale, vertical and horizontal lines are the $1\sigma$ experimental range.

[6] Morisi et al., Phys.Rev.D84:036003,2011. King et. al., Phys.Lett. B 724 (2013) 68-72. These models consider $A_4$ as the underlying flavor symmetry.

# "Questions"

- Is it possible to obtain the previous mass relation with another group?
- Are there other mass relations in the charged sector?

# "What we found"

- **Is it possible to obtain the previous mass relation with another group?**
  Yes, for instance with $T_7$ [7].
- **Are there other mass relations in the charged sector?**

[7] Some works with $T_7$, Luhn et al. Phys.Lett. B652 (2007) 27-33, Cao et al. Phys.Rev.Lett. 106 (2011) 131801.

# "What we found"

- **Is it possible to obtain the previous mass relation with another group?**
  Yes, for instance with $T_7$ [8].
- **Are there other mass relations in the charged sector?**
  We still do not know.

[8] Some works with $T_7$, Luhn et al. Phys.Lett. B652 (2007) 27-33, Cao et al. Phys.Rev.Lett. 106 (2011) 131801.

# $T_7$ group[9]

$T_7$ is a $SU(3)$ subgroup with 21 elements. It contains three singlets ($\mathbf{1}_i$ with $i = 1, 2, 3$) and two triplets ($\mathbf{3}$ and $\bar{\mathbf{3}}$).
Multiplication rules:

- $\mathbf{3} \times \bar{\mathbf{3}} = \sum_i \mathbf{1}_i + \mathbf{3} + \bar{\mathbf{3}}$
- $\mathbf{3} \times \mathbf{3} = \mathbf{3} + \bar{\mathbf{3}} + \bar{\mathbf{3}}$
- $\bar{\mathbf{3}} \times \bar{\mathbf{3}} = \bar{\mathbf{3}} + \mathbf{3} + \mathbf{3}$
- $\mathbf{3} \times \mathbf{3} \times \mathbf{3} \supset \mathbf{1} + \mathbf{1}$

[9] Ishimori et al. Prog.Theor.Phys.Suppl. 183 (2010) 1163, arXiv:1003.3552 [hep-th].

# Matter assignments of the model

| | $\overline{L}$ | $\ell_R$ | $N_R$ | $\nu_R$ | $\overline{Q}$ | $d_R$ | $u_{R_i}$ | $H$ | $\varphi_\nu$ | $\varphi_u$ | $\varphi_d$ | $\xi_\nu$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T_7$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{1}_0$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{1}_i$ | $\mathbf{1}_0$ | $\mathbf{3}$ | $\bar{\mathbf{3}}$ | $\mathbf{3}$ | $\mathbf{1}_0$ |
| $\mathbb{Z}_7$ | $a^3$ | $a^3$ | $a^5$ | $a^2$ | $a^3$ | $a^3$ | $a^2$ | $1$ | $a^4$ | $a^2$ | $a^1$ | $a^3$ |

Table : Where $a^7 = 1$.

• **The Yukawa Lagrangian for the charged sector is given by,**

$$\mathcal{L} = \frac{Y^\ell}{\Lambda}\overline{L}\ell_R H_d + \frac{Y^d}{\Lambda}\overline{Q}d_R H_d + \frac{Y^u}{\Lambda}\overline{Q}u_R H_u + h.c. \qquad (5)$$

where $H_d = H\varphi_d$, $H_u = \tilde{H}\varphi_u$ and $\tilde{H} = i\sigma_2 H^*$.

After EWSB,

$$M_u = \begin{pmatrix} y_1^u u_1 & y_2^u u_1 & y_3^u u_1 \\ y_1^u u_2 & \omega y_2^u u_2 & \omega^2 y_3^u u_2 \\ y_1^u u_3 & \omega^2 y_2^u u_3 & \omega y_3^u u_3 \end{pmatrix}$$

and

$$M_f = \begin{pmatrix} 0 & e^{i\theta_f} y_1^f v_3 & y_2^f v_2 \\ y_2^f v_3 & 0 & e^{i\theta_f} y_1^f v_1 \\ e^{i\theta_f} y_1^f v_2 & y_2^f v_1 & 0 \end{pmatrix}, \qquad (6)$$

where $\omega^3 = 1$,

$$\frac{\langle \varphi_u \rangle \langle H \rangle}{\Lambda} \approx (u_1, u_2, u_3) \text{ and } \frac{\langle \varphi_d \rangle \langle H \rangle}{\Lambda} \approx (v_1, v_2, v_3)$$

# Mass relation between quarks and leptons

The mass matrix of the charged leptons (down-type quarks) can be rewritten as,

$$M_f = \begin{pmatrix} 0 & e^{i\theta_f} a^f \alpha^f & b^f \\ b^f \alpha^f & 0 & e^{i\theta_f} a^f r^f \\ e^{i\theta_f} a^f & b^f r^f & 0 \end{pmatrix}, \tag{7}$$

where

$$a^f = y_1^f v_2, \quad b^f = y_2^f v_2, \quad \alpha^f = v_3/v_2 \quad \text{and} \quad r^f = v_1/v_2. \tag{8}$$

# Mass relation between quarks and leptons

Taking the invariants of $M_f M_f^\dagger$ and assuming

$$\boxed{r^f \gg a^f \Leftrightarrow u_1 \gg u_{2,3}}$$

then one obtains,

$$\boxed{r^f \approx \frac{m_3^f}{\sqrt{m_1^f m_2^f}} \sqrt{\alpha^f}.}$$

By definition,

$$r^\ell = r^d \quad \text{and} \quad \alpha^\ell = \alpha^d$$

and then

$$\boxed{\frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_s m_d}}.}$$

This was obtained **without** the need of **any GUT.**

# Neutrino Sector

| | $\bar{L}$ | $\ell_R$ | $N_R$ | $\nu_R$ | $\bar{Q}$ | $d_R$ | $u_{R_i}$ | $H$ | $\varphi_\nu$ | $\varphi_u$ | $\varphi_d$ | $\xi_\nu$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T_7$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{1}_0$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{1}_i$ | $\mathbf{1}_0$ | $\mathbf{3}$ | $\bar{\mathbf{3}}$ | $\mathbf{3}$ | $\mathbf{1}_0$ |
| $\mathbb{Z}_7$ | $a^3$ | $a^3$ | $a^5$ | $a^2$ | $a^3$ | $a^3$ | $a^2$ | $1$ | $a^4$ | $a^2$ | $a^1$ | $a^3$ |

Table : Where $a^7 = 1$.

•**The Lagrangian for the neutrino sector is given by,**

$$\mathcal{L}_\nu = \frac{Y_1^\nu}{\Lambda}\bar{L}N_R\tilde{H}_d + \frac{Y_2^\nu}{\Lambda}\bar{L}\nu_R H_u + \kappa_1 N_R N_R \varphi_\nu + \kappa_2 \nu_R \nu_R \xi_\nu \qquad (9)$$

where, $\tilde{H}_d = \tilde{H}\overline{\varphi}_d$.

•The neutrino masses come from the **Type-I seesaw**,

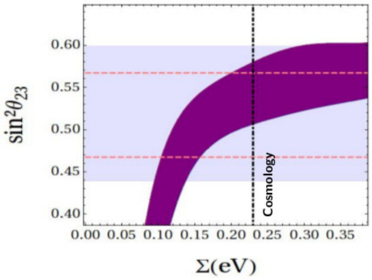
$$M_\nu = -M_D M_{RR}^{-1} M_D^T \tag{10}$$

$$M_\nu \quad = \kappa \begin{pmatrix} \epsilon_1 - 2e^{-i\theta_\nu}\alpha_1^2\epsilon_2 & -\alpha^d - 2e^{-i\theta_\nu}\alpha_1\alpha_2\epsilon_2 & -\epsilon_3 - 2e^{-i\theta_\nu}\alpha_1\epsilon_2 \\ \cdot & \frac{\alpha^{d\,2}}{\epsilon_1} - 2e^{-i\theta_\nu}\alpha_2^2\epsilon_2 & -\frac{\alpha^d\epsilon_3}{\epsilon_1} - 2e^{-i\theta_\nu}\alpha_2\epsilon_2 \\ \cdot & \cdot & -2e^{-i\theta_\nu}\epsilon_2 + \frac{\epsilon_3^2}{\epsilon_1} \end{pmatrix},$$

which is symmetric and $\alpha_1 = 2.14\lambda^4$, $\alpha_2 = 1.03\lambda^2$, $\lambda = 0.2$ and we have defined,

$$\kappa \equiv \frac{(Y^\nu v)^2}{M}, \quad \epsilon_2 \equiv \frac{M(Y_2^\nu u)^2}{M_4(Y_1^\nu v)^2}, \quad \epsilon_3 \equiv \frac{r^d}{R}$$
$$\text{and} \quad \theta_\nu \equiv -2\theta_1 + \theta_2. \tag{11}$$

## Model with the $T_7$ flavor group



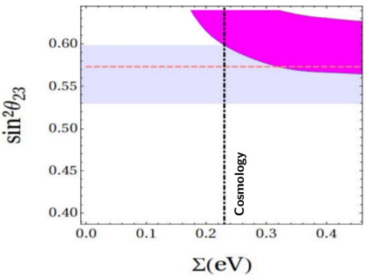




Figure : Left: Assuming normal hierarchy (NH). Right: Assuming inverted hierarchy (IH). Oscillation constraints are taken at $3\sigma$ [11].

[10] D. Forero, M. Tortola, and J. Valle, Neutrino oscillations refitted, Phys. Rev. D90, 093006 (2014), arXiv:1405.7540 [hep-ph],

[11] D. Forero, M. Tortola, and J. Valle, Neutrino oscillations refitted, Phys. Rev. D90, 093006 (2014), arXiv:1405.7540 [hep-ph],

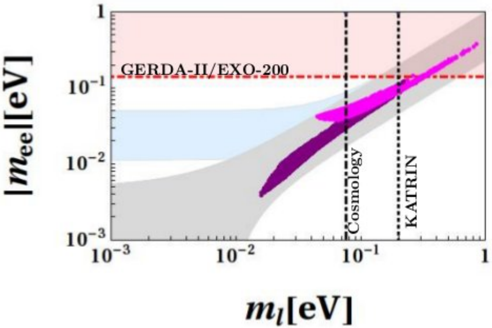



Figure : Effective neutrino mass parameter $|m_{ee}|$ versus the lightest neutrino mass for NH and IH. The vertical dotdashed line and labeled as "Cosmology" denotes the bound from the combination of CMB and BAO data [14]. The vertical dotted line is the future sensitivity of KATRIN, [15].

[12] Planck Collaboration, P. Ade et al., " Planck 2013 results. XVI. Cosmological parameters," arXiv:1303.5076 [astro-ph.CO].

[13] KATRIN Collaboration Collaboration, L. Bornschein, KATRIN: Direct measurement of neutrino masses in the sub-Ev region, eConf C030626 (2003) FRAP14, arXiv:hep-ex/0309007 [hep-ex].

- We have a model which predicts a mass relation between quarks and leptons.
- We have a lower bound for the lightest neutrino mass.
- $\boxed{\frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_s m_d}}}$ because of the group structure. It be obtained with other groups containing three-dimensional irreducible representations (irreps) such as, for example, $T_n \cong Z_n \rtimes Z_3$ (with $n = 13, 19, 31, 43, 49$) as well as $T'$.