UNIFYING DARK ENERGY THEORIES

EFFECTIVE FIELD THEORY OF DARK ENERGY AND CONSTRAINTS IN QUINTESSENCE MODELS

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collaboration
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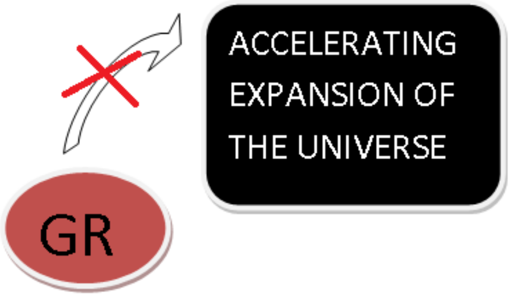
Outline

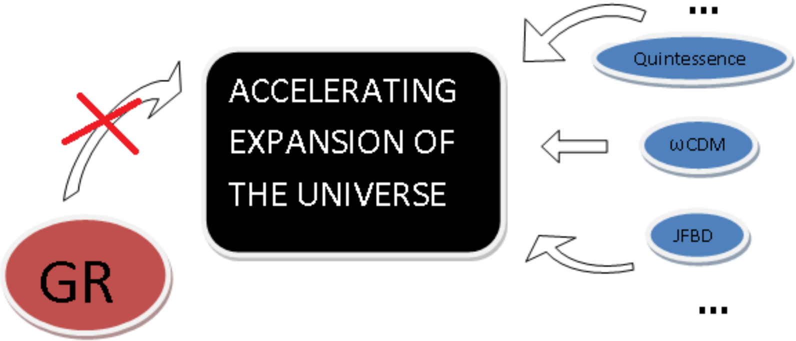
Motivation

ACCELERATING
EXPANSION OF
THE UNIVERSE

Motivation

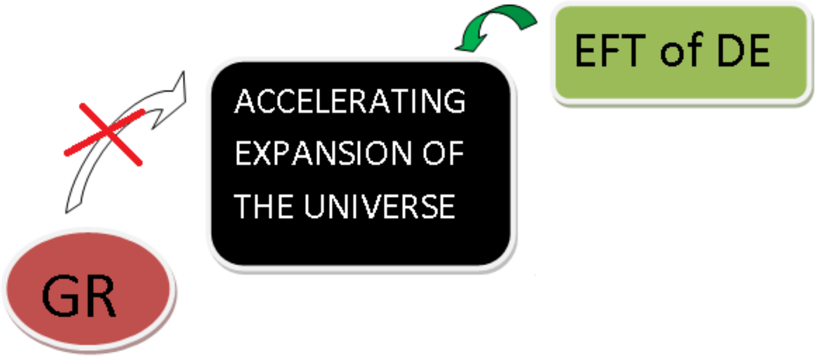


Motivation



GR
ACCELERATING EXPANSION OF THE UNIVERSE
...
Quintessence
ωCDM
JFBD
...

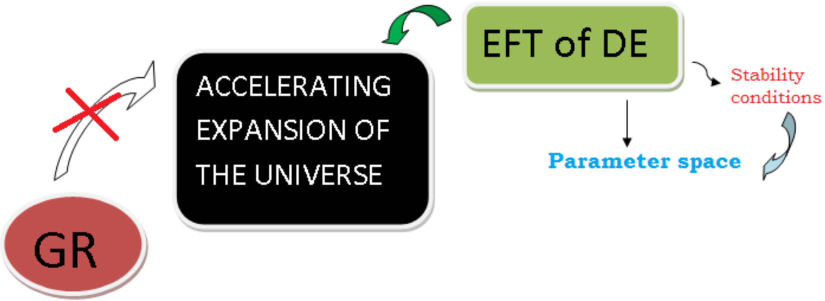
Motivation



Motivation

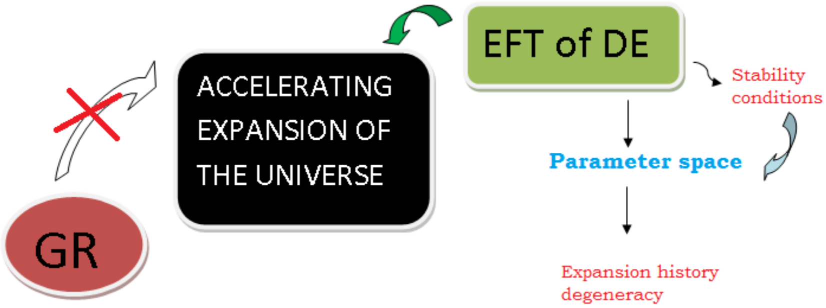

GR
ACCELERATING
EXPANSION OF
THE UNIVERSE
EFT of DE
Parameter space

Motivation



EFT of DE
ACCELERATING EXPANSION OF THE UNIVERSE
Stability conditions
Parameter space
GR

Motivation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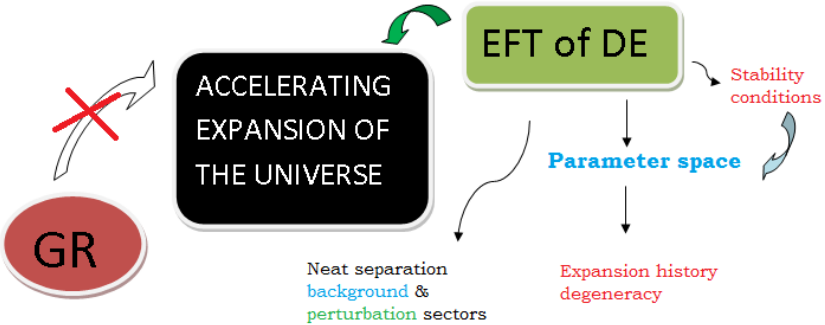
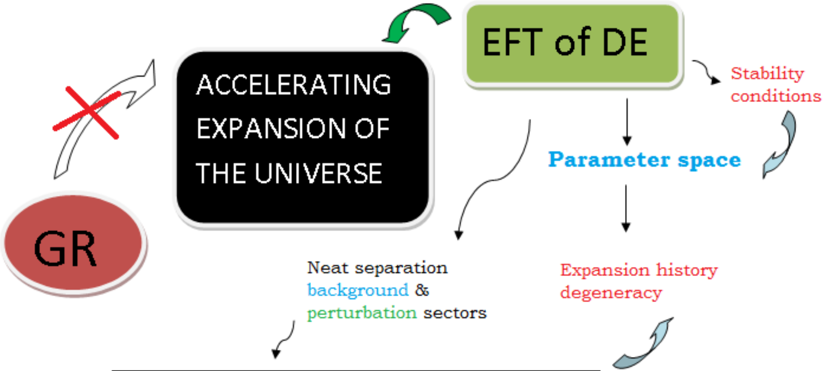
ACCELERATING EXPANSION OF THE UNIVERSE
EFT of DE
Stability conditions
Parameter space
Expansion history degeneracy

Motivation


GR
ACCELERATING EXPANSION OF THE UNIVERSE
EFT of DE
Stability conditions
Parameter space
Neat separation background & perturbation sectors
Expansion history degeneracy

Motivation



Observation	Background	Perturbation
SNe Ia	✓	✗
CMB	✓	✗
Growth of structures	✓	✓

Effective Field Theory of Dark Energy

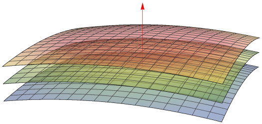
EFTofDE -Action-

G. Gubitosi, F. Piazza and F. Vernizzi, JCAP **1302** (2013) 032 [arXiv :1210.0201 [hep-th]].

- FRW background

Unitary gauge :

Scalar field is taken as the time coordinate $t \equiv t(\phi)$, spatial coordinates unfixed.



Time slicing $\phi \equiv const$, normal vector to the Cauchy hypersurfaces

$$n_\mu \equiv -\frac{\partial_\mu \phi}{\sqrt{-(\partial\phi)^2}} \rightarrow -\frac{\delta^0_\mu}{\sqrt{-g^{00}}} \quad (1)$$

and the induced spatial metric $h_{\mu\nu} \equiv g_{\mu\nu} + n_\mu n_\nu$.

- General perturbed FRW spacetime
  Scalar field $\phi(t, \vec{x}) = \phi_0(t) + \delta\phi(t, \vec{x})$
  Unitary gauge $\Rightarrow \delta\phi = 0$.
- Weak equivalence principle $\Rightarrow$ matter fields couple to the metric through a covariant action.

EFTofDE -Action-

F. Piazza, H. Steigerwald and C. Marinoni, arXiv : 1312.6111v1.

Action in unitary gauge :

$$S = \int d^4x\sqrt{|g|}\frac{M^2(t)}{2}\left[R - 2\lambda(t) - 2C(t)g^{00} \right.$$
$$\left. + \mu_2^2(t)\left(\delta g^{00}\right)^2 - \mu_3(t)\delta K\delta g^{00} + \epsilon_4(t)(\dot{K}^{\mu}_{\ \nu}\dot{K}^{\nu}_{\ \mu} - \delta K^2 + {}^{(3)}R\,\delta g^{00}/2)\right] + S_m[g_{\mu\nu};\psi]$$

Structural functions, $\delta g^{00} \equiv g^{00} + 1$ lapse component, $K_{\mu\nu}$ extrinsic curvature on hypersurfaces, its trace K and ${}^{(3)}R$ the 3D Ricci scalar.

- Stückelberg diffeomorphism $t \to t + \varphi(x^\mu)$ restores gauge invariance of the action.
- Parametrization of models such as ΛCDM, ωCDM, Quintessence, JFBD.

Theory	$\mu = \frac{dlog(M^2(t))}{dt}$	$\lambda(t)$	$C(t)$	$\mu_2^2(t)$	$\mu_3(t)$	$\epsilon_4(t)$
ΛCDM	0	*const.*	0	0	0	0
ωCDM	0	✓	0	0	0	0
Quintessence	0	✓	✓	0	0	0
JFBD	✓	✓	✓	0	0	0

EFTofDE -Stability-

J. Gleyzes, D. Langlois and F. Vernizzi, Int. J. Mod. Phys. D **23** (2014) 3010 [arXiv :1411.3712 [hep-th]].

Linear order

$$\mathcal{L} \supseteq A\dot{\varphi}^2 - B(\nabla\varphi)^2$$

where φ scalar field fluctuation.

1. $A \geq 0$ positive kinetic energy $\Rightarrow$ **Ghost free**
2. $c_s^2 \equiv \frac{B}{A} \geq 0$ positive sound speed $\Rightarrow$ **Gradient instability free**

Implement the Stückelberg trick $t \to t + \varphi(x^\mu)$

Stability conditions :

$$A = (C + 2\mu_2^2)(1 + \epsilon_4) + \frac{3}{4}(\mu - \mu_3) \geq 0, \checkmark \quad (2)$$

$$B = (C + \tilde{\mu}_3/2 - \dot{H}\epsilon_4 + H\tilde{\epsilon}_4)(1 + \epsilon_4) - (\mu - \mu_3)\left(\frac{\mu - \mu_3}{4(1 + \epsilon_4)} - \mu - \tilde{\epsilon}_4\right) \geq 0. \checkmark \quad (3)$$

Theoretical restriction on the parameter space.

- $c_s^2 > c^2$?
- Large values of $\mu_2^2(t)$ can prevent superluminal propagation of DE.

EFTofDE -Relevant sectors-

G. Gubitosi, F. Piazza and F. Vernizzi, JCAP **1302** (2013) 032 [arXiv :1210.0201 [hep-th]].

Unitary gauge ⇒ neat separation between

- **background evolution :** $M^2(t)$, $\lambda(t)$ and $C(t)$.

$$C = \frac{1}{2}(H\mu - \dot{\mu} - \mu^2) + \frac{1}{2M^2}(\rho_{\rm DE} + p_{\rm DE}), \quad \lambda = \frac{1}{2}(5H\mu + \dot{\mu} + \mu^2) + \frac{1}{2M^2}(\rho_{\rm DE} - p_{\rm DE}). \quad (4)$$

- and **perturbation sector :** $M^2(t)$, $C(t)$, $\mu_3(t)$ and $\epsilon_4(t)$.

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}\frac{G_{eft}}{G}\Omega_m(t)\delta = 0, \quad (5)$$

being $\Omega_m(t)$ the matter content and

$$\frac{G_{eft}}{G} = \frac{M^2_{Planck}}{M^2(1+\epsilon_4)} \frac{2C + \tilde{\mu}_3 - 2\dot{H}\epsilon_4 + 2H\tilde{\epsilon}_4 + 2(\mu + \tilde{\epsilon}_4)^2 + Y_{IR}(t,k)}{2C + \tilde{\mu}_3 - 2\dot{H}\epsilon_4 + 2H\tilde{\epsilon}_4 + 2\frac{(\mu+\tilde{\epsilon}_4)(\mu-\mu_3)}{1+\epsilon_4} - \frac{(\mu-\mu_3)^2}{2(1+\epsilon_4)^2} + Y_{IR}(t,k)} \quad (6)$$

where $\tilde{\mu}_3$, $\tilde{\epsilon}_4$ and $Y_{IR}(t,k)$ are combinations of the structural functions.
IR corrections relevant at scales ∼ Hubble scale.
Quintessence models ⇒ $G_{eft} = G$.

Surveys and Datasets

Background evolution -Supernovae Ia-

G. Barro Calvo and A. L. Maroto, Phys. Rev. D **74** (2006) 083519 [astro-ph/0604409]

- Distance Modulus : difference between the apparent magnitude, m, and the absolute magnitude, M,

$$\tilde{\mu}(\tau) = m - M = 5\log d_L(\tau) + 5\log\left(\frac{cH_0^{-1}}{Mpc}\right) + 25 \\ = 5\log d_L(\tau) + \tilde{M} \tag{7}$$

$\tau = H_0 t$ dimensionless cosmic time and $\tilde{M}$ nuissance parameter which minimizes the analytical expression of χ^2.

- Directly related to the luminosity distance,

$$d_L(\tau) = \frac{a(\tau_0)}{a(\tau)} \int_{\tau}^{\tau_0} \frac{d\tau'}{a(\tau')} \tag{8}$$

- **Union2** [1] supernovae dataset.

Background evolution -CMB-

W. Hu and N. Sugiyama, Astrophys. J. **471** (1996) 542 [astro-ph/9510117].

- The distance prior method uses two distances ratios measured by means of the CMB temperature power spectrum
  1. the acoustic scale, l_A, which is defined as the ratio of the angular diameter distance, $d_A(z)$the comoving sound horizon, $r_s(z)$, evaluated at the decoupling epoch, z_*,

  $$l_A \equiv \frac{d_A(z_*)}{r_s(z_*)}, \tag{9}$$

  2. and the shift parameter, R, given by

  $$R = \sqrt{\Omega_m} H_0 d_A(z_*). \tag{10}$$

- $\chi^2 = \chi^T C^{-1} \chi$, where the vector χ saves the difference between the theoretical values and the observed ones of l_A, R and z_*.

$$\begin{pmatrix} l_A \\ R \\ z_* \end{pmatrix} = \begin{pmatrix} 302.10 \pm 0.86 \\ 1.710 \pm 0.019 \\ 1090.04 \pm 0.93 \end{pmatrix}, \quad C^{-1} = \begin{pmatrix} 1.800 & 27.968 & -1.103 \\ 27.968 & 5667.577 & -92.263 \\ -1.103 & -92.263 & 2.923 \end{pmatrix}$$

Cosmological perturbations

H. Steigerwald, J. Bel and C. Marinoni, JCAP **1405** (2014) 042 [arXiv :1403.0898 [astro-ph.CO]]
S. Basilakos, S. Nesseris and L. Perivolaropoulos, Phys. Rev. D **87** (2013) 12, 123529 [arXiv :1302.6051 [astro-ph.CO]].

- Growth structure function defined as $f(z)\sigma_{0,8}\delta(z)$, being $f(z) = \frac{d\ln\delta}{d\ln a}$ the growth rate.
- Galaxy power spectra observational datasets

Survey	Redshift, z	$f\sigma_8(z)$	Reference
THF	0.02	0.360 ± 0.040	[2]
6dFGS	0.067	0.423 ± 0.055	[4]
2dFGRS	0.17	0.510 ± 0.060	[5, 6]
SDSS	0.35	0.440 ± 0.050	[7]
VVDS	0.77	0.490 ± 0.180	[6, 15]
SDSS LRG	0.25	0.351 ± 0.058	[12]
	0.37	0.460 ± 0.036	
WiggleZ	0.22	0.420 ± 0.070	[8]
	0.41	0.450 ± 0.040	
	0.78	0.380 ± 0.040	
BOSS	0.57	0.43 ± 0.07	[13]
SDSS	0.30	0.407 ± 0.055	[9]
	0.50	0.427 ± 0.043	
WiggleZ	0.20	0.40 ± 0.13	[14]
	0.40	0.39 ± 0.08	
	0.60	0.40 ± 0.07	
	0.76	0.48 ± 0.09	
2SLAQ	0.55	0.45 ± 0.05	[10]
VIPERS	0.80	0.47 ± 0.08	[16]

Quintessence

Quintessence

S. Tsujikawa, Class. Quant. Grav. **30** (2013) 214003 [arXiv :1304.1961 [gr-qc]].
G. Barro Calvo and A. L. Maroto, Phys. Rev. D **74** (2006) 083519 [astro-ph/0604409]

Action

$$S_\phi = \int d^4x\sqrt{|g|}\left[\frac{M_{Pl}^2}{2}R + \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi)\right] \quad (11)$$

Background and field equations

$$\left(\frac{a'(\tau)}{a(\tau)}\right)^2 = \frac{\tilde{\phi}'^2}{6} + \tilde{V}(\tilde{\phi}) + \Omega_{m,0}a^{-3}(\tau), \qquad \tilde{\phi}'' + 3\frac{a'}{a}\tilde{\phi}' + \frac{\partial\tilde{V}(\tilde{\phi})}{\partial\tilde{\phi}} = 0 \quad (12)$$

Matter density perturbation $G_{eft} = G$

$$\delta'' + 2H\delta' - \frac{3}{2}\Omega_m(\tau)\delta = 0 \quad (13)$$

Double Exponential Potential (2EP) model of Quintessence

$$\tilde{V}(\tilde{\phi}) = A(e^{\alpha\tilde{\phi}} + e^{\beta\tilde{\phi}}) \quad (14)$$

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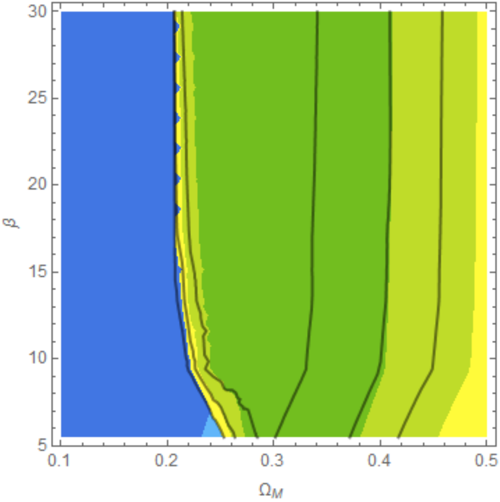
Double Exponential Potential (2EP) model of Quintessence

$$\tilde{\mathbf{V}}(\tilde{\phi}) = \mathbf{A}(\mathbf{e}^{\alpha\tilde{\phi}} + \mathbf{e}^{\beta\tilde{\phi}}) \tag{14}$$

Results

Preliminary plot



χ^2 contourplots, marginalized over α. Supernovae blue, perturbations green. Black contours refers total analysis. Up to 3 σ of confidence level.

Results

Theory	χ^2_{red}	$\Omega_{m,0}$	Second parameter	Third parameter
ΛCDM	0.9898	$0.270^{+0.013}_{-0.012}$	$\omega = -1$	$-$
ωCDM	0.9911	$0.300^{+0.070}_{-0.070}$	$\omega = -1.007^{+0.090}_{-0.091}$	$-$
2EP	1.158	$0.236^{+0.064}_{-0.016}$	$\alpha = 0.1$	$\beta = 22.8^{+7.2}_{-17.3}$

Total χ^2 values and the best range of parameters at one σ.

Conclusions

1. **Effective Field Theory of Dark Energy** formalism
   - gathers Modified Gravity and Dark Energy theories creating a parameter space,
   - neat separation between background and perturbation sectors,
   - controls any kind of instabilities.
2. To constrain the parameter space
   - We focus on **Double Exponential Potential** model of Quintessence.
   - We perform a χ^2 analysis using supernovae, CMB and growth structure datasets.
   - We compare results with $\Lambda\mathrm{CDM}$ and $\omega\mathrm{CDM}$. $\Lambda\mathrm{CDM}$ the best model.
3. **Prospects** :
   - To expand the analysis to further models of Quintessence and Brans-Dicke theory.
   - The analysis can be done in a more general way by constraining the EFT parameter space. ■

References

[1] R. Amanullah, C. Lidman, D. Rubin, G. Aldering, P. Astier, K. Barbary, M. S. Burns and A. Conley *et al.*, Astrophys. J. **716** (2010) 712 [arXiv :1004.1711 [astro-ph.CO]].

[2] S. J. Turnbull, M. J. Hudson, H. A. Feldman, M. Hicken, R. P. Kirshner and R. Watkins, Mon. Not. Roy. Astron. Soc. **420** (2012) 447 [arXiv :1111.0631 [astro-ph.CO]].

[3] M. Davis, A. Nusser, K. Masters, C. Springob, J. P. Huchra and G. Lemson, arXiv :1011.3114 [astro-ph.CO].

[4] F. Beutler, C. Blake, M. Colless, D. H. Jones, L. Staveley-Smith, G. B. Poole, L. Campbell and Q. Parker *et al.*, Mon. Not. Roy. Astron. Soc. **423** (2012) 3430 [arXiv :1204.4725 [astro-ph.CO]].

[5] W. J. Percival *et al.* [2dFGRS Collaboration], Mon. Not. Roy. Astron. Soc. **353** (2004) 1201 [astro-ph/0406513].

[6] Y. -S. Song and W. J. Percival, JCAP **0910** (2009) 004 [arXiv :0807.0810 [astro-ph]].

[7] M. Tegmark *et al.* [SDSS Collaboration], Phys. Rev. D **74** (2006) 123507 [astro-ph/0608632].

[8] C. Blake, S. Brough, M. Colless, C. Contreras, W. Couch, S. Croom, T. Davis and M. J. Drinkwater *et al.*, Mon. Not. Roy. Astron. Soc. **415** (2011) 2876 [arXiv :1104.2948 [astro-ph.CO]].

[9] R. Tojeiro, W. J. Percival, J. Brinkmann, J. R. Brownstein, D. Eisenstein, M. Manera, C. Maraston and C. K. McBride *et al.*, Mon. Not. Roy. Astron. Soc. **424** (2012) 2339 [arXiv :1203.6565 [astro-ph.CO]].

[10] J. da Angela, T. Shanks, S. M. Croom, P. Weilbacher, R. J. Brunner, W. J. Couch, L. Miller and A. D. Myers *et al.*, Mon. Not. Roy. Astron. Soc. **383** (2008) 565 [astro-ph/0612401].

[11] A. Cabre and E. Gaztanaga, Mon. Not. Roy. Astron. Soc. **393** (2009) 1183 [arXiv :0807.2460 [astro-ph]].

[12] L. Samushia, W. J. Percival and A. Raccanelli, Mon. Not. Roy. Astron. Soc. **420** (2012) 2102 [arXiv :1102.1014 [astro-ph.CO]].

[13] B. A. Reid, L. Samushia, M. White, W. J. Percival, M. Manera, N. Padmanabhan, A. J. Ross and A. G. Sanchez *et al.*, arXiv :1203.6641 [astro-ph.CO].

[14] C. Contreras *et al.* [WiggleZ Collaboration], Monthly Notices of the Royal Astronomical Society, Advance Access. 10 pp. Feb 2013 [arXiv :1302.5178 [astro-ph.CO]].

[15] L. Guzzo, M. Pierleoni, B. Meneux, E. Branchini, O. L. Fevre, C. Marinoni, B. Garilli and J. Blaizot *et al.*, Nature **451**

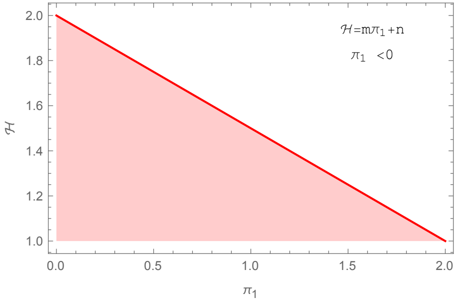
Appendices

Ostrogradski instability

Euler-Lagrange equation

$$\mathcal{L} = \mathcal{L}(\phi, \dot{\phi}, \ddot{\phi}) \Rightarrow \frac{\partial \mathcal{L}}{\partial \phi} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{\phi}} + \frac{\mathrm{d}^2}{\mathrm{d}^2 t}\frac{\partial \mathcal{L}}{\partial \ddot{\phi}} = 0 \qquad (15)$$

Define new variables and their conjugate momenta



Ostrogradski instability implies no lower limit of the energy of a system.

$$\phi_1 \equiv \phi \qquad \Pi_1 = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} + \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \ddot{\phi}}$$

$$\phi_2 \equiv \dot{\phi} \qquad \Pi_2 = \frac{\partial \mathcal{L}}{\partial \ddot{\phi}}$$

Hamiltonian

$\mathcal{H} = \Pi_1 \dot{\phi}_1 + \Pi_2 \dot{\phi}_2 - \mathcal{L}(\phi, \dot{\phi}, \ddot{\phi})$

assume $\ddot{\phi} = \dot{\phi}_2 = F(\phi_1, \phi_2, \Pi_2)$

(F invertible)

Then

$$\mathcal{H} = \Pi_1 \dot{\phi}_1 + \Pi_2 F(\phi_1, \phi_2, \Pi_2) - \mathcal{L}(\phi_1, \phi \qquad (16)$$

Cosmological perturbations in GR

V. Mukhanov, Cambridge University Press.18 (2005).

- The matter density perturbation equation. Density contrast $\delta = \frac{\rho - \rho_0}{\rho_0}$.
  1. RW metric in *longitudinal gauge*

$$ds^2 = a^2(\eta)\{(1+2\Phi)d\eta^2 - (1-2\Psi)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]\}, \quad (17)$$

     Φ and Ψ Bardeen's potentials.
  2. We obtain the perturbed equations of motions up to linear order :

$$\delta G^{\mu}_{\nu} = -8\pi G \delta T^{\mu}_{\nu}. \quad (18)$$

  3. We assume :
     - Perfect fluid behavior.
     - Adiabatic perturbations (entropy is constant).
     - Quasi-static approximation (QSA). Time derivatives are small with respect to spatial derivatives.
  4. Fourier space :

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G \rho_m(t)\delta = 0. \quad (19)$$

     Valid for sub-Hubble modes, $k >> H$.
- The growth structure function is defined as $f(z)\sigma_{0,8}\delta(z)$, being $f(z) = \frac{d\ln\delta}{d\ln a}$ the growth rate and $\sigma_{0,8} \equiv 0.8$.