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Dynamics of non-minimally coupled perfect fluids

Dario Bettoni
Technion

Refs:

DB, S. Liberati, L. Sindoni, 2011,
DB, V. S. Liberati, C. Baccigalupi, 2012,
DB, M. Colombo, S. Liberati, 2014
DB, S. Liberati, arXiv:1502.06613

Motivations

Why fluids?

- Useful language in many cosmological/astrophysical systems
- Alternative and complementary description of dark sector (DE as fluid)
- Understanding of (general)relativistic fluids challenging

Perfect fluid and beyond

- Think of fluids as a derivative expansion, add
- Couple fluids to other fields (e.g., coupled DE)
- Couple fluids to curvature

Why non-minimal coupling?

Flat space-time		Curved space-time	
$(\rho,\ p,\ s,\ \ldots)$	Fluid scale L_f	$(\rho,\ p,\ s,\ \ldots)$	Fluid scale L_f
$\eta_{\mu\nu}$	No scale	$g_{\mu\nu} \Rightarrow R_{\mu\nu\rho}{}^{\sigma}$	Gravity scale L_g

Example: Bose-Einstein Condensate [DB, Colombo, Liberati, 2014]

Non-minimally coupled fluid

[DB, Liberati, Sindoni, 2011]

$$S = \frac{c^3}{16\pi G}\int d^4x\sqrt{-g}R\,[1+\alpha_s F_C(n,s)] + \frac{\alpha_R c^3}{16\pi G}\int d^4x\sqrt{-g}F_D(n,s)R_{\mu\nu}u^\mu u^\nu + S_{fluid}[g,m]$$

Conformal coupling

Disformal coupling

[Brown,1993]
[Schutz & Sorkin,1977]

$$S_{fluid} = \int d^4x\,\left[-\sqrt{-g}F(n,s) + J^\alpha L_\alpha\right]$$

$$J^\alpha = \sqrt{-g}nu^\alpha$$

Lagrangian multipliers

What should we expect

- Not a scalar-tensor theory: $F_i(|J|/\sqrt{-g},s)$
- Higher derivatives of fluid variables
- Modified TD properties

Cosmological background

- Consider a FLRW metric $ds^2 = -dt^2 + a(t)^2 \left(dr^2 + r^2 d\Omega^2\right)$

Continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0$$

- Unchanged with respect to the minimally coupled case (for both conformal and disformal couplings).

Friedmann equations

$$H^2 = \frac{8\pi G}{3} \frac{\rho}{1 + \alpha_C (2\rho_C + 3p_C)} \,,$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho \left[\frac{1 - \alpha_C \left(4\rho_C + 3p_C + 9(\rho_C + p_C)c_C^2\right)}{(1 + \alpha_C(2\rho_C + 3p_C))^2} \right] .$$

- Constraints on density and coupling from positivity of Newton constant
- ρ can be thought also as the total density of the Universe. Universality of NMC
- Second equation, has a parenthesis that may become negative, thus providing positive acceleration even if the minimally coupled matter fields satisfy the strong energy condition
- Similar results holds for disformal coupling

Newtonian limit

[DB, Liberati, Sindoni, 2011]

- Consider small perturbation around flat space-time $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$

Modified Poisson equation

$$\nabla^2 \phi_N = 4\pi G_N \left(\rho - \frac{\alpha_D}{2} \nabla^2 F_D - \frac{\alpha_C}{2} \nabla^2 F_C \right)$$

- For flat fluid distributions or other fluids dominance effects are negligible

More degrees of freedom

$$-\frac{1}{2} \nabla^2 \gamma_{ij} = \frac{8\pi G_N}{c^2} \left[\alpha_C \left(\frac{1}{2} \eta_{ij} \nabla^2 F_C + \partial_i \partial_j F_C \right) - \frac{\alpha_D}{2} \eta_{ij} \nabla^2 F_D \right]$$

- Deviation from GR. Constraints from $1 - \psi/\phi \ll 1$

Conclusions

What we know

- Interesting alternative/complementary description of dark sector
- Continuity equation is valid (in general not only con FLRW)
- Extra force acting on fluid (not shown here)
- Newtonian limit: gradient correction to Poisson and $\phi \neq \psi$

What is up next

- Look for viable cosmologies and constrain the free functions
- What about cosmological perturbation?
- Screening?
- What about thermodynamic properties of the NMC fluid?
- Two fluids system: curvature mediated DE-DM interaction?

Thanks!

BACKUP SLIDES

Perfect fluid action

$$S_{fluid} = \int d^4x \left[-\sqrt{-g}\rho(n,s) + J^\alpha \left(\varphi_{,\alpha} + s\theta_{,\alpha} + \beta_A \alpha^A_{\ ,\alpha}\right)\right]$$

$$J^\alpha = \sqrt{-g} n u^\alpha$$

Einstein equations

$$G_{\mu\nu} = 8\pi G T^{pf}_{\mu\nu} \qquad T^{pf}_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}$$

Fluid equations

$$\dot{\rho} + \theta(\rho + p) = 0 \qquad (\rho + p)\dot{u} + h^\sigma{}_\nu \nabla^\nu p = 0$$

with the definitions

$$\rho = -F\,, \qquad p = F - n\frac{\partial F}{\partial n}$$

Fluid evolution

[DB, Pettorino, Liberati, Baccigalupi, 2012]
[DB, Liberati, arXiv1502.06613]

Conformally coupled fluid

$$\dot{\rho} + \theta(\rho + p) = 0$$

$$\dot{u}^\sigma = -\frac{1}{\rho + p} H^{\sigma\nu}\nabla_\nu p + \underline{H^{\sigma\nu}\nabla_\nu \ln(1 - \alpha_C \zeta' R)}$$

Disformally coupled fluid

$$\dot{\rho} + \theta(\rho + p) = 0$$

$$\dot{u}^\sigma \left[\rho - \alpha_R \left(\frac{1}{2}\xi'\rho - \xi\right)\langle R\rangle\right] = -\alpha_R h^{\sigma\beta}\underline{\left\{R_{\beta\alpha}u^\alpha\left(\dot{\xi} + \theta\xi\right) + R_{\beta\alpha}\dot{u}^\alpha\right\}}$$

$$+\alpha_R\left\{\xi - \rho\xi'\right\} u^\alpha R_{\alpha\gamma} h^{\sigma\beta}\nabla_\beta u^\gamma$$

$$+\alpha_R u^\alpha u^\gamma h^{\sigma\beta}\underline{\left\{\xi\nabla_\alpha R_{\beta\gamma} - \frac{1}{2}\rho\xi'\nabla_\beta R_{\alpha\gamma}\right\}}$$

$$\underline{+\frac{1}{2}\alpha_R\left\{\rho\xi'\langle R\rangle\right\} h^{\sigma\beta}\nabla_\beta\xi'}$$

Effective pressure —

Mixing terms —

Higher derivatives —

Einstein equations

$$\left(M_{Pl}^2 + 2\alpha_{Scal}\psi\right) G_{\mu\nu} = T_{\mu\nu} + 2\alpha_{Scal}\left[-g_{\mu\nu}\Box\psi + \nabla_\mu\nabla_\nu\psi - \frac{R}{2}(\rho+p)\psi' H_{\mu\nu}\right]$$

$$+\alpha_{Ric}\left[\langle R\rangle\left(\xi - (\rho+p)\xi'\right) H_{\mu\nu} + \langle R\rangle \xi u_\mu u_\nu\right.$$

$$\left. - \Box t_{\mu\nu} + 2\nabla_\sigma\nabla_{(\mu} t_{\nu)}{}^{\sigma} - g_{\mu\nu}\nabla_\rho\nabla_\sigma t^{\sigma\rho}\right]$$

Conformally coupled Einstein equations

Conformally coupled EFE

$$M_*^2 G_{\mu\nu} = T_{\mu\nu} + 2\alpha_C \left[-g_{\mu\nu}\Box\zeta + \nabla_\mu \nabla_\nu \zeta - \frac{R}{2}(\rho + p)\zeta' H_{\mu\nu} \right]$$

$$-M_*^2 R = T - 6\alpha_C \left(\Box\zeta + \frac{R}{2}(\rho + p)\zeta' \right)$$