# A GRAVITATIONAL ARROW OF TIME


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**X IBERIAN COSMOLOGY MEETING**

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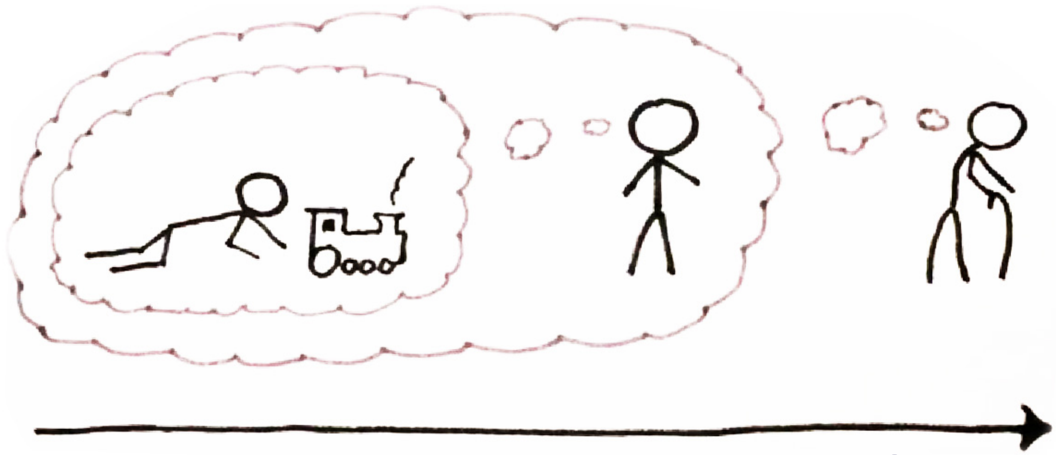
[ *'A Shape Dynamics Tutorial'* (upcoming book) arXiv:1409.0105 ]

[ *'Identification of a Gravitational Arrow of Time'* PRL **113**, 181101 ]

# The Arrow of Time Problem



ⓒ Henry Reich

The laws of physics are time-reversal invariant.

However:

- The universe is not in equilibrium (thermodynamic arrow)

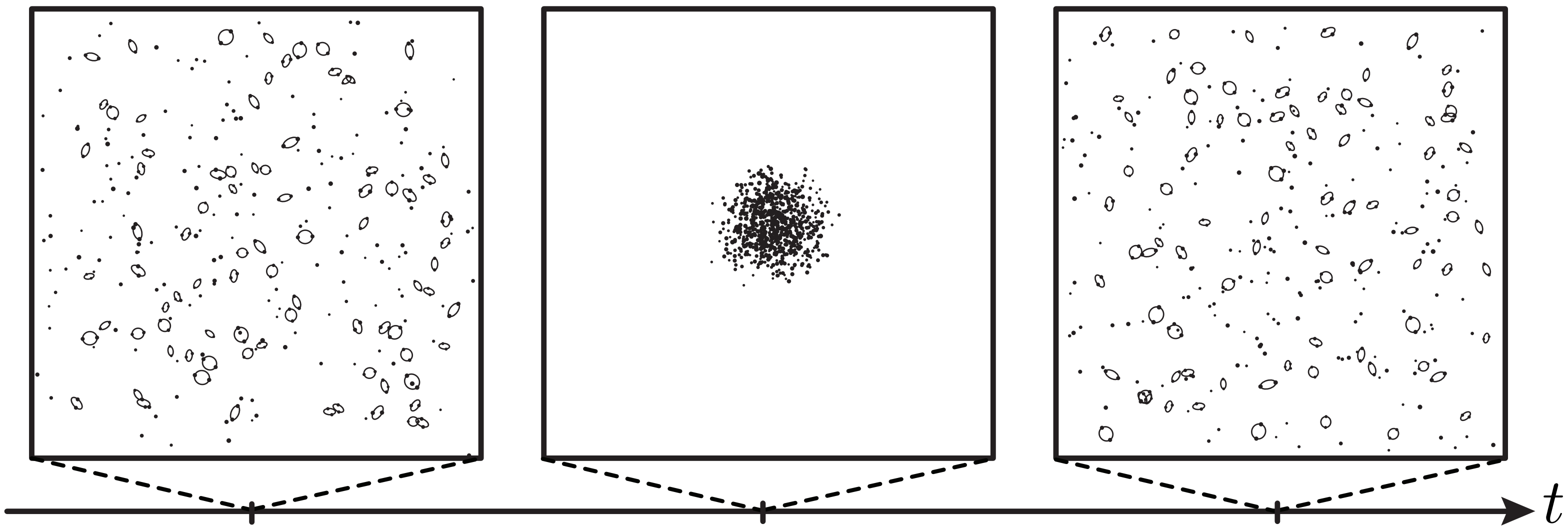
- Only retarded, no advanced waves are observed (electromagnetic arrow)

- Only black holes, no white holes (horizon arrow)

...

# The $E = J = 0$ N-body problem

$N$ point particles interacting with Newton's potential
No extraneous frame or scale $\mathbf{J}_{\text{tot}} = 0, \quad \mathbf{P}_{\text{tot}} = 0, \quad E_{tot} = 0,$




A 'one-past–two-futures' scenario: two sides look like expanding universes

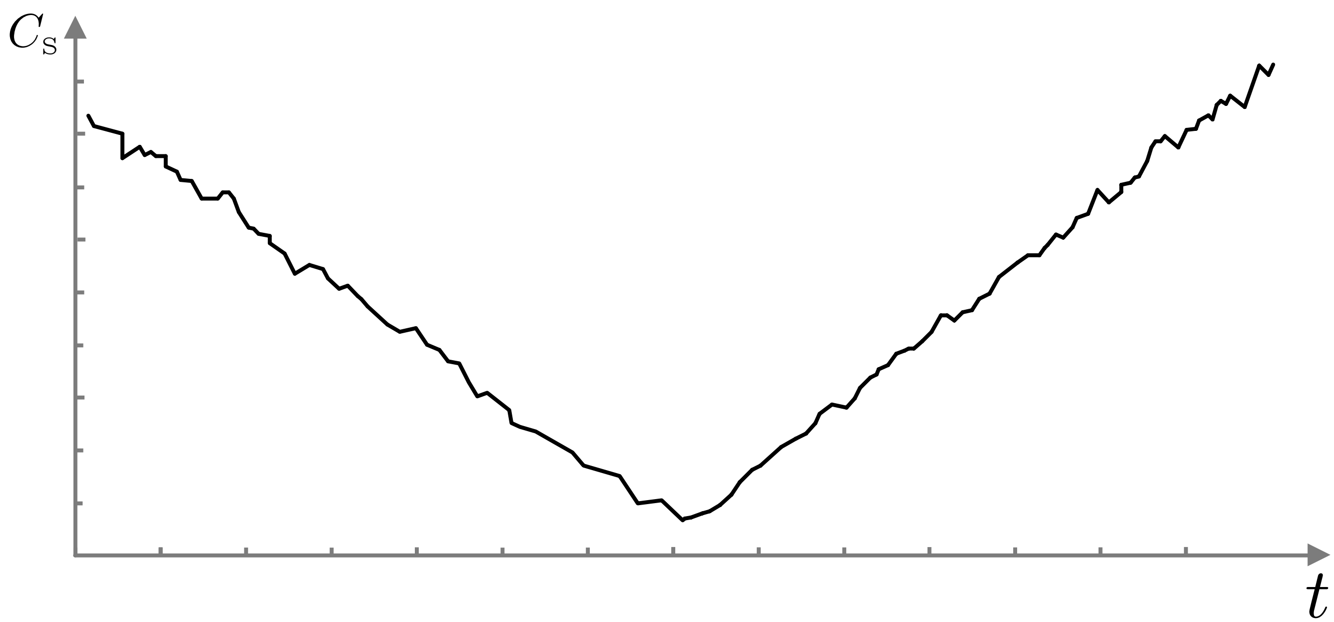
The explanation of this involves discarding the overall **scale** of the system and describing everything in terms of the **shapes** of the universe.

# Complexity measure

$$-V_{\text{Newton}}/m_{\text{tot}}^2 = \frac{1}{m_{\text{tot}}^2}\sum_{a<b}\frac{m_a\, m_b}{r_{ab}} = \frac{1}{\ell} \quad \rightarrow \quad \text{'mean harmonic length' } \ell$$

$$I_{\text{cm}}/m_{\text{tot}} = \frac{1}{m_{\text{tot}}^2}\sum_{a<b} m_a m_b\, r_{ab}^2 = L^2 \quad \rightarrow \quad \text{'root mean square length' } L$$

'Complexity' $\boxed{C_{\text{S}} = \frac{L}{\ell}}$ a sensitive measure of clustering

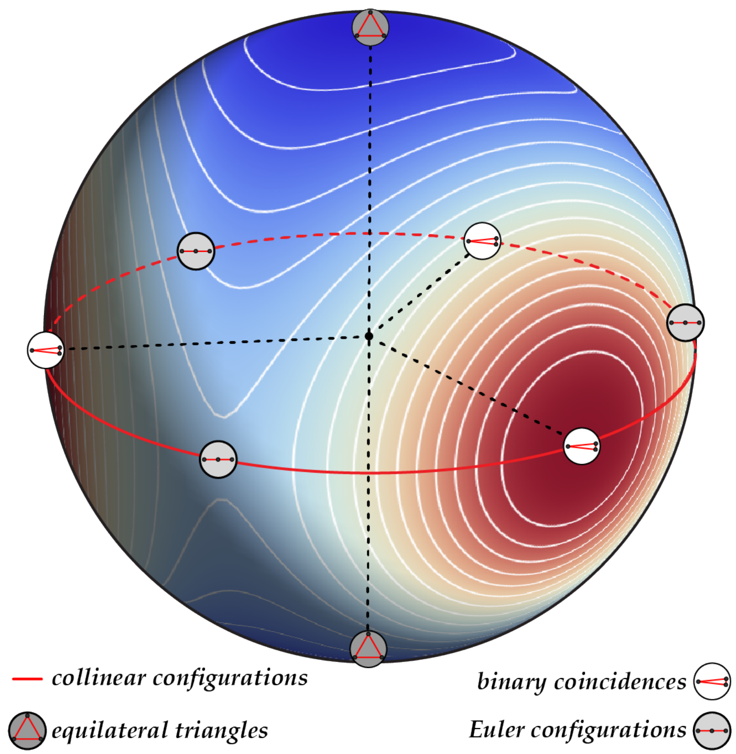


# The shape-dynamical description (3-body case)

$6N - 12$ dofs. Two are **dilatational momentum** and **moment of inertia**:

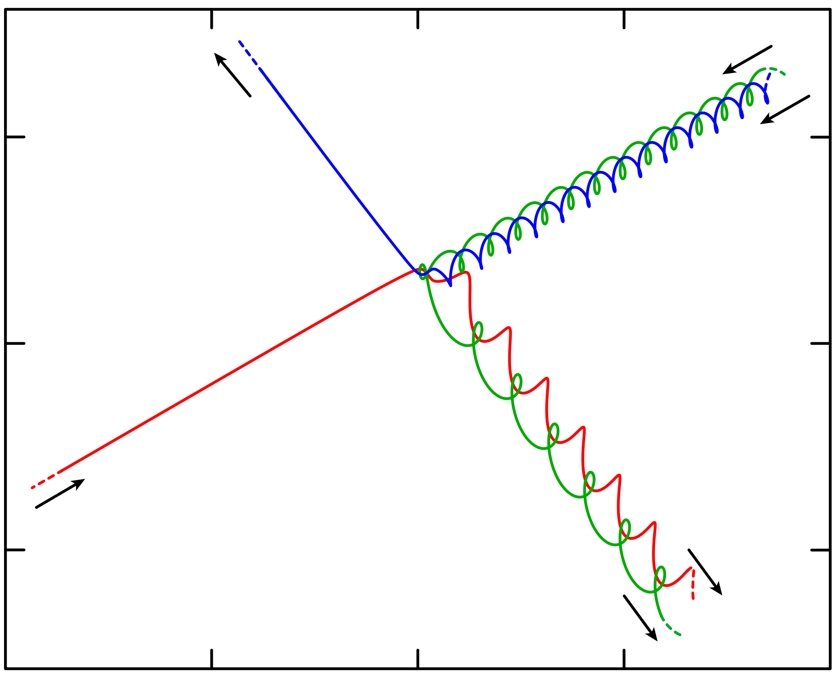
$$D = \sum_{a=1}^{N} \mathbf{r}_a \cdot \mathbf{p}^a \,, \qquad I_{\text{cm}} = \sum_{a<b} m_a m_b \|\mathbf{r}_a - \mathbf{r}_b\|^2 \,,$$

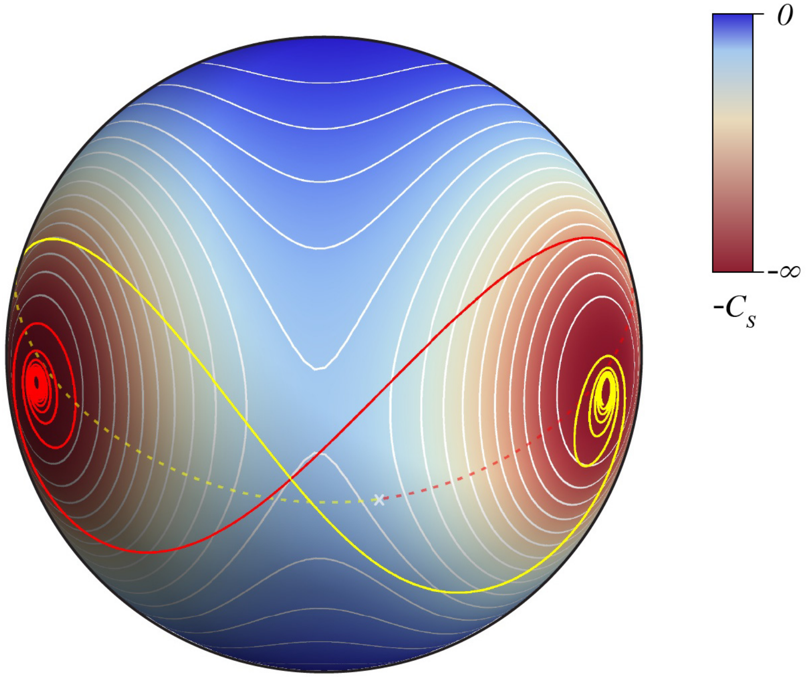
What remains are the $6N - 14$ *shape* (scale-invariant) degrees of freedom, forming *shape space* and shape momenta:




If $N = 3$ shape space is the space of triangles. 2 internal angles characterize a triangle: shape space is 2D.

# The generic 3-body solution






In the SD description, $\boxed{-\log C_{\mathsf{S}}}$ acts as a potential on shape space and the dynamics appears dissipative (therefore $C_{\mathsf{S}}$ grows secularly)

## Typicality & the second law (work in progress)

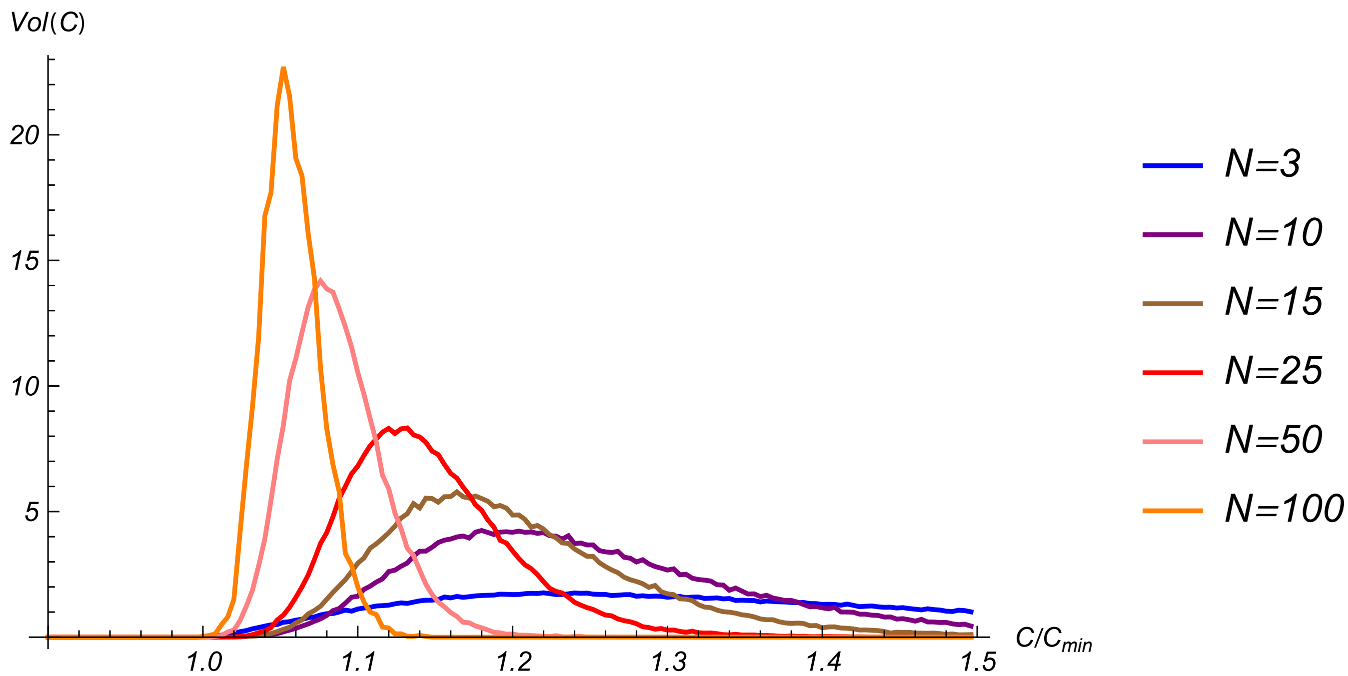
Carroll ('10): in time-reversal invariant theories, more appropriate to use **mid-point conditions** than initial conditions.

Natural place to set initial conditions: $\boxed{D = 0}$. **Unique point** in each solution.

Moreover $D = 0$ is a scale-invariant statement.

Natural measure on $D = 0$ surface: induced **Liouville measure**.

Idea: treat $C_{\mathsf{S}}$ as an **order parameter**. Define $\mathrm{vol}(C_{\mathsf{S}})$ =volume of shape phase space occupied by states of same complexity.




This quantity grows rapidly to a maximum, and then decreases with $C_S$.

At $D = 0$, it measures the probability of a solution to be randomly chosen.

If measured along the solution, it **decreases** away from $D = 0$.
It measures the **typicality** of the current state of the universe.
We called it **Entaxy** ('en' = towards, 'taxos' = order).

# Summary

- A hint that the arrow of time is explained solely by the form of the dynamical law and not a special initial condition. Established for the N-body problem. Remarkable that the simplest dimensionless measure of complexity is the gravitational shape potential.

- The universe's complexity increases, and its entaxy decreases. This means that the probability for the universe to have been created in its current state by a random choice is ever decreasing.

- How to generalize this to geometrodynamics?

  proposal: $$C_S = \inf_{\phi>0} \frac{\int d^3x \sqrt{g}\,\phi(R\,\phi - 8\Delta\phi)}{\int d^3x \sqrt{g}\,\phi^6}\,.$$