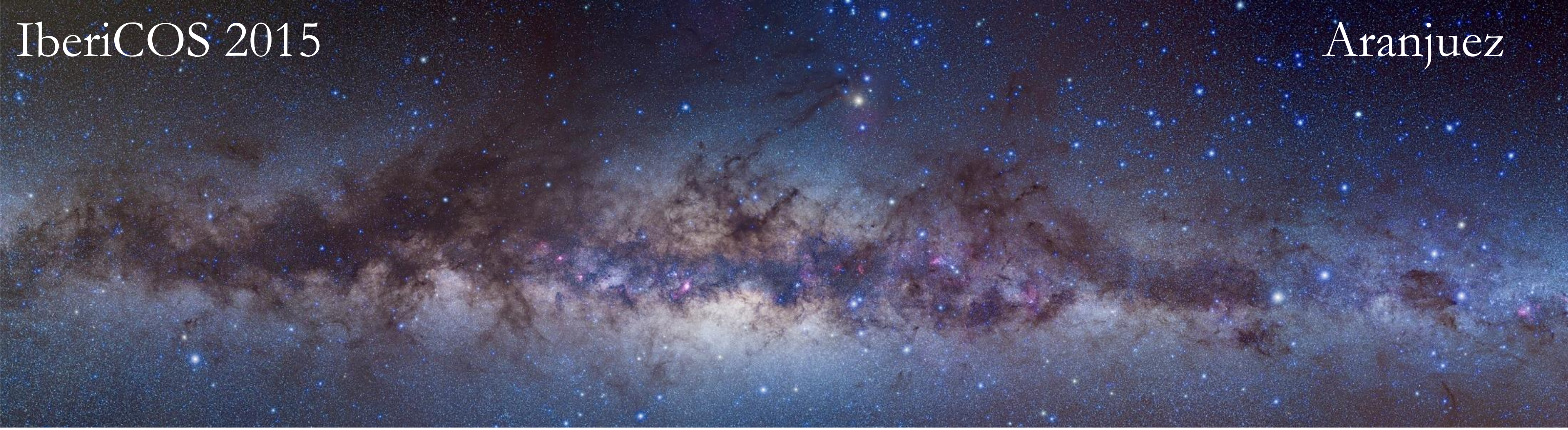

On the consistency of universally non-minimally coupled $f(R, T, R_{\mu\nu}T^{\mu\nu})$ theories

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MultiDark
Multimessenger Approach for Dark Matter Detection

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Outline

1. Introduction
    - Motivation
2. Recently developed theory
    - The Multi-Scalar representation
    - Conformal transformation in these theories
3. Dependence with the Lagrangian matter
    - Canonical Scalar Field
    - Vector Fields
4. Particular models for scalar field
5. Conclusions

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Why these theories?

• Possible extensions of Einsteinian gravity resides in the coupling of gravity and matters fields → possibility of non-minimal coupling in cosmological scales.

But… are all models possible?

No. We need criteria which aims to guarantee the absence of instabilities.

$$S = \int d^4x \sqrt{-g} \left[f(R, T, R_{\mu\nu} T^{\mu\nu}) + \mathcal{L}_m(g_{\mu\nu}, \Psi) \right]$$

<u>Example of instability:</u>

<u>Ostrogradski:</u>

It is that there is a linear instability in the Hamiltonian associated with Lagrangians which depend upon more than one time derivative in such a way that the dependence cannot be eliminated by partial integration.

M. Ostrogradski, Mem. Ac. St. Petersbourg VI **4**, 385 (1850)

How can we avoid the presence of Ostrogradski instability?

Requiring the Euler-Lagrange equations to be second order.

C. Deffayet, G. Esposito-Farese, A. Vikman, Phys. Rev. D **79**, 084003 (2009) [arXiv:0901.1314]

G. W. Horndeski, Int. J. Theor. Phys. **10** (1974) 363-384

A first great leap: the Horndeski's theorem.

$$X \equiv \frac{1}{2}\partial_\mu \phi \, \partial^\mu \phi$$

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = G_3(\phi, X)\Box\phi$$

$$\mathcal{L}_4 = G_4(\phi, X)R - G_{4,X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi + \frac{1}{6}G_{5,X}(\phi, X)\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

✓**f(R) case**: avoid the Ostrogradski instability through a conformal transformation and not with the Horndeski's theorem.

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Recently developed theory :

$$S = \int \mathrm{d}^4x\sqrt{-g}\Big[f(R,T,R_{\mu\nu}T^{\mu\nu}) + \mathcal{L}_m(g_{\mu\nu},\Psi)\Big]$$

Z. Haghani, T. Harko, F. S. N. Lobo, H. R. Sepangi and S. Shahidi, Phys. Rev. D **88** (2013) 4, 044023 [arXiv:1304.5957 [gr-qc]]

S. D. Odintsov and D. Sáez-Gómez, Phys. Lett. B **725** (2013) 437 [arXiv:1304.5411 [gr-qc]].

Recently developed theory :

$$S = \int d^4x\sqrt{-g}\left[f(R,T,R_{\mu\nu}T^{\mu\nu}) + \mathcal{L}_m(g_{\mu\nu},\Psi)\right]$$

Multi-scalar representation $\chi_1 = R \quad \chi_2 = T \quad \chi_3 = R_{\mu\nu}T^{\mu\nu}$

$$S = \int d^4x\sqrt{-g}\left[f(\chi_1,\chi_2,\chi_3) + \sum_{i=1}^{3} f_{\chi_i}\left(P_i - \chi_i\right) + \mathcal{L}_m\right]$$

Condition: the determinant $\frac{\partial^2 f}{\partial\chi_i\partial\chi_j}$ is non zero.

$$\varphi_i = -f_{\chi_i}$$

$$S = \int d^4x\sqrt{-g}\left[\mathcal{U}(\varphi_1,\varphi_2,\varphi_3) - \varphi_1 R - \varphi_2 T - \varphi_3 R_{\mu\nu}T^{\mu\nu} + \mathcal{L}_m\right]$$

An useful tool: the conformal transformation

$$g_{\mu\nu} = \mathrm{e}^{2\Omega}\tilde{g}_{\mu\nu}$$

$$R_{\mu\nu} = \tilde{R}_{\mu\nu} - 2\tilde{\nabla}_\mu\tilde{\nabla}_\nu\Omega + 2\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega - \left(2\tilde{g}^{\alpha\beta}\partial_\alpha\Omega\partial_\beta\Omega + \tilde{\Box}\Omega\right)\tilde{g}_{\mu\nu}$$

$$R = \mathrm{e}^{-2\Omega}\left(\tilde{R} - 6\tilde{g}^{\alpha\beta}\partial_\alpha\Omega\partial_\beta\Omega - 6\tilde{\Box}\Omega\right)$$

$$\tilde{T}_{\mu\nu} = \mathrm{e}^{2\Omega}T_{\mu\nu}$$

An useful tool: the conformal transformation

$$g_{\mu\nu} = \mathrm{e}^{2\Omega}\tilde{g}_{\mu\nu}$$

$$R_{\mu\nu} = \tilde{R}_{\mu\nu} - 2\tilde{\nabla}_\mu\tilde{\nabla}_\nu\Omega + 2\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega - \left(2\tilde{g}^{\alpha\beta}\partial_\alpha\Omega\partial_\beta\Omega + \tilde{\Box}\Omega\right)\tilde{g}_{\mu\nu}$$

$$R = \mathrm{e}^{-2\Omega}\left(\tilde{R} - 6\tilde{g}^{\alpha\beta}\partial_\alpha\Omega\partial_\beta\Omega - 6\tilde{\Box}\Omega\right)$$

$$\tilde{T}_{\mu\nu} = \mathrm{e}^{2\Omega}T_{\mu\nu}$$

The result for $\Omega = \log\frac{1}{\sqrt{16\pi G\varphi_1}}$ — Minimal coupling

$$S = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\left\{\mathrm{e}^{4\Omega}\mathcal{U}(\Omega,\varphi_2,\varphi_3) - \frac{1}{16\pi G}\left(\tilde{R} - 6\tilde{g}^{\alpha\beta}\partial_\alpha\Omega\partial_\beta\Omega\right) - \varphi_2\tilde{T}\right.$$
$$- \mathrm{e}^{-2\Omega}\varphi_3\left[\tilde{R}_{\mu\nu} - 2\tilde{\nabla}_\mu\tilde{\nabla}_\nu\Omega + 2\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega - \left(2\tilde{g}^{\alpha\beta}\partial_\alpha\Omega\partial_\beta\Omega + \tilde{\Box}\Omega\right)\tilde{g}_{\mu\nu}\right]\tilde{T}^{\mu\nu}$$
$$\left. + \mathrm{e}^{4\Omega}\mathcal{L}_m(\mathrm{e}^{2\Omega}\tilde{g}_{\mu\nu},\Psi)\right\}$$

Potentially problematic terms

Problem!
Non-minimal coupling of the Ricci tensor to the energy-momentum tensor

For a fixed curved background, this coupling will modify the kinetic term → could turn into a ghost.

For dynamical gravitational fields, this will introduce additional propagating degrees of freedom → Ostrogradski instability.

$$\begin{aligned} S &= \int \mathrm{d}^4x\sqrt{-\tilde{g}}\Big\{\hat{\mathcal{U}}(\Omega,\tilde{T},\varphi_3) - \frac{1}{16\pi G}\Big(\tilde{R} - 6\tilde{g}^{\alpha\beta}\partial_\alpha\Omega\partial_\beta\Omega\Big) \\ &- \mathrm{e}^{-2\Omega}\varphi_3\Big[\tilde{R}_{\mu\nu}\tilde{T}^{\mu\nu} - \Big(2\tilde{T}^{\mu\nu} + \tilde{T}\tilde{g}^{\mu\nu}\Big)\tilde{\nabla}_\mu\tilde{\nabla}_\nu\Omega + 2\Big(\tilde{T}^{\mu\nu} - \tilde{T}\tilde{g}^{\mu\nu}\Big)\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega\Big] \\ &+ \mathrm{e}^{4\Omega}\mathcal{L}_m(\mathrm{e}^{2\Omega}\tilde{g}_{\mu\nu},\Psi)\Big\} \end{aligned}$$

Problem!
It contains first derivatives of the matter fields so it will lead to higher-order equations of motion and the propagation of additional degrees of freedom → Ostrogradski instability

Outline

CANONICAL SCALAR FIELD

$$\mathcal{L}_m = \mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

$$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - g_{\mu\nu}\mathcal{L}_\phi$$

$$T = -(\partial\phi)^2 + 4V(\phi)$$

$$R_{\mu\nu}T^{\mu\nu} = G_{\mu\nu}\partial^\mu\phi\partial^\nu\phi + RV(\phi)$$

With Horndeski:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\Big(c_1 + c_2V(\phi)\Big)R + \frac{1}{2}\Big(g^{\mu\nu} + c_2G^{\mu\nu}\Big)\partial_\mu\phi\,\partial_\nu\phi - V(\phi) + f(T)\right]$$

With Multiscalar-tensor representation:

$$S = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\left\{\hat{\mathcal{U}}(\Omega,\tilde{T},\varphi_3) - \frac{1}{16\pi G}\left[\tilde{R} - 6(\partial\Omega)^2\right]\right.$$
$$\left. -\varphi_3\left[\tilde{G}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 2\left(\partial_\mu\Omega\partial^\mu\phi\right)^2 + (\partial\Omega)^2(\partial\phi)^2 + 2\Big(\tilde{g}^{\mu\nu}(\partial\phi)^2 - \partial^\mu\phi\partial^\nu\phi\Big)\tilde{\nabla}_\mu\tilde{\nabla}_\nu\Omega\right]\right\}$$

CANONICAL SCALAR FIELD

$$\mathcal{L}_m = \mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

$$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - g_{\mu\nu}\mathcal{L}_\phi$$

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With Horndeski:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\Big(c_1 + c_2V(\phi)\Big)R + \frac{1}{2}\Big(g^{\mu\nu} + c_2G^{\mu\nu}\Big)\partial_\mu\phi\,\partial_\nu\phi - V(\phi) + f(T)\right]$$

With Multiscalar-tensor representation:

$$S = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\left\{\hat{\mathcal{U}}(\Omega,\tilde{T},\varphi_3) - \frac{1}{16\pi G}\left[\tilde{R} - 6(\partial\Omega)^2\right]\right.$$
$$\left. -\alpha\left[\tilde{G}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 2\left(\partial_\mu\Omega\partial^\mu\phi\right)^2 + (\partial\Omega)^2(\partial\phi)^2 + 2\Big(\tilde{g}^{\mu\nu}(\partial\phi)^2 - \partial^\mu\phi\partial^\nu\phi\Big)\tilde{\nabla}_\mu\tilde{\nabla}_\nu\Omega\right]\right\}$$

VECTORS FIELDS

$$\mathcal{L}_m = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M^2A^2$$

$$T_{\mu\nu} = -F_{\mu\alpha}F_{\nu}{}^{\alpha} + \frac{1}{4}g_{\mu\nu}F^2 - \frac{M^2}{2}g_{\mu\nu}A^2 + M^2A_\mu A_\nu$$

$$T = -M^2A^2$$

$$R_{\mu\nu}T^{\mu\nu} = \frac{1}{4}\left(RF_{\mu\nu}F^{\mu\nu} - 4R_{\mu\nu}F^{\mu\alpha}F^{\nu}{}_{\alpha}\right) + M^2G_{\mu\nu}A^\mu A^\nu$$

→ Don't have the Horndeski's terms

→ Multi-scalar representation:

Almost Horndeski term

$$\begin{aligned}S = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\Big\{&\mathrm{e}^{4\Omega}\hat{\mathcal{U}} - \frac{1}{16\pi G}\left[\tilde{R} - 6(\partial\Omega)^2\right] - \varphi_3 M^2\tilde{G}^{\mu\nu}A_\mu A_\nu - \varphi_3\mathrm{e}^{-2\Omega}\left(\frac{1}{4}\tilde{F}^2\tilde{R} - \tilde{R}_{\mu\nu}\tilde{F}^{\mu\alpha}\tilde{F}^{\nu}{}_{\alpha}\right)\\
&+ 2\varphi_3\left[\mathrm{e}^{-2\Omega}\left(\frac{1}{4}\tilde{F}^2\tilde{g}^{\mu\nu} - \tilde{F}^{\mu\alpha}\tilde{F}^{\nu}{}_{\alpha}\right) - M^2\left(\tilde{A}^2\tilde{g}^{\mu\nu} - \tilde{A}^\mu\tilde{A}^\nu\right)\right]\tilde{\nabla}_\mu\tilde{\nabla}_\nu\Omega\\
&- 2\varphi_3\left[\mathrm{e}^{-2\Omega}\left(\frac{1}{4}\tilde{F}^2\tilde{g}^{\mu\nu} - \tilde{F}^{\mu\alpha}\tilde{F}^{\nu}{}_{\alpha}\right) + M^2\left(\tilde{A}^2\tilde{g}^{\mu\nu} + \tilde{A}^\mu\tilde{A}^\nu\right)\right]\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega + \mathrm{e}^{4\Omega}\mathcal{L}_m\left(\mathrm{e}^{2\Omega}\tilde{g}_{\mu\nu}, \tilde{A}\right)\Big\}\end{aligned}$$

VECTORS FIELDS

Direct coupling of vector fields with the scalar curvature. The coupling through the Einstein tensor guarantees the absence of an extra mode

Almost Horndeski term

$$
\begin{aligned}
S = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\,\Big\{ & \mathrm{e}^{4\Omega}\hat{\mathcal{U}} - \frac{1}{16\pi G}\left[\tilde{R} - 6(\partial\Omega)^2\right] - \varphi_3 M^2 \tilde{G}^{\mu\nu}A_\mu A_\nu - \varphi_3\mathrm{e}^{-2\Omega}\left(\frac{1}{4}\tilde{F}^2\tilde{R} - \tilde{R}_{\mu\nu}\tilde{F}^{\mu\alpha}\tilde{F}^{\nu}{}_{\alpha}\right) \\
& + 2\varphi_3\left[\mathrm{e}^{-2\Omega}\left(\frac{1}{4}\tilde{F}^2\tilde{g}^{\mu\nu} - \tilde{F}^{\mu\alpha}\tilde{F}^{\nu}{}_{\alpha}\right) - M^2\left(\tilde{A}^2\tilde{g}^{\mu\nu} - \tilde{A}^\mu\tilde{A}^\nu\right)\right]\tilde{\nabla}_\mu\tilde{\nabla}_\nu\Omega \\
& - 2\varphi_3\left[\mathrm{e}^{-2\Omega}\left(\frac{1}{4}\tilde{F}^2\tilde{g}^{\mu\nu} - \tilde{F}^{\mu\alpha}\tilde{F}^{\nu}{}_{\alpha}\right) + M^2\left(\tilde{A}^2\tilde{g}^{\mu\nu} + \tilde{A}^\mu\tilde{A}^\nu\right)\right]\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega + \mathrm{e}^{4\Omega}\mathcal{L}_m\left(\mathrm{e}^{2\Omega}\tilde{g}_{\mu\nu},\tilde{A}\right)\Big\}
\end{aligned}
$$

Coupling of the conformal mode to F is pathological → we can find higher-order equations of motion

We conclude these theories lead to Ostrogradski instabilities in a very general manner when coupled to vector fields

Outline

PARTICULAR MODELS

1. Model: $f(R,T,R_{\mu\nu}T^{\mu\nu}) = \alpha R^n + \beta(R_{\mu\nu}T^{\mu\nu})^m$

- n=1 and m=1.
- Arbitrary n and m=1.

Free of instabilities

- Arbitrary n and m. → Instabilities appear!
- Case $f(R,T,R_{\mu\nu}T^{\mu\nu}) = -\frac{R}{16\pi G} + \beta(R_{\mu\nu}T^{\mu\nu})^m$

$$S = \int d^4x\sqrt{-\tilde{g}}\Big\{e^{4\Omega}\mathcal{U}(\Omega,\phi) - \frac{1}{16\pi G}\Big(R + 6(\partial\Omega)^2\Big)$$
$$+ \frac{1-e^{-2\Omega}}{16\pi G V(\phi)}\Big[\tilde{G}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 2(\partial^\alpha\Omega\partial_\alpha\phi)^2 + (\partial\Omega)^2(\partial\phi)^2 + 2(\tilde{g}^{\mu\nu}(\partial\phi)^2 - \partial^\mu\phi\partial^\nu\phi)\tilde{\nabla}_\mu\tilde{\nabla}_\nu\Omega\Big]$$
$$+ e^{4\Omega}\mathcal{L}_m(\phi, e^{2\Omega}\tilde{g}_{\mu\nu})\Big\}$$

Free of instabilities

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CONCLUSIONS

- We have considered a class of universal non-minimally coupled where the gravitational Lagrangian is of the form: $f(R, T, R_{\mu\nu}T^{\mu\nu})$

- We have studied instabilities in these theories. We have found **two sources of instabilities**:

    1. Derivative non-minimal coupling of the matter fields to curvature.
    2. Conformal mode with second derivatives in the action.

- We have analyzed some cases for the matter sector and we found **conditions** for these theories

- The universal nature of the non-minimal coupling should be abandoned because, although it is possible to obtain stable models for scalar fields, it is troublesome to have couplings to vector fields