Horndeski's Vector-Tensor Theory

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The Maxwell vector

- Standard kinetic term conformally invariant: $\mathcal{L}_M = -\frac{1}{4}\sqrt{-g}F_{\mu\nu}F^{\mu\nu} = -\frac{1}{4}\sqrt{-g}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$
- FLRW conformally flat
- Thus: $(\mathcal{L}_M)_{\text{FLRW}} = -\frac{1}{4}\eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$
- Field energy decays adiabatically (a^{-4})
- Needs to break the conformal invariance...

History

Vectors in cosmology:

- Varying fine-structure constant.
  Beckenstein 1982:
  $\mathcal{L}_{\text{int}} = -(1/4)I^2(\phi)F_{\mu\nu}F^{\mu\nu}$
- Production of primordial magnetic fields
  Turner and Widrow (1988) considered:
  $RF_{\mu\nu}F^{\mu\nu}$, $R_{\mu\nu}F^{\mu\alpha}F^{\nu}{}_{\alpha}$, $R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$.
  $RA_{\mu}A^{\mu}$, $R_{\mu\nu}A^{\mu}A^{\nu}$
- Acceleration driven by vectors.
  Ford (1989)
- Statistical anisotropies
  Ackerman, Carroll and Wise (2008)

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[2] S. M. Carroll, T. R. Dulaney, M. I. Gresham, and H. Tam, *Instabilities in the Aether*, *Phys.Rev.* **D79** (2009) 065011, [arXiv:0812.1049].

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[4] T. S. Koivisto, D. F. Mota, and C. Pitrou, *Inflation from N-Forms and its stability*, *Journal of High Energy Physics* **2009** (Mar., 2009) 24, [arXiv:0903.4158].

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[6] A. Golovnev, *Linear perturbations in vector inflation and stability issues*, *Physical Review D* **81** (Oct., 2009) 11, [arXiv:0910.0173].

[7] G. Esposito-Farese, C. Pitrou, and J.-P. Uzan, *Vector theories in cosmology*, *Phys.Rev.* **D81** (2010) 063519, [arXiv:0912.0481].

Horndeski's theory

Horndeski's Vector-Tensor Theory:

- Second order field equations, gauge invariant, reduces to Maxwell theory in flat spacetime:

$$\mathcal{L} = \frac{1}{2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4M^2}F^{\mu\nu}F^{\rho\sigma} * R *_{\mu\nu\rho\sigma} .$$

- Remarkably simple containing only one free parameter M^2.

- Totally neglected in the literature!

Example: inhomogeneous electromagnetic field in FLRW

Equations of motion:

$$\nabla \cdot \mathbf{B} = 0,$$
$$\nabla \cdot \mathbf{E} = 0,$$
$$\nabla \times \mathbf{B} = \frac{1 + 2H^2/M^2}{1 - 2qH^2/M^2}\dot{\mathbf{E}} + 2H\mathbf{E},$$
$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}} - 2H\mathbf{B},$$

Energy-density and pressure components:

$$\rho = \frac{1}{2}\left(\mathbf{E}^2 + \mathbf{B}^2\right) + \frac{H^2}{M^2}\left(3\mathbf{E}^2 - 2\mathbf{B}^2\right) - \frac{2H}{M^2}\mathbf{E} \cdot (\nabla \times \mathbf{B})$$
$$- \frac{1}{M^2}\nabla \cdot [(\mathbf{B} \cdot \nabla)\mathbf{B}]\,,$$
$$p = \frac{1}{6}(\mathbf{E}^2 + \mathbf{B}^2) - \frac{1}{3}\frac{H^2}{M^2}\left((3 - 2q)\mathbf{E}^2 + (2 + 4q)\mathbf{B}^2\right)$$
$$- \frac{2H}{3M^2}\left(\mathbf{B} \cdot (\nabla \times \mathbf{E}) + 2\mathbf{E} \cdot \dot{\mathbf{E}}\right)$$
$$- \frac{1}{3M^2}\left(\dot{\mathbf{B}}^2 + (\nabla \times \mathbf{E})^2 - 2\dot{\mathbf{E}} \cdot (\nabla \times \mathbf{B}) + \nabla \cdot [(\mathbf{E} \cdot \nabla)\mathbf{E}]\right).$$

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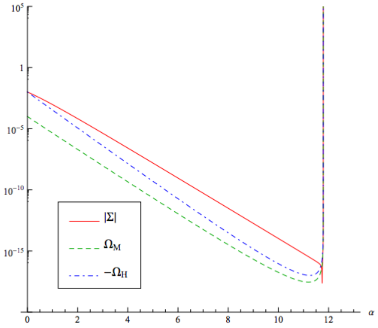
Cosmologies in Horndeski's second-order vector-tensor theory,
John D. Barrow, Mikjel Thorsrud, Kei Yamamoto (1211.5403).

We considered a non-linear dynamics

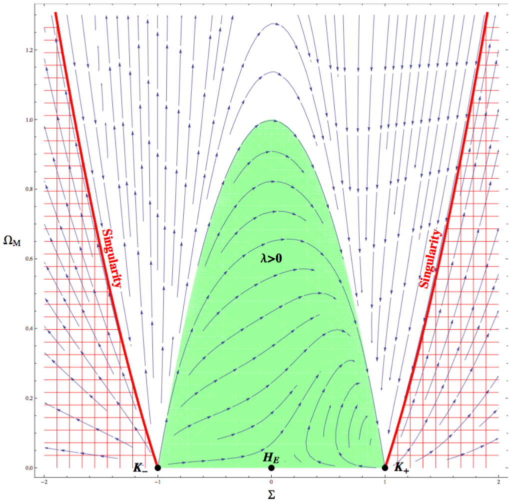
- Homogenous vector
- Axisymmetric Bianchi I
- Perfect fluid $p = w\rho$
- Hamiltonian constraint:
  $1 = \Sigma^2 + \Omega_H + \Omega_M + \Omega_m$
- Dynamical system approach, but H does not couple off...

For $M^2 < 0$ a finite time singularity inevitable if $|\Omega_H| > 3\Omega_M$.

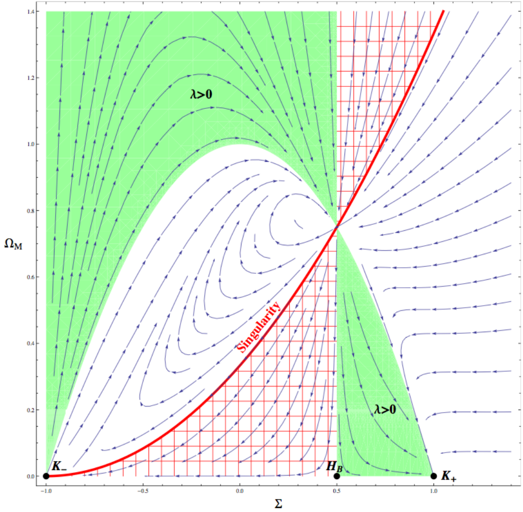


$(w = -0.9)$

Phase space portrait:



Similar conclusions for magnetic case:

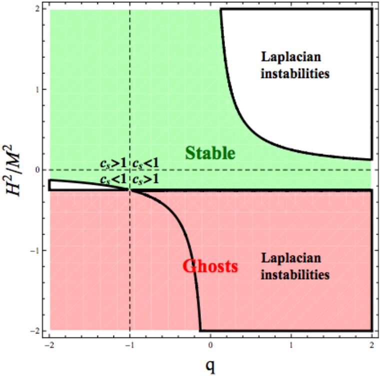


Stability of Horndeski vector-tensor interactions,
J. B. Jimenez, R. Durrer, L. Heisenberg, M. Thorsrud (1308.1867)

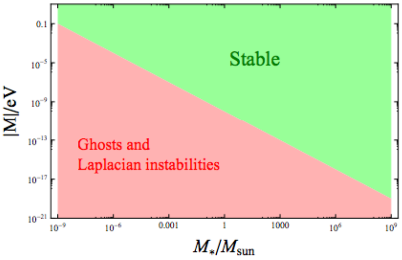
Flat FLRW:

$$S = \frac{1}{2}\int d^3k d\eta \left[\left(1 + \frac{4\mathcal{H}^2}{a^2 M^2}\right) (\vec{\mathcal{A}}'_{\perp,\vec{k}})^2 + \left(1 - \frac{4q\mathcal{H}^2}{a^2 M^2}\right) k^2 \left(\vec{\mathcal{A}}_{\perp,\vec{k}}\right)^2 \right],$$

Hamiltonian stability in flat FLRW:



Hamiltonian stability in Schwarzschild:



Free of ghosts and Laplacian instabilities if

$$-\frac{1}{4}R_s^2 < \frac{1}{M^2} < \frac{1}{2}R_s^2$$

At horizon:

$$-1/2 \lesssim \left(\frac{\mathcal{L}_H}{\mathcal{L}_M}\right)_{r=R_s} \lesssim 1,$$

Propagation speed depends on direction

$$
\begin{aligned}
c_r^2 &= 1, \\
c_{\Omega,1}^2 &= 1 - 6R_s/(M^2 r^3) \\
c_{\Omega,2}^2 &= 1 + 6R_s/(M^2 r^3)
\end{aligned}
$$

Summary

Summary:

- The best constraints on the non-minimal coupling comes from stability of black holes, $|M| > 10^{-10}$ eV.
- Maxwell theory has a stable neighborhood, but various types of instabilities are present in the regime where the non-minimal interaction energy dominate.
- Phenomenology seems NOT interesting in regimes where the theory is healthy!