The Initial State of a Primordial Anisotropic Stage of Inflation

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Primordial anisotropic stage of inflation

- Inflationary models can nicely fit WMAP/*Planck* data.

  But, they also observed large-scale *anomalies which* deserve great attentions.

  Many models introduced matter fields, in order to sustain spatial anisotropy.

- Here, we consider the possibility without invoking new anisotropic source.
  - Primordial stage of inflation could be highly anisotropic, as in the presence of a cosmological constant / potential energy the universe approaches de Sitter within a few e-folds. Wald (84)
  - Effects of the initial state can be seen at the largest scales today, if inflation lasted just enough to solve the flatness and horizon problems, say $N\sim 50-60$.

Modification in the large-scale power could explain the observed rotational symmetry violation in the sky.

- In the absence of matter, the initial geometry before the onset of inflation is naturally assumed to be the Kasner solution

$$ds^2 = -dt^2 + \sum_{i=1}^{3} t^{2p_i} dx_i^2, \qquad \sum_i p_i = \sum_i p_i^2 = 1.$$

- For a cosmological constant $\Lambda > 0$, the Kasner-de Sitter solution describes universe being isotropic within $(a\ few) \times H^{-1}$.

Gumrukcuoglu, Contaldi & Peloso (07)

$$ds^2 = -dt^2 + \sum_{i=1}^{3} \sinh^{\frac{2}{3}}(3Ht)\left\{\tanh\left(\frac{3Ht}{2}\right)\right\}^{2(p_i - \frac{1}{3})} dx_i^2, \qquad H := \sqrt{\frac{\Lambda}{3}}$$

Only the initially regular Kasner branch of $p_1 = 1, p_2 = p_3 = 0$

$$ds^2 = -dt^2 + \left(\frac{2}{3}H^{-1}\sinh\frac{3Ht}{2}\left(\cosh\frac{3Ht}{2}\right)^{-\frac{1}{3}}\right)^2 dr^2 + \left(\cosh\frac{3Ht}{2}\right)^{\frac{4}{3}} dx_\perp^2$$

$\approx Milne_2 \times R^2$ at $t \approx 0$: $ds^2 \approx -dt^2 + t^2 dr^2 + dx_\perp^2$.

A part of 4d Minkowski spacetime

- The adiabatic vacuum state in the Kasner-de Sitter background

Kim & Minamitsuji (10,12)

Conformal vacuum of 2d Milne.

$$\sinh(3Ht) = \frac{1}{\sinh(-3H\eta)}$$

$$f^{(c)}_{k_\perp,k}(\eta) = \sqrt{\frac{\pi}{2H_{2d}\sinh(\pi\tilde{k})}} J_{-i\tilde{k}}\left(2\tilde{k}_\perp e^{H_{2d}\eta}\right)$$

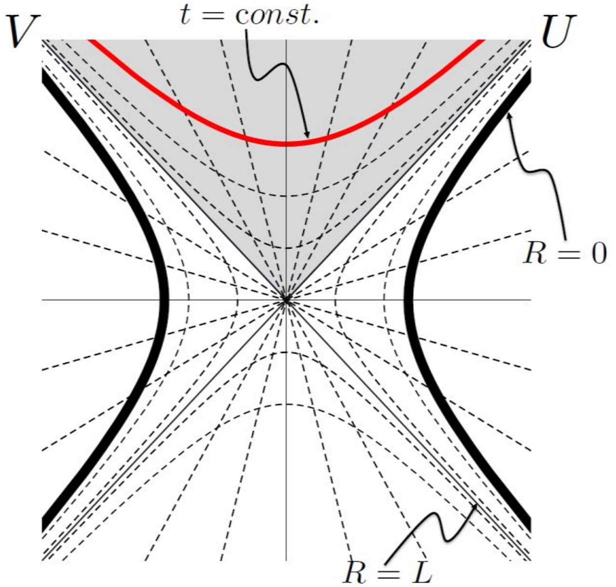
$$\tilde{k} = \frac{k}{H} \qquad \tilde{k}_\perp = \frac{k_\perp}{H}$$

The final power spectrum diverges on the plane $k \longrightarrow 0$, which leads to large backreaction, making our choice of the initial vacuum *questionable*.

Dey & Paban (11,13)

- The existence of the singularity spoils the predictability of quantum state.

Horowitz & Marolf (95)



⇒need a way to make the singularity invisible.

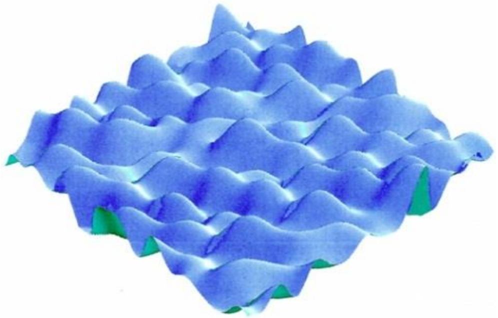
Kasner-de Sitter bubble nucleation

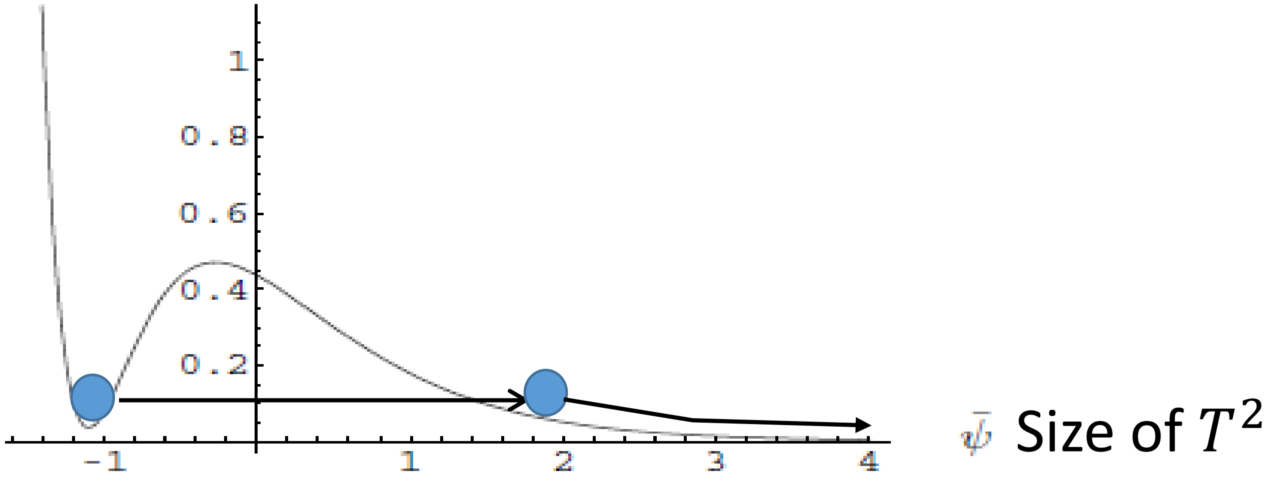
- We give a new interpretation of Kasner-de Sitter universe as an outcome of quantum tunneling from a universe with a compactified space.
  - ➢ $R^3 \to R \times T^2$: $ds^2 \approx -dt^2 + t^2 dr^2 + dx_\perp^2. \Rightarrow T^2$

Stabilized before tunneling and initially static.

Decompactification

Blanco-Pillado, Schwartz-Perlov & Vilenkin (09)



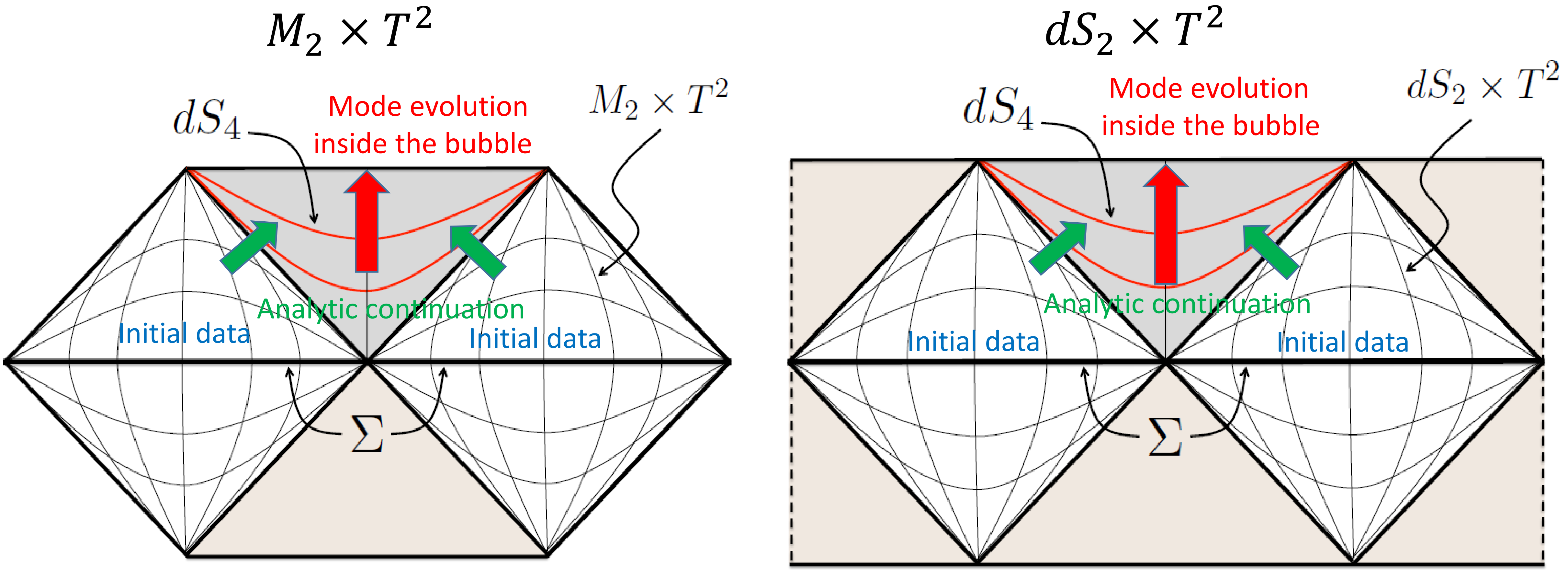


- Models with finite spatial curvature induced the late-time quadrupole anisotropy in the CMB, leading to a large lower bound of the e-folding.

Demianski & Doroshkevich (07), Blanco-Pillado & Salem (10), Adamek, Campo & Niemeyer (10)

Our model is free from this problem of quadrupole anisotropy.

- The Kasner-de Sitter bubble nucleation inside $M_2 \times T^2$ / $dS_2 \times T^2$ vacuum.



- The initial quantum state is set on the global Cauchy surface Σ.
- The initial data inside the bubble is obtained via analytic continuation.
- The final power spectrum in dS_4 has a variation with angle on large scales, depending on the choice of the parent vacuum.

Evolution inside the bubble

- The time evolution of each mode inside the Kasner de Sitter bubble follows

$$\left[\frac{d^2}{d\eta^2} + \Omega^2(k_\perp, k, \eta)\right] f_{k_\perp,k}(\eta) = 0$$

$$\Omega^2(k_\perp, k, \eta) = \alpha^{-4} \sinh^{-4/3}(-H_{2d}\eta) \;\; e^{2H_{2d}\eta/3} \left(\alpha^6 k_\perp^2 + e^{-2H_{2d}\eta} k^2\right) \; .$$

$$-\infty < \eta < 0$$

Kasner de Sitter

- Power spectrum obtained by integrating *numerically* toward $\eta \to 0$

$$\mathcal{P} = \frac{1}{2\pi^2}\left(\alpha^{-4}k^2 + \alpha^2 k_\perp^2\right)^{\frac{3}{2}} \times \begin{cases} \left|f^{(M)}_{k_\perp,k}(\eta \to 0)\right|^2 & (M_2 \times T_2) \\ \sum_{i=1}^{2} \left|f^{(i)}_{k_\perp,k}(\eta \to 0)\right|^2 & (dS_2 \times T_2) \end{cases}$$

Bubble nucleation from $M_2 \times T^2$

- The initial mode functions inside at $\eta \to -\infty$, obtained by the analytic continuation from 2d Minkowski vacuum.

$$f^{(M)}_{k_\perp,k}(\eta) = \frac{1}{2}\sqrt{\frac{\pi}{H_{2d}}} e^{\pi\tilde{k}/2}\ H^{(2)}_{i\tilde{k}}\left(2\tilde{k}_\perp e^{H_{2d}\eta}\right)$$

$\Rightarrow$ regular for $k \ll k_\perp$

...related to that in the conformal vacuum of $Milne_2$ by

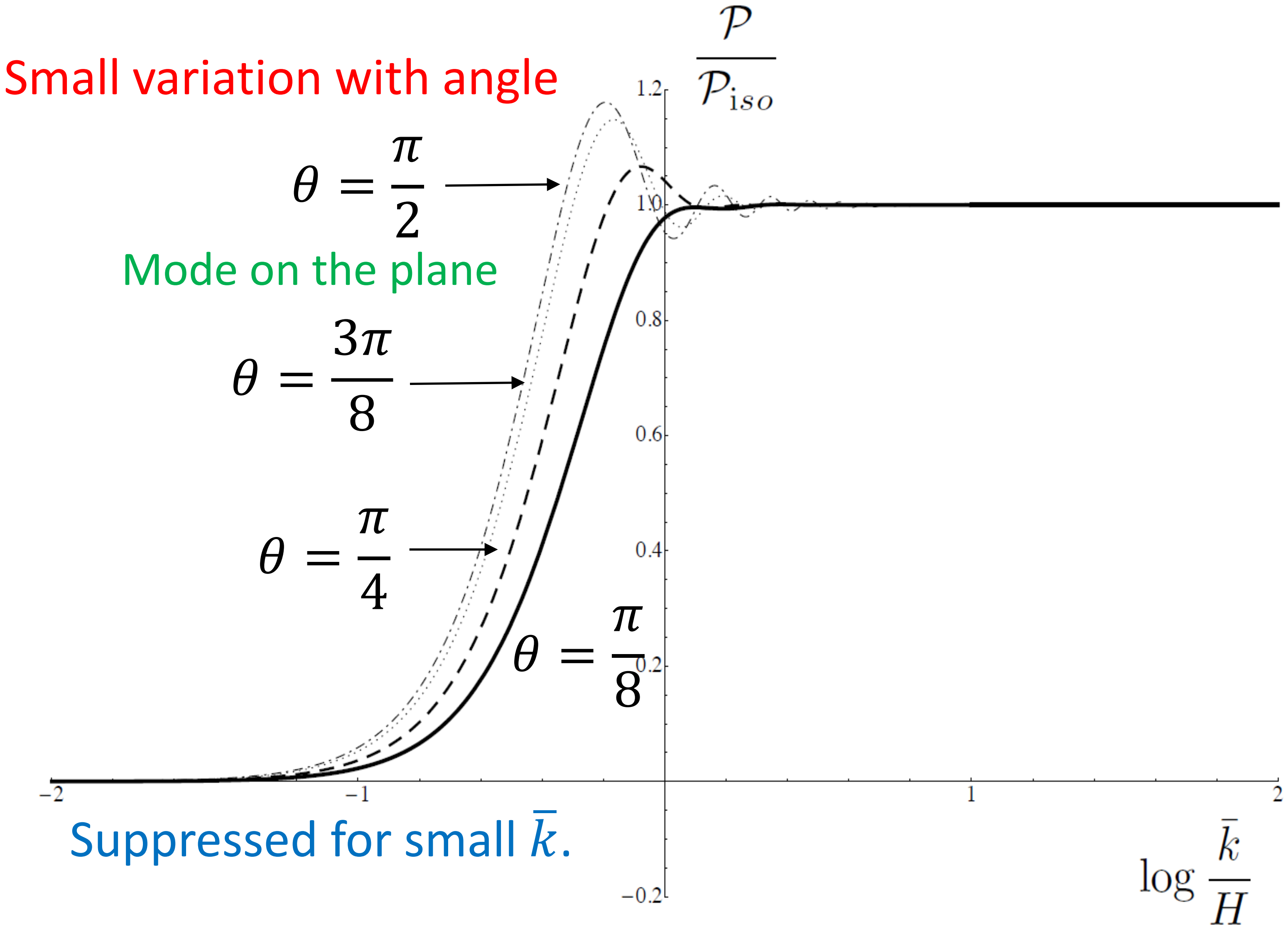
$$f^{(M)}_{k_\perp,k} = \alpha_k f^{(c)}_{k_\perp,k} + \beta_k \left(f^{(c)}_{k_\perp,k}\right)^*$$

$$f^{(c)}_{k_\perp,k}(\eta) = \sqrt{\frac{\pi}{2H_{2d}\sinh(\pi\tilde{k})}} J_{-i\tilde{k}}\left(2\tilde{k}_\perp e^{H_{2d}\eta}\right)$$

$\Rightarrow$ divergent behavior of the final power spectrum for $k \ll k_\perp$

Kim & Minamitsuji (10,12),
Dey & Paban (11,13)

$$\alpha_{\tilde{k}} = \frac{e^{\pi\tilde{k}/2}}{\sqrt{e^{\pi\tilde{k}} - e^{-\pi\tilde{k}}}} \qquad \beta_{\tilde{k}} = -\frac{e^{-\pi\tilde{k}/2}}{\sqrt{e^{\pi\tilde{k}} - e^{-\pi\tilde{k}}}}$$



Approaching the scale-invariant and isotropic spectrum for large $\bar{k}$.

$$k = \bar{k}\cos\theta \text{ and } k_\perp = \bar{k}\sin\theta$$

The power spectrum contains a small variation with angle in the intermediate regime and is regular on the plane $\theta = \frac{\pi}{2}$.

Bubble nucleation from $dS_2 \times T^2$

- The initial condition at $\eta \to -\infty$ is obtained by the analytic continuation from two normalized modes in the BD vacuum of dS_2.

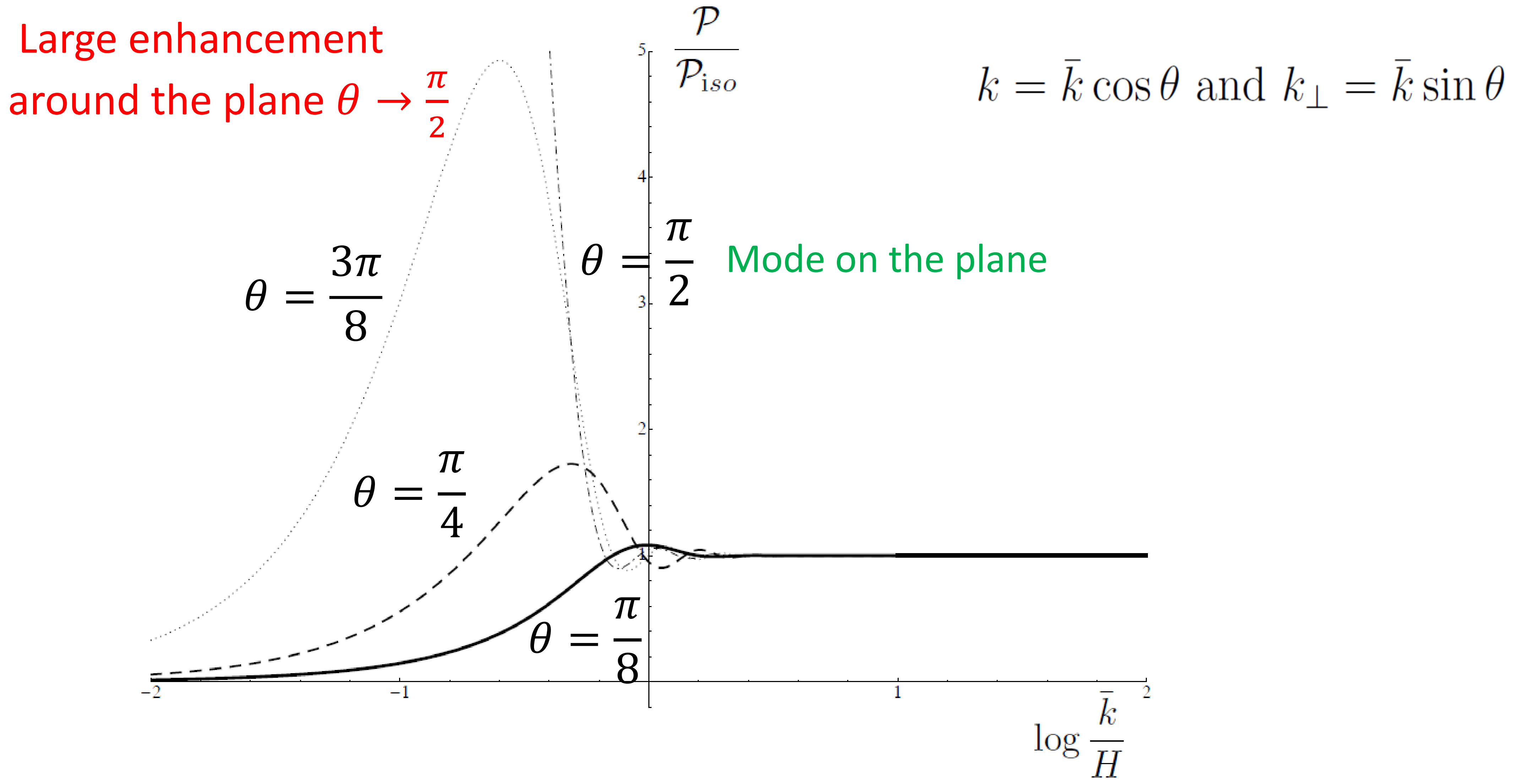
$$f^{(1)}_{k_\perp,k}(\eta) = \frac{1}{\sqrt{2k}} \frac{e^{\pi k/2H_{2d}}}{\sqrt{2\sinh(\pi k/H_{2d})}} N(k,k_\perp)\ \tilde{f}^{(1)}_{k_\perp,k}(\eta)$$

$$f^{(2)}_{k_\perp,k}(\eta) = \frac{1}{\sqrt{2k}} \frac{e^{\pi k/2H_{2d}}}{\sqrt{2\sinh(\pi k/H_{2d})}} \left(L(k,k_\perp)\ \tilde{f}^{(1)}_{k_\perp,k}(\eta) + e^{-\pi k/H_{2d}} \tilde{f}^{(2)}_{k_\perp,k}(\eta)\right)$$

$$\tilde{f}^{(1)}_{k_\perp,k}(\eta) = e^{-ik\eta} F\left[-\nu, \nu+1, 1-\mu, \frac{1+\xi_i}{2}\right] \qquad N(k,k_\perp) = \frac{\Gamma(1+\nu-\mu)\Gamma(-\mu-\nu)}{\Gamma(1-\mu)\Gamma(-\mu)}$$

$$\tilde{f}^{(2)}_{k_\perp,k}(\eta) = e^{ik\eta} F\left[-\nu, \nu+1, 1+\mu, \frac{1+\xi_i}{2}\right] \qquad L(k,k_\perp) = -\frac{\Gamma(1+\mu)\Gamma(1+\nu-\mu)\Gamma(-\mu-\nu)}{\Gamma(1-\mu)\Gamma(-\nu)\Gamma(1+\nu)}.$$

$$\xi_i = \coth(H_{2d}\eta) \quad ; \quad \mu = i\left(\frac{k}{H_{2d}}\right) \quad ; \quad \nu(\nu+1) = -\left(\frac{k_\perp}{H_{2d}}\right)^2$$



- A large power enhancement in the intermediate scales does not lead to a singular behavior along the lightcone, but contradiction with CMB data.

Summary

- Large-scale anomalies in the CMB may be caused by the nontrivial modifications of initial quantum states *before* the onset of inflation.

- Quantization in the conformal vacuum of $Milne_2$ universe leads to divergences in the spectrum and makes the choice of initial state questionable.

- We took the new picture that the initial Kasner universe is an outcome of quantum tunneling from the universe with a stabilized direction, making the initial state regular along the planar direction.

- For $M_2 \times T^2$, the tunneling leads to suppression of the power as well as small variation with angle, which could be related to low-ℓ anomalies of CMB.

Thank you.