Gravitational contraction in $f(R)$ theories: curvature singularities, particle production and repulsive gravity

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- E.V. Arbuzova, A.D. Dolgov and **LR**, *Eur. Phys. J. C* 72, 2247 (2012) [arXiv:1211.5011]
- **LR**, *Phys. Rev. D* 87, 084005 (2013) [arXiv:1212.2870]
- E.V. Arbuzova, A.D. Dolgov and **LR**, *Phys. Rev. D* 88 024035 (2013) [arXiv:1305.5668]
- E.V. Arbuzova, A.D. Dolgov and **LR**, *Astrop. Phys.* 54, 44 (2014) [arXiv:1306.5694]

$f(R)$ Gravity

Action

$$\mathcal{S}_{\text{grav}} = \int d^4x \sqrt{-g}\,(R + 2\Lambda) \quad \rightarrow \quad \mathcal{S}_{\text{grav}} = \int d^4x \sqrt{-g}\, f(R)$$

Field Equations

$$R_{\mu\nu} - \frac{R}{2}\, g_{\mu\nu} - \Lambda\, g_{\mu\nu} = T_{\mu\nu}$$

$$\downarrow$$

$$f_{,R}(R)\, R_{\mu\nu} - \frac{f(R)}{2}\, g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu) f_{,R}(R) = T_{\mu\nu}$$

- one new scalar degree of freedom: **SCALARON** $\xi \sim f_{,R}$
- equations are 4th order in $\partial g_{\mu\nu}$, but:
- theory free of **Ostrogradski** ghost(s)

$f(R)$ Models of Dark Energy

Viability Conditions

$$\begin{aligned} f_{,R} &> 0 \quad \text{no ghosts} \\ f_{,RR} &< 0 \quad \text{no tachyons} \end{aligned} \qquad \begin{cases} f \to R \\ f_{,R} \to 1 \\ f_{,RR} \to 0 \end{cases} \quad \text{for } |R| \gg |R_c|$$

This usually corresponds to finite values of the scalaron field ξ and of its potential at the singular point $R \to \infty$.

Several models have been proposed that meet early Universe and Solar System constraints, and that have a stable de Sitter attractor at late times, e.g. Starobinsky 2007, Hu-Sawicki 2007, Appleby-Battye 2007, etc.

Too good?

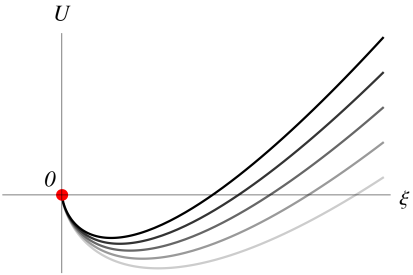
Models fulfilling these conditions may be indistinguishable from ΛCDM using just cosmological expansion tests. New probes are necessary!

Curvature Evolution in Contracting Systems

Consider an astronomical cloud under the following assumptions:

- "High" Density: $\varrho_m \gg \varrho_c$
- "Low" Gravity: $|g_{\mu\nu} - \eta_{\mu\nu}| \ll 1$
- Sph. Symmetry + Homogeneity
- Pressureless Dust: $T \propto \varrho$

$$\ddot{\xi} + R(\xi) + T = 0 \quad \Leftrightarrow \quad \ddot{\boldsymbol{\xi}} + \boldsymbol{U'(\xi, T) = 0}$$



- GR solution: $U'(\xi) = R + T = 0$
- $\xi = \xi(R = R_{\mathrm{GR}}) + |\boldsymbol{\xi}_{\mathbf{osc}}| \sin \boldsymbol{\omega t}$
- Starobinsky/Hu-Sawicki: $\xi \sim R^{-(2n+1)}$
- **Oscillations** may allow $\xi = 0$:

SINGULARITY $R \to \infty$

High-Curvature Corrections

$$f(R) = f_\Lambda(R) - \frac{R^2}{6m^2}$$

Infrared + **Ultraviolet**

Starobinsky – Hu-Sawicki

$$\xi(R) \sim R^{-(2n+1)} - g\,R \qquad (g \ll 1)$$

$$U \simeq \begin{cases} T\,\xi - \alpha\,\xi^{\frac{2n}{2n+1}} & \xi > 0 \\ T\,\xi + \beta\,\xi^2 & \xi < 0 \end{cases}$$

NO SINGULARITY!

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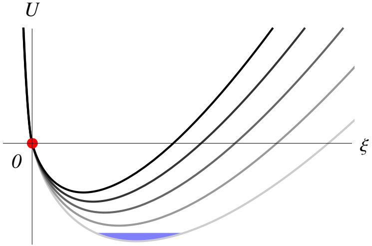

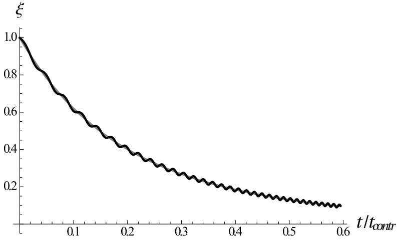

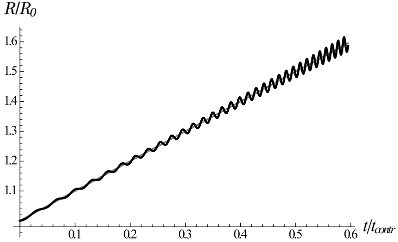

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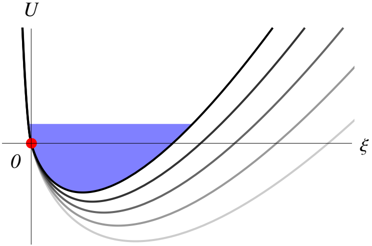
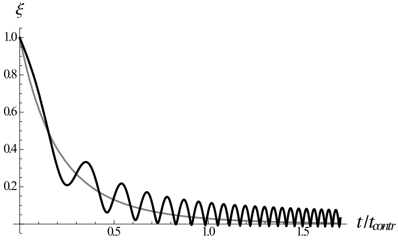
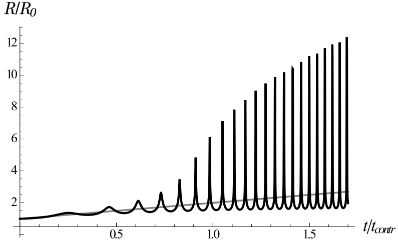
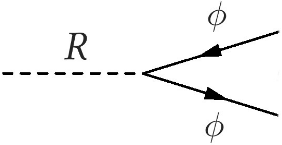






Gravitational Particle Production

- Massless ($m_\phi \ll m_\xi$) scalar field ϕ
- Minimal coupling to gravity: $\mathcal{L}_\phi = (1/2)\, g^{\mu\nu} \nabla_\mu \phi \, \nabla_\nu \phi$
- Coupling with curvature in e.o.m: $(\Box + R/6)\, \phi = 0 \;\rightarrow\; V_{\text{int}} \sim R\, \phi^2$



Given an arbitrary external $R(t)$ with Fourier transform $\mathcal{R}(\omega)$, the production of ϕ particles is

$$\dot{\varrho}_\phi = \frac{1}{576\pi^2 \Delta t} \int d\omega\, \omega \left|\mathcal{R}(\omega)\right|^2 \sim \omega_\xi \, \Delta_R^2$$

Cosmic Rays

Harmonic Region

$$\frac{L}{\text{GeV s}^{-1}} \simeq 7.3 \times 10^{-74}\, \widetilde{C}_1(n)\, N_s \left[\frac{M}{10^{11} M_\odot}\right] \left[\frac{\varrho}{\varrho_0}\right]^{4n+3} \left[\frac{\varrho_c}{\varrho_0}\right]^{n} \left[\frac{10^{10}\text{ ys}}{t_{contr}}\right]^2$$

Even for high density and short contraction times, this value is practically always negligible. However, the produced particles are $\sim$ monochromatic, perhaps detectable signal in some range of parameters.

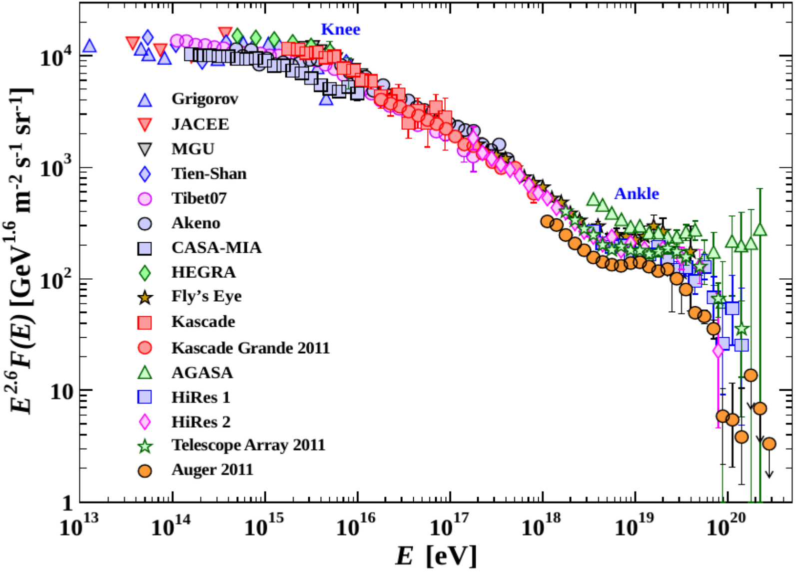
Spike Region

$$\frac{L}{\text{GeV s}^{-1}} \simeq 6.0 \times 10^{20}\, \widetilde{C}_2(n)\, N_s \left[\frac{m}{10^5\text{GeV}}\right]^2 \left[\frac{M}{10^{11} M_\odot}\right] \left[\frac{\varrho}{\varrho_0}\right]^{2n+1} \left[\frac{\varrho_c}{\varrho_0}\right]^{3n+2} \left[\frac{10^{10}\text{ys}}{t_{contr}}\right]^2$$

Potentially large luminosity, particles produced at energies up to 10^5 GeV $\lesssim m \lesssim m_{Pl}$

$\Rightarrow$ implications for the UHECR "ankle" problem?

“Ankle” in UHECR Spectrum



Spherical Symmetry

A spherically-symmetric metric can always be cast in the simple form

$$ds^2 = [1 + A(t,r)]\, dt^2 - [1 + B(t,r)] dr^2 - r^2 d\Omega$$

We assume small perturbations from Minkowski ($A, B \ll 1$), finding

$$\begin{cases} A'' - \dfrac{A'}{r} = -\dfrac{3B}{r^2} + \ddot{B} + T_{00} - 2T_{rr} + \dfrac{T_{\theta\theta}}{r^2} + \dfrac{T_{\phi\phi}}{r^2 \sin^2\theta} \equiv S_A(T_{\mu\nu}, B; t, r) \\ B' + \dfrac{B}{r} = T_{00}\, r \end{cases}$$

The corresponding solutions are:

- $A(t,r) = C_{A1}(t) r^2 + C_{A2}(t) + \int_r^R dr_1\, r_1 \int_{r_1}^R \frac{dr_2}{r_2} S_A(t, r_2)$
- $B(t,r) = \frac{C_B(t)}{r} + \frac{1}{r}\int_0^r dr_1\, r_1^2\, T_{00}(t, r_1) = \boldsymbol{B_{\mathrm{GR}}(t,r) = \frac{2GM(r,t)}{r}}$

Schwarzschild Limit

External Solution: $r > r_M$

$$A = -\frac{2GM}{r} + \left[C_{A1}(t) - \frac{2GM}{2r_M^3}\right] r^2 + \left[C_{A2}(t) + \frac{6GM}{2r_M}\right]$$

$$\boldsymbol{A \to A_{\mathrm{GR}}(r > r_M) = -\frac{2GM}{r}}$$

Internal Solution: $r < r_M$

Analogous solution, but this time

$$R \approx A'' + \frac{2A'}{r} \quad \Rightarrow \quad C_{A1} \approx \frac{R}{6}$$

which yields

$$A \to A_{\mathrm{GR}}(r < r_M) + \frac{R\, r^2}{6} \approx \frac{GM(r) r^2}{r_M^3} + \boldsymbol{\frac{R\, r^2}{6}}$$

Anti-Gravity?

Dynamics of test particle governed by

$$\ddot{r} = -\frac{A'}{2} = -\frac{1}{2}\left[\frac{2G_N M(r)r}{r_M^3} + \frac{R\,r}{3}\right]$$

Gravitational Repulsion for

$$|R| > 8\pi G\,\varrho = |R_{GR}|$$

- In the derivation of this result we have assumed that $|R| \gg 8\pi G\varrho$, so this is not truly a limit value
- still, time-dependent repulsive behaviour is possible
- (how) does this affect structure formation?

Conclusions

- $f(R)$ theories are a viable alternative to ΛCDM
  - cosmological viability conditions well understood, but:
  - hard to distinguish between viable $f(R)$ and GR
  - Finding alternative ways to test these theories is paramount
- The possibility of curvature singularities in contracting systems is rather general
- Ultraviolet corrections protect from such singularities, but still allow $R \gg R_{\mathrm{GR}}$
- Large amplitude oscillations lead to potentially detectable gravitational particle production. UHECR "ankle"?
- Spherically symmetric solutions are modified accordingly: repulsive behaviour?

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THANK YOU! ¡GRACIAS!