Three-form Cosmology

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What is a three-form?

Totally antisymmetric tensor with three indexes

$$A_{ijk} = -A_{jik}$$

For example, a three-form defines the cross product

$$(\vec{a} \times \vec{b})_i = \epsilon_{ijk}\, a_j b_k$$

where ϵ_{ijk} is the Levi-Civita symbol.

Three-form action

Action for the three-form $A_{\mu\nu\rho}$

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} - \frac{1}{48}F^2(A) - V(A^2)\right)$$

where

$$F_{\mu\nu\rho\sigma} = 4\nabla_{[\mu}A_{\nu\rho\sigma]} = \nabla_\mu A_{\nu\rho\sigma} - \nabla_\sigma A_{\mu\nu\rho} + \nabla_\rho A_{\sigma\mu\nu} - \nabla_\nu A_{\rho\sigma\mu}$$

We have the equations of motion:

$$\nabla \cdot F = 12V'(A^2)A$$

and due to antisymmetry we have the additional constraints:

$$\nabla \cdot V'(A^2)A = 0$$

The dual theory

We define duals as:

$$(\star F) = \frac{1}{4!}\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta\gamma\delta} \equiv \Phi \qquad F_{\alpha\beta\gamma\delta} = -\epsilon_{\alpha\beta\gamma\delta}\Phi$$

$$(\star A)_\alpha = \frac{1}{3!}\epsilon_{\alpha\beta\gamma\delta}A^{\beta\gamma\delta} \equiv B_\alpha \qquad A_{\beta\gamma\delta} = -\epsilon_{\alpha\beta\gamma\delta}B^\alpha$$

which allows us to write the equivalent formulations of the Lagrangians:

$$\mathcal{L}_{IV}(F, \nabla\cdot F) = -\frac{1}{48}F^2 + 2A^2(\nabla\cdot F)V'\left(A^2(\nabla\cdot F)\right) - V(A^2(\nabla\cdot F))$$

$$\mathcal{L}_{III}(A, \nabla A)) = -\frac{1}{3}\left[\nabla A\right]^2 - V(A^2)$$

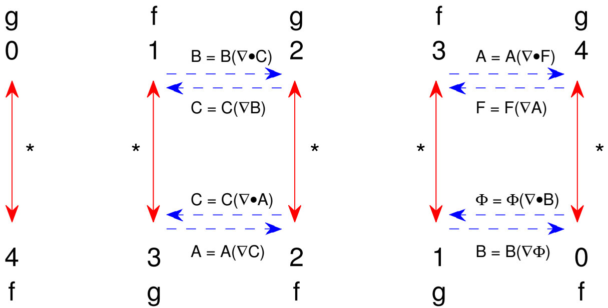
$$\mathcal{L}_I(B, \nabla\cdot B) = \frac{1}{2}(\nabla\cdot B)^2 - V\left(-6B^2\right)$$

$$\mathcal{L}_0(\Phi, \nabla\Phi) = -\frac{1}{2}\Phi^2 - 12B^2(\nabla\Phi)V'\left(-6B^2(\nabla\Phi)\right) - V\left(-6B^2(\nabla\Phi)\right)$$

We recognise $\mathcal{L}_0$ as a $p(X,\Phi)$ theory with $X = -\nabla^\mu\Phi\nabla_\mu\Phi$.

Equivalent formulations

Equivalence between Lagrangian descriptions:



Faraday formulation

$$\mathcal{L}_f = f\left(F^2(x)\right) - V(x^2)\,,$$

Gauge fixing formulation

$$\mathcal{L}_g = g\left((\nabla \cdot x)^2\right) - U(x^2)$$

Gauge invariance and stability

$$\mathcal{L} = -\frac{1}{48}F^2(A) - V(A^2)$$

F^2 is invariant under $A \to A + \nabla C$.
$V(A^2)$ breaks this symmetry resulting in extra degrees of freedom.
To see this we can make an expansion in Stückelberg fields s.t.
$A = \tilde{A} + 4[\nabla\Sigma]$

$$\mathcal{L}' = -\frac{1}{48}F^2(\tilde{A}) - V((\tilde{A} + F(\Sigma))^2)$$

is now invariant under $\tilde{A} \to \tilde{A} + [\nabla C]$; $\Sigma \to \Sigma - C/4$.
Expanding the potential around $\tilde{A}$

$$\mathcal{L}' = \mathcal{L} - V'(\tilde{A}^2)F^2(\Sigma)$$

Presence of ghost field for

$$V'(\tilde{A}^2) < 0 \quad \Leftrightarrow \quad V_{,\chi\chi} < 0$$

Equations of motion

Consider flat FRW cosmology:

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

Most general three-form compatible with FRW:

$$A_{ijk} = a^3(t)\epsilon_{ijk}\chi(t)$$

Equations of motion of the field χ:

$$\ddot{\chi} + 3H\dot{\chi} + V_{,\chi} + 3\dot{H}\chi = 0$$

Equation of motion of background fluid:

$$\dot{\rho}_B = -3\gamma H\rho_B$$

Equations of motion

Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{3}\left[\frac{1}{2}(\dot{\chi} + 3H\chi)^2 + V(\chi) + \rho_B\right]$$

can also write:

$$H^2 = \frac{\kappa^2}{3}\frac{V + \rho_B}{1 - \kappa^2(\chi' + 3\chi)^2/6}$$

with $\,' = d/d\ln a$.

Evolution of the Hubble rate and equation of state parameter of χ

$$\dot{H} = -\frac{\kappa^2}{2}\left(V_{,\chi}\chi + \gamma\rho_B\right), \qquad w_\chi = -1 + \frac{V_{,\chi}\chi}{\rho_\chi}$$

Universe de Sitter with $V = 0$; Superinflation when $V_{,\chi}\chi < 0$.

Critical points

[Koivisto, NN (2009, 2010)]
[Felice, Karwan, Wongjun (2012)]

Rewriting the equations of motion in the form of system of first order differential equations:

$$x' = 3\left(\sqrt{\frac{2}{3}}y - x\right)$$

$$y' = -\frac{3}{2}\lambda(x)\left(1 - y^2 - w^2\right)\left[xy - \sqrt{\frac{2}{3}}\right] + \frac{3}{2}\gamma w^2 y$$

$$w' = -\frac{3}{2}w\left(\gamma + \lambda(x)\left(1 - y^2 - w^2\right)x - \gamma w^2\right)$$

$$x \equiv \kappa\chi\,, \quad y \equiv \frac{\kappa}{\sqrt{6}}(\chi' + 3\chi)\,, \quad z^2 = \frac{\kappa^2 V}{3H^2} \quad w^2 \equiv \frac{\kappa^2 \rho_B}{3H^2}\,, \quad \lambda(x) \equiv -\frac{1}{\kappa}\frac{V_{,\chi}}{V}$$

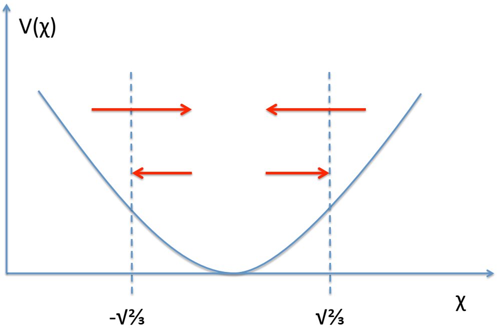
The Friedmann equation $\Rightarrow$ $y^2 + z^2 + w^2 = 1$

	x	y	w	$\dot{H}/H^2$	λ	description
A	0	0	± 1	$-3\gamma/2$	any	matter domination
$\mathrm{B}_\pm$	$\pm\sqrt{2/3}$	± 1	0	0	any	kinetic domination
C	$x_{\rm ext}$	$\sqrt{3/2}\,x_{\rm ext}$	0	0	0	potential extrema

The intuitive picture

$$H^2 = \frac{\kappa^2}{3}\,\frac{V + \rho_B}{1 - \kappa^2(\chi' + 3\chi)^2/6}$$

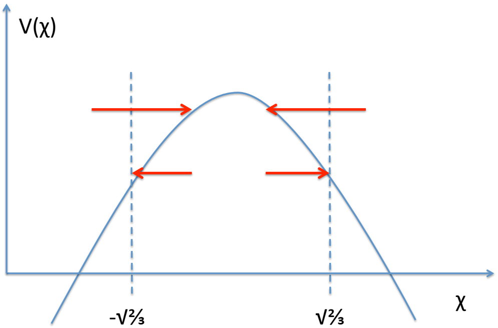
$$\Rightarrow \quad \kappa|\chi' + 3\chi| < \sqrt{6}$$



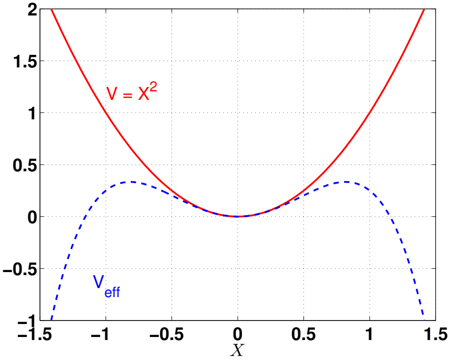
The intuitive picture

$$H^2 = \frac{\kappa^2}{3} \frac{V + \rho_B}{1 - \kappa^2(\chi' + 3\chi)^2/6}$$

$$\Rightarrow \quad \kappa|\chi' + 3\chi| < \sqrt{6}$$



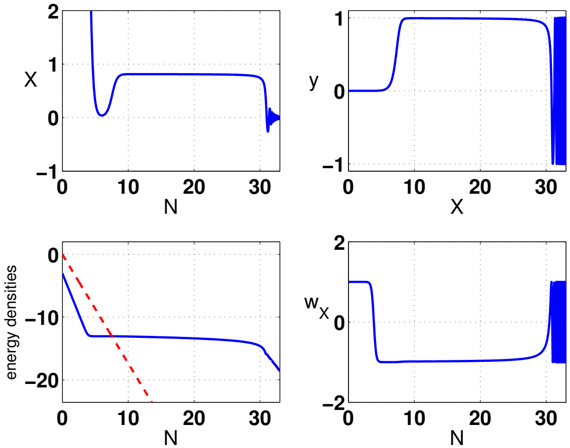
Effective potentials



$$V_{\text{eff},\chi} = V_{,\chi} + 3\dot{H}\chi = V_{,\chi}\left(1 - \frac{3}{2}(\kappa\chi)^2\right) - \frac{3}{2}\gamma\kappa^2\rho_B\chi$$

[More in Bruno Barros talk]

Example evolutions: $V = \chi^2$



Example evolutions: $V = \chi^n$

- Epoch of tracking:

$$N_s = \frac{1}{3n} \ln \left(\frac{V_i}{3H_i^2 y_i^2} \right)$$

- Point of turn around:

$$N_t = \frac{1}{3\left(1 + \gamma/2\right)} \ln \left(\frac{2}{\gamma} \frac{B}{A} \right)$$

where $A = \sqrt{2/3}\, y_i/(1 + \gamma/2)$ and $B = \chi_i - A$.

- Epoch of inflation:

$$\Delta N = \frac{2}{9n} \frac{1}{2/3 - (\kappa \chi_{\rm init})^2} - \frac{1}{2}$$

- Oscillations:

$$\langle w_\chi \rangle = \frac{n-2}{n+2}$$

Thus for $n = 2$ the field behaves as dust, $\langle w_\chi \rangle = 0$ and for $n = 4$ it mimics radiation, $\langle w_\chi \rangle = 1/3$.

Part I:

Inflation

Cosmological perturbations

[Mulryne, Noller, NN (2012)]

General perturbations about FRW background:

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + h_{ij}\left(\mathrm{d}x^i + N^i\mathrm{d}t\right)\left(\mathrm{d}x^j + N^j\mathrm{d}t\right)$$

$$h_{ij} = a^2 e^{2\zeta}\delta_{ij}$$

ζ is the curvature perturbation, and we expand N_i and N as:

$$N_i = \psi_{,i} + \tilde{N}_i \qquad\qquad N = 1 + \tilde{\alpha}$$

Perturbations of the three-form:

$$A_{0ij} = a(t)\epsilon_{ijk}\,\alpha_{,k} \qquad\qquad A_{ijk} = a^3(t)\epsilon_{ijk}(\chi(t) + \alpha_0)$$

Vector perturbations are decaying and can be ignored.

The second order action

The second order action in scalar perturbations:

$$S_2 = \int \mathrm{d}t\mathrm{d}^3x \left[a^3 \frac{\Sigma}{H^2}\dot{\zeta}^2 - a\epsilon(\partial\zeta)^2\right]$$

where the speed of sound

$$c_s^2 = \frac{V_{,XX}X}{V_{,X}}$$

and

$$\epsilon = -\frac{\dot{H}}{H^2} \qquad \Sigma = \frac{H^2\epsilon}{c_s^2}$$

ζ is conserved on the large scales.

$$\dot{\zeta} = \frac{c_s^2}{\epsilon}\frac{\nabla^2}{a^2}\left(\psi + \frac{\zeta}{H}\right)$$

Power spectrum of scalar perturbations

The 2-point correlation function

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = (2\pi)^5 \delta^3(\mathbf{k}_1 + \mathbf{k}_2)\frac{P_\zeta(k_1)}{2k_1^3} \,,$$

The power spectrum

$$P_\zeta \equiv \frac{1}{2\pi^2} k^3 \left|\zeta_k\right|^2 = \frac{1}{2(2\pi)^2 \epsilon c_s} \frac{H^2}{M_{\rm Pl}^2}\bigg|_*$$

$*$ indicates horizon crossing $c_s k = aH$.
The spectral index n_s is

$$1 - n_s = 2\epsilon + \frac{\dot{\epsilon}}{\epsilon H} + \frac{\dot{c}_s}{c_s H}$$

Power spectrum of tensor perturbations

Since the three-form does not generate tensor perturbations, their evolution equation is as usual,

$$\ddot{h} + 3H\dot{h} - \frac{\nabla^2}{a^2}h = 0$$

Tensor power spectrum:

$$\mathcal{P}_T = \frac{2}{\pi^2}\frac{H^2}{M_{\rm Pl}^2}\bigg|_*$$

The tensor spectral index is then

$$n_T = -2\epsilon$$

Consistency relation

Ratio of tensor to scalar perturbations

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\zeta} = 16\, c_S\, |\epsilon|$$

Thus it is in principle possible to distinguish the three-form inflation from scalar field already from the spectra of linear perturbations.

The third order action

The third order action of scalar perturbations:

$$S_3 = \int \left\{ dt d^3x [-\epsilon a \zeta (\partial\zeta)^2 - a^3(\Sigma + 2\lambda)\frac{\dot{\zeta}^3}{H^3} + \frac{3a^3\epsilon}{c_s^2}\zeta\dot{\zeta}^2 \right.$$
$$\left. + \frac{1}{2a}\left(3\zeta - \frac{\dot{\zeta}}{H}\right)(\partial_i\partial_j\psi\partial_i\partial_j\psi - \partial^2\psi\partial^2\psi) - 2a^{-1}\partial_i\psi\partial_i\zeta\partial^2\psi \right\}$$

where λ is

$$\lambda = -\frac{1}{12}\frac{V_{,\chi}^3 V_{,\chi\chi\chi}}{V_{,\chi\chi}^3}$$

At tree level in quantum field theory, and in the interaction picture, the In-In (equal time) three-point correlation function is given by the expression

$$\langle \zeta(t,\mathbf{k}_1)\zeta(t,\mathbf{k}_2)\zeta(t,\mathbf{k}_3)\rangle = -i\int_{t_0}^{t} \mathrm{d}t' \langle [\zeta(t,\mathbf{k}_1)\zeta(t,\mathbf{k}_2)\zeta(t,\mathbf{k}_3), H_{\text{int}}(t')]\rangle$$

The bispectrum and non-Gaussianity

The non-Gaussianity of the CMB in the WMAP observations is analyzed by assuming

$$\zeta = \zeta_L - \frac{3}{5} f_{\rm NL} \zeta_L^2$$

where ζ_L is the linear Gaussian part the perturbations, and $f_{\rm NL}$ is an estimator parameterizing the size of the non-Gaussianity.
The three-point correlation function:

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7 \delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{1}{k_1^3 k_2^3 k_3^3} {P_\zeta}^2 \mathcal{A}(k_1,k_2,k_3)$$

$$f_{\rm NL} = \frac{10}{3}\frac{1}{k_1^3+k_2^3+k_3^3}\mathcal{A} \qquad f_{\rm NL}^{\rm equil} = 30\frac{1}{K^3}\mathcal{A}$$

where $K/3 = k_1 = k_2 = k_3$.

$$f_{\rm NL}^{\rm equil} \approx \frac{5}{81}\left(\frac{1}{c_s^2} - 1 - 2\frac{\lambda}{\Sigma}\right) - \frac{35}{108}\left(\frac{1}{c_s^2} - 1\right) + \dots$$

Example I: Power law potential

In the original three-form theory

$$\mathcal{L} = -\frac{1}{48}F^2 - V_0 A^{2p}$$

and in the $p(X,\phi)$ theory

$$\mathcal{L}_\phi = (2p-1)\left(\frac{1}{V_0}\right)^{1/(2p-1)}\left(\frac{X}{24p^2}\right)^{\frac{p}{2p-1}} - \frac{1}{2}\phi^2$$

In either approach we obtain: $c_s^2 = 2p-1$

N e-folds before the end of inflation:

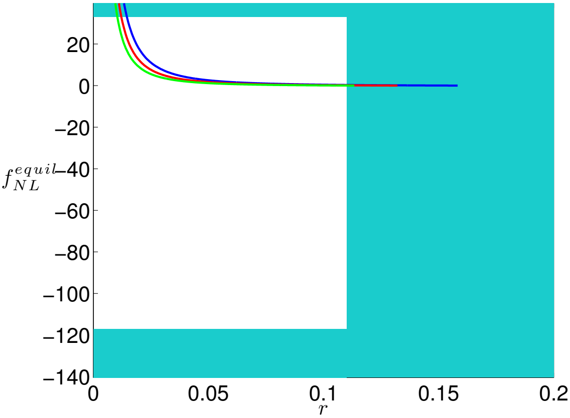
$$\chi_N^2 = \frac{2}{3} - \frac{4}{18p}\frac{1}{1+2N} \qquad \epsilon_N \approx \frac{1}{1+2N}$$

The spectral index for $N = 60$ gives

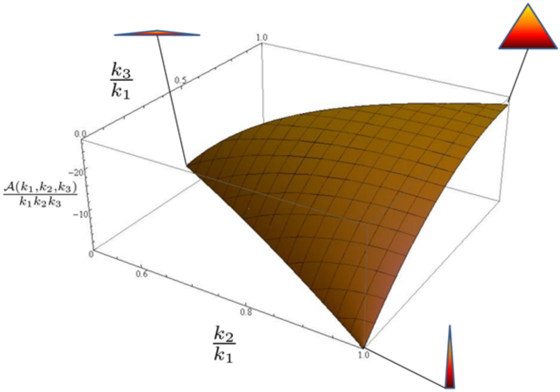
$$n_s \approx -4\epsilon \approx 0.97$$

Power law potential

Bounds from Planck in blue. Lines are for N = 50, 60, 70.



Power law potential



Amplitude dominant in the equilateral shape.

Example II: Exponential potential

In the original three-form theory

$$\mathcal{L} = -\frac{1}{48}F^2 - V_0 \exp(\beta A^2)$$

and in the $p(X, \phi)$ theory:

$$\mathcal{L}_0 = (W(x) - 1)\, V_0 \exp\left(\frac{1}{2}W(x)\right) - \frac{1}{2}\phi^2$$

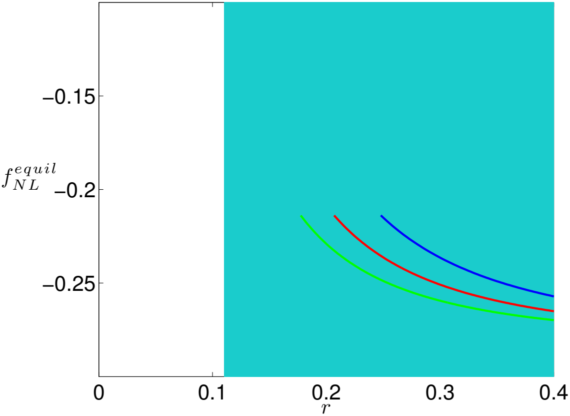
where $W(x)$ is the Lambert-W function and $x = X/12\beta V_0^2$.
N e-folds before the end of inflation

$$\chi_N^2 = \frac{2}{3} - \frac{1}{18\beta}\frac{1}{1+\sqrt{6}N}$$

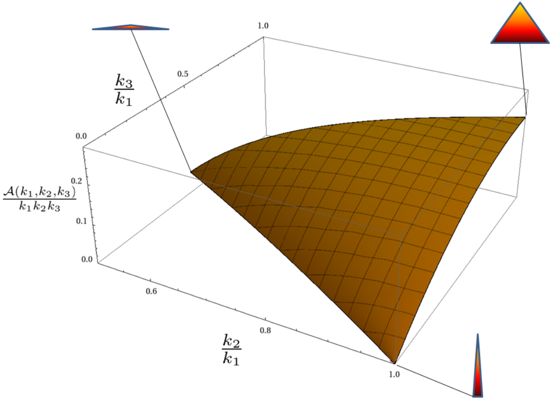
and the spectral index gives for $N = 60$

$$n_s \approx 0.97$$

Exponential potential



Exponential potential



Amplitude dominant in the equilateral shape.

Multifield three-form inflation

[Kumar, Marto, NN, Moniz (2014)]

$$S = -\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \sum_{n=1}^{\mathbb{N}}\left(\frac{1}{48}F_n^2 + V_n(A_n^2)\right)\right]$$

For two fields:

$$x_1' = 3\left(\sqrt{2/3}w_1 - x_1\right)$$

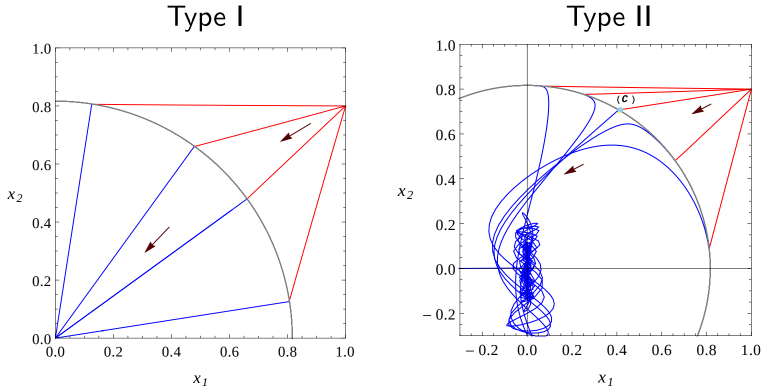
$$w_1' = \frac{3}{2}\left(1-(w_1^2+w_2^2)\right)\left(\lambda_1\left(x_1w_1-\sqrt{2/3}\right)+\lambda_2x_2w_1\right)$$

and a similar equation for x_2', w_2'. $\lambda_n = V_{(n),x_n}/V$.

Critical points: - extrema of the potential and

- for $w_1^2+w_2^2=1$ and $x_1^2+x_2^2=2/3$.

Multifield three-form inflation



$$V = x_1^2 + x_2^2 \qquad\qquad V = x_1^2 + x_2^4$$

Entropy perturbations

- Curved trajectories in field space usually lead to the growth of entropy perturbations.
- Curvature perturbations are sourced by entropy perturbations $\Rightarrow$

Corrections to spectral index and ratio of tensor to scalar perturbations.

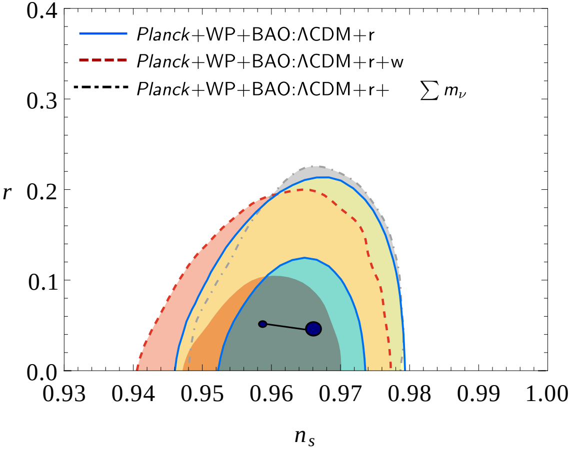
$$n_s \equiv \frac{d\ \ln \mathcal{P}_\mathcal{R}}{d\ \ln k} = n_s(t_*) + \frac{1}{H_*}\left(\frac{\partial \mathcal{T}_{\mathcal{RS}}}{\partial t_*}\right)\sin(2\Delta)$$

$$r \equiv \frac{\mathcal{P}_\mathsf{T}}{\mathcal{P}_\mathcal{R}} = 16\epsilon c_s\Big|_* \cos^2\Delta$$

$$\cos\Delta = \frac{1}{\sqrt{1+\mathcal{T}_{\mathcal{RS}}^2}}$$

Comparison with Planck (2013)

$$V = V_{10}(x_1^2 + bx_1^4) + V_{20}(x_2^2 + bx_2^4)$$



Three-form couplings with curvature

[Germani, Kehagias (2009)]

$$S = \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}R - \frac{1}{48}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} - \frac{1}{12}m^2 A_{\mu\nu\rho}A^{\mu\nu\rho} +$$

$$+\frac{1}{8}RA_{\mu\nu\rho}A^{\mu\nu\rho} - \frac{1}{2}A_{\mu\nu\kappa}R^{\kappa\lambda}A^{\mu\nu}_{\lambda}\Big)$$

This is equivalent to a scalar field theory with non-minimal kinetic terms coupled to gravity.

$$\mathcal{L} = \frac{1}{2\kappa^2}R - \frac{1}{2}\Lambda^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi - \frac{1}{2}m^2\Phi^2$$

with

$$\Lambda^{\alpha\mu}\Big[\Big(1 + \frac{R}{2m^2}g_{\alpha\nu}\Big) - \frac{2}{m^2}R_{\alpha\nu}\Big] = \delta^\mu_\nu$$

For de Sitter, $\Lambda^{\alpha\beta} = g^{\alpha\beta}$.

Three-form couplings with curvature

The second order action:

$$\delta S \simeq \int d^4x \frac{9\dot{\phi}^2}{2m^2}\left[a^3\dot{\zeta}^2 - a^3 3H^2\zeta^2 - \frac{m^2}{9H^2}a(\partial_i\zeta)^2\right]$$

Curvature perturbation at super-horizon scales

$$\zeta \sim a^{-3/2}$$

- ζ decays at super-horizon scales;
- Adiabaticity of linear perturbations is lost due to the non-minimal couplings;
- This three-form action cannot be responsible for the CMB temperature fluctuations;

Part II:

Reheating and Preheating

The action

[Felice, Karwan, Wongjun (2012)]

$$S = \int d^4x\sqrt{-g}\Big(\frac{R}{2\kappa^2} \quad -\frac{1}{48}F^2(A) - V(A^2)$$
$$- \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{\lambda}{2}\phi F^2\Big)$$

The reheating temperature can be estimated as

$$T_{\rm rh} \lesssim \lambda\left(\frac{9g_*}{640\pi^4}\right)^{1/4}\left(\frac{m}{M_{\rm Pl}}\right)^{1/2} M_{\rm Pl},$$

It is 10^5 times larger than $T_{\rm rh}$ for $f(R) = R + R^2/(6m^2)$!

Parametric resonance

One can write the equation for ϕ_k in the form of a Mathieu equation for $y_k = a^{3/2}\phi_k$

$$\frac{d^2 y_k}{dz^2} + \left(A_k - 2q\cos(2z)\right) y_k = 0$$

with

$$A_k = \frac{4k^2}{a^2m^2} + \frac{4m_\phi^2}{m^2}, \qquad q = \frac{4\sqrt{8}\lambda M_{\rm Pl}}{\sqrt{3}m^2(t-t_{\rm os})}$$

Instability bands in which the y_k grows exponentially. To guarantee enough efficiency in the production of particles we must have broad-resonance ($A_k \sim 1, 2, 3, \ldots$ and $q \gg 1$).

- For 3-forms this is easily achieved as $q \propto \lambda M_{\rm Pl}/m$.
- Also, broad resonance lasts longer than for scalar field inflation.

Still to do...

- Backreaction on the inflaton background field;
- Consider the coupling to another 3-form instead of a scalar field:

$$\mathcal{L} \sim A^2_{(2)} F^2$$

Part III:

Dark Energy

Three-form action, couplings to dark matter

Action for the three-form $A_{\mu\nu\rho}$

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} - \frac{1}{48}F^2(A) - V(A^2) - \sum_a m_a(A^2)\delta(x - x(\lambda))\sqrt{\frac{\dot{x}^2}{-g}}\right)$$

where

$$F_{\mu\nu\rho\sigma} = 4\nabla_{[\mu}A_{\nu\rho\sigma]} = \nabla_\mu A_{\nu\rho\sigma} - \nabla_\sigma A_{\mu\nu\rho} + \nabla_\rho A_{\sigma\mu\nu} - \nabla_\nu A_{\rho\sigma\mu}$$

We have the equations of motion:

$$\nabla \cdot F = 12\left(V'(A^2) + 2\rho\frac{m'(A^2)}{m(A^2)}\right) A$$

and due to antisymmetry we have the additional constraints:

$$\nabla \cdot \left(V'(A^2) + 2\rho\frac{m'(A^2)}{m(A^2)}\right) A = 0$$

Equations of motion

Consider flat FRW cosmology:

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

Most general three-form compatible with FRW:

$$A_{ijk} = a^3(t)\epsilon_{ijk}\chi(t)$$

Equations of motion of the field χ with $f \equiv 2m_{,\chi}/m$

$$\ddot{\chi} + 3H\dot{\chi} + V_{,\chi} + 3\dot{H}\chi = -\kappa\rho_m f$$

Equation of motion of dark matter fluid:

$$\dot{\rho}_m + 3H\rho_m = \kappa\rho_m f$$

Equations of motion

Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}(\dot{\chi} + 3H\chi)^2 + V(\chi) + \rho_m\right)$$

can also write:

$$H^2 = \frac{\kappa^2}{3}\,\frac{V + \rho_m}{1 - \kappa^2(\chi' + 3\chi)^2/6}$$

Evolution of the Hubble rate:

$$\dot{H} = -\frac{\kappa^2}{2}\left(V_{,\chi}\chi + (1 + \kappa f\chi)\rho_m\right)$$

Equation of state parameter of χ:

$$w_\chi = -1 + \frac{V_{,\chi} + \kappa f \rho_m}{\rho_\chi}\chi$$

Effective potential

$$\ddot{\chi} + 3H\dot{\chi} + V_{,\chi} + 3\dot{H}\chi = -\kappa\rho_m f$$

$$V_{\text{eff},\chi} = V_{,\chi}\left(1 - \frac{3}{2}(\kappa\chi)^2\right) - \frac{3}{2}\kappa^2\rho_m\chi + \kappa f\rho_m\left(1 - \frac{3}{2}(\kappa\chi)^2\right)$$

We are going to study 4 cases:

(i) $V = 0,\ f = 0;$

(ii) $V = 0,\ f \neq 0;$

(iii) $V \neq 0,\ f = 0;$

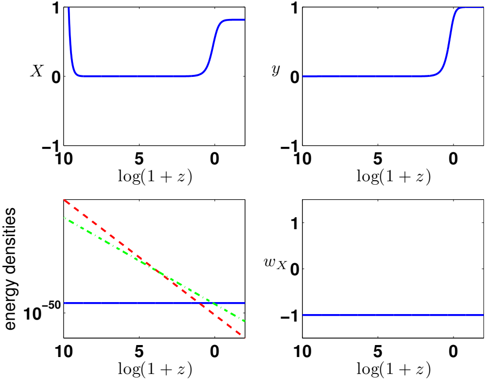
(iv) $V \neq 0,\ f \neq 0.$

Case: $V = 0,\ f = 0;$

$y_i \equiv \chi_i' + 3\chi_i \neq 0$ otherwise $\rho_\chi \equiv 0$.

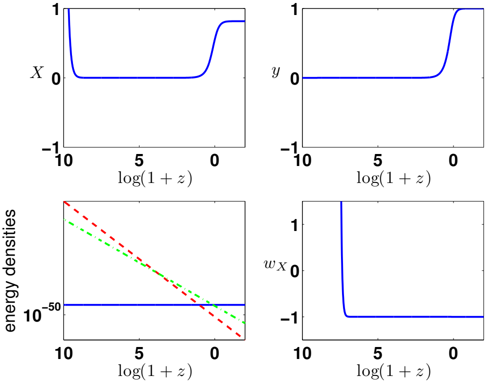
$$w_\chi \equiv -1$$

It is a cosmological constant!



Case: $V = 0,\ f \neq 0;$

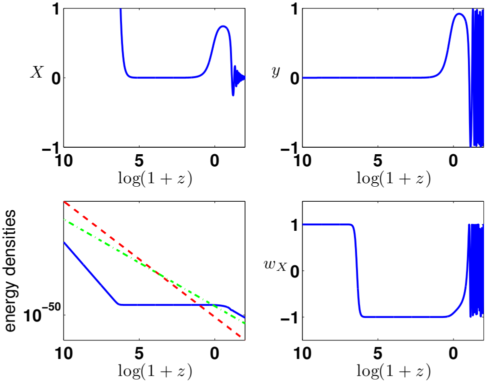
$$w_\chi = -1 + \frac{\kappa f \rho_m}{\rho_\chi}$$



Case: $V \neq 0,\ f = 0;$

With $V = V_0\chi^2;$

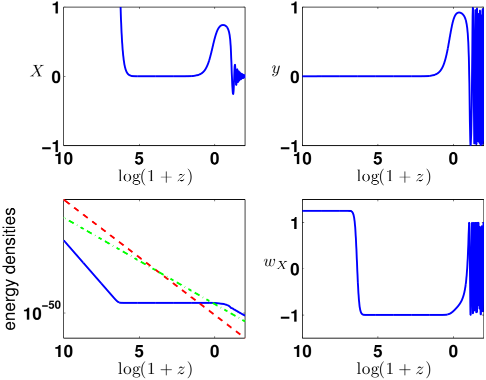
$$w_\chi = -1 + \frac{V_{,\chi}}{\rho_\chi}\chi$$



Case: $V \neq 0,\ f \neq 0$;

With $V = V_0 \chi^2$;

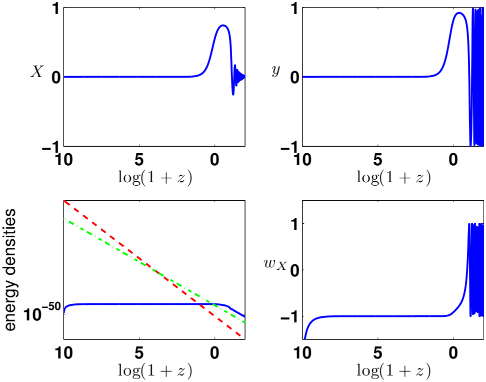
$$w_\chi = -1 + \frac{V_{,\chi} + \kappa f \rho_m}{\rho_\chi} \chi$$



Case: $V \neq 0,\ f \neq 0;$

With $V = V_0\chi^2$ and $\chi_i = \chi_i' = 0$

$$w_\chi = -1 + \frac{V_{,\chi} + \kappa f \rho_m}{\rho_\chi}\chi$$



Newtonian limit of linear perturbations

Linear evolution of matter density perturbations

$$\ddot{\delta}_m + \left(2H + \kappa f \dot{\chi} - \frac{2\dot{F}}{1-F}\right)\dot{\delta}_m = \left(\frac{\kappa_{\rm eff}^2}{2}\rho_m - \frac{k^2}{a^2}c_{\rm eff}^2\right)\delta_m$$

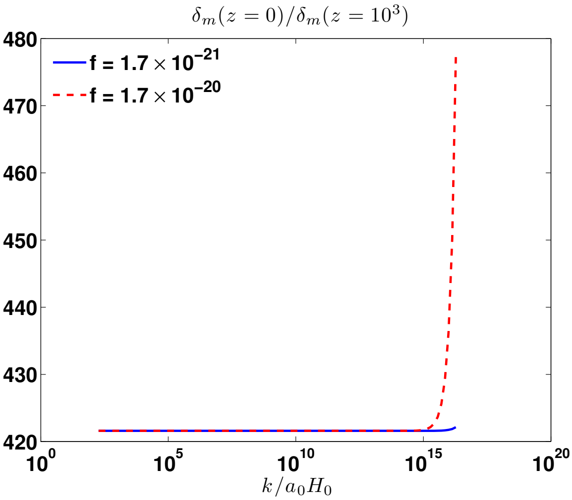
$$\kappa_{\rm eff}^2 = \frac{\kappa^2}{1-F}\left[1 + \frac{2}{\kappa^2\rho_m}\left(\ddot{F} + (2H + \kappa f\dot{\chi})\dot{F} - \frac{\kappa^2}{2}\frac{V_{,\chi}\kappa f\rho_m}{V_{,\chi\chi} + \kappa f_{,\chi}\rho_m}\right)\right]$$

$$c_{\rm eff}^2 = \frac{F}{1-F}\,, \qquad F \equiv -\frac{\kappa^2 f^2\rho_m}{V_{,\chi\chi} + \kappa f_{,\chi}\rho_m}$$

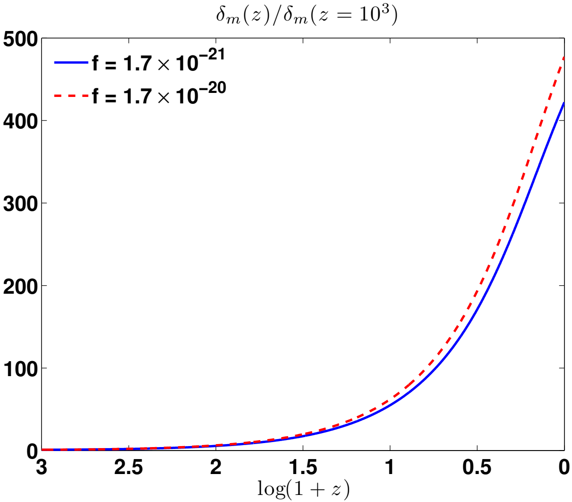
$F < 0 \Rightarrow c_{\rm eff}^2 < 0 \Rightarrow$

for sufficiently large modes there is extra source term for growth of perturbations!

Growth at a given time



Growth for a given mode



Other couplings

[Ngampitipana, Wongjuna (2011)]

$$\ddot{\chi} + 3H\dot{\chi} + 3\dot{H}\chi + V_{,\chi} = \frac{Q}{\dot{\chi} + 3H\chi}$$

- $Q_1 = \sqrt{2/3}\kappa\beta\rho_c(\dot{\chi} + 3H\chi)$
- $Q_2 = \alpha H\rho_c$
- $Q_3 = \Gamma\rho_c$
- $Q_4 = \sqrt{6/\kappa^2}\Gamma(\dot{\chi} + 3H\chi)$

They study of dynamical systems for $V = V_0 e^{-\eta\chi}$, $V = V_0 e^{-\eta\chi^2}$, $V = V_0\chi^{-n}$.

Part IV:

Magnetic Fields

The issue

- Evidence for magnetic fields from galaxies, clusters, filaments;
- Origin and nature of these magnetic fields is still unclear;
- Possibility I: Magnetogenesis during scalar field inflation;
- Backreaction problem: EM energy density catches up with energy density of inflation and brings inflation to an end;
- Possibility II: Three-Magnetogenesis [Koivisto, Urban (2012)].

The action

[Koivisto, Urban (2012)]

$$\mathcal{L} = -\frac{1}{4}F^2(\mathcal{A}) - \frac{1}{48}F^2(A) - V(A^2) - \frac{1}{2}\alpha F_{\mu\nu}(\mathcal{A})F^{\mu\nu}(B)$$

$\mathcal{A}^\mu$ = photon vector potential,
$A^{\alpha\beta\gamma}$ = three-form,
B_α = dual of three-form.

In Fourier space, the solution for $\mathcal{A}$ is

$$\mathcal{A}(\eta) = \mathcal{A}_1 \cos(k\eta) + \mathcal{A}_2 \sin(k\eta) + \mathcal{A}_3 e^{\Gamma k\eta} + \mathcal{A}_4 e^{-\Gamma k\eta}$$

If $\Gamma^2 > 0$, we have exponentially growing/decaying solution.

Stability

$$V = V_0 \exp(-\beta\chi^2/M_{\rm Pl})$$

- $\beta > 0$, critical points at $\chi = \pm\sqrt{2/3}$:

$$\Gamma^2 = \frac{\kappa_\Lambda - \kappa}{\Lambda^2\kappa_\Lambda^2 + \kappa^2}$$

with $\kappa = k/\mathcal{H}_e$ and $\Lambda \approx 8\alpha^2/3$ and $\kappa_\Lambda^2 \approx \beta V/(\alpha^2 M_{\rm Pl}^2)$.
Instability for $k < k_\Lambda$.

- $\beta < 0$, critical points at the minima of the potential:

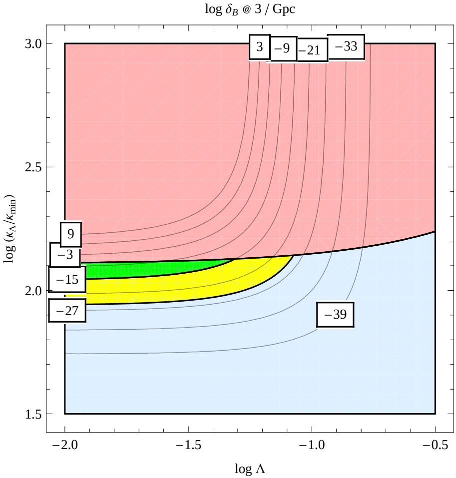
$$\Gamma^2 = -\frac{|\beta| V_0}{\alpha^2 M_{\rm Pl}^2 k^2} - 1 < 0$$

There is no instability.

Allowed parameter space

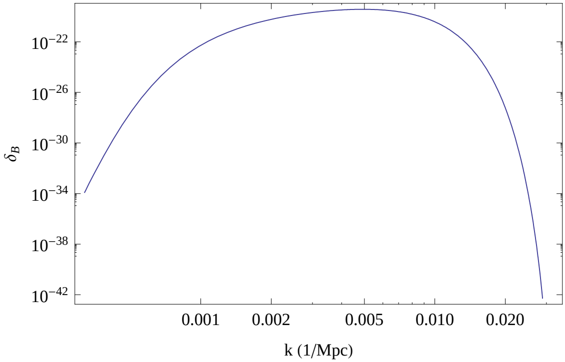
green: magnetic fields observed in intergalactic medium;

yellow: for successful seed to be fed to the magnetohydrodynamic plasma.



Power spectrum

δ_B^0 (Gauss), $\Lambda = 10^{-2}$, $k_\Lambda / k_{\min} = 10^2$.



Only a few orders of magnitude in k are efficiently magnified.

Part V:

Screening Mechanisms

Screening with vector fields

[Beltrán-Jiménez, Fróes, Mota (2012)]

Massive vector field with a gauge fixing term

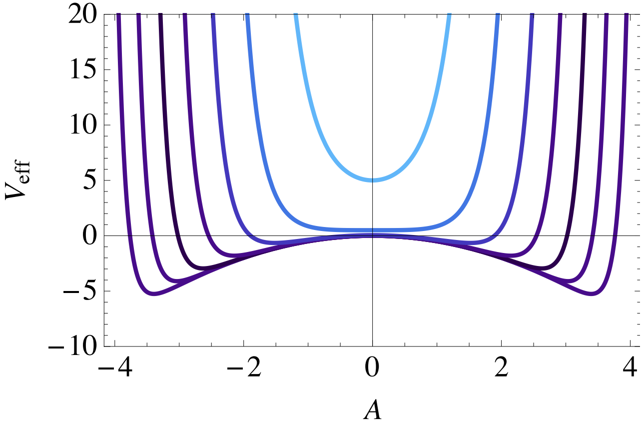
$$S = \int d^4x\sqrt{-g}\left[-\frac{R}{2\kappa^2} - \frac{1}{4}F^2 - \frac{1}{2}(\nabla_\mu B^\mu)^2 - \frac{M^2}{2}B^2\right] + \\ + \int d^4x \mathcal{L}_m[\tilde{g}_{\mu\nu}, \psi]$$

Matter fields couple to gravity via $\tilde{g}_{\mu\nu} = \Omega^2(B^2)g_{\mu\nu}$

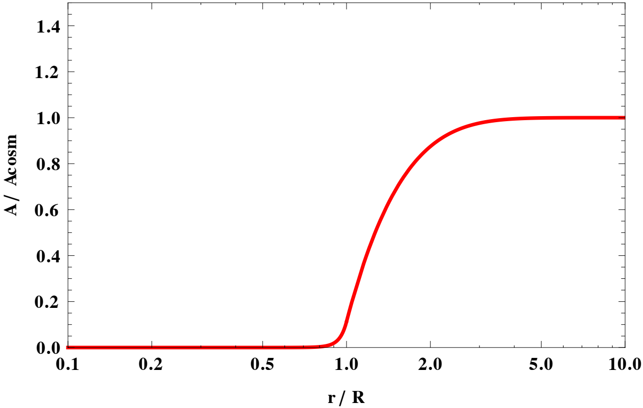
Equations of motion give

$$\Box B_\mu = \left(M^2 + \frac{2\beta\rho}{M_p^2}e^{\beta B^2/M_p^2}\right)B_\mu$$

Screening with vector fields



Screening with vector fields



Screening with three-form fields

[Barreiro, Bertello, NN (in progress)]

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{48}F^2 - V(A^2)\right] + \int d^4x\tilde{\mathcal{L}}_m$$

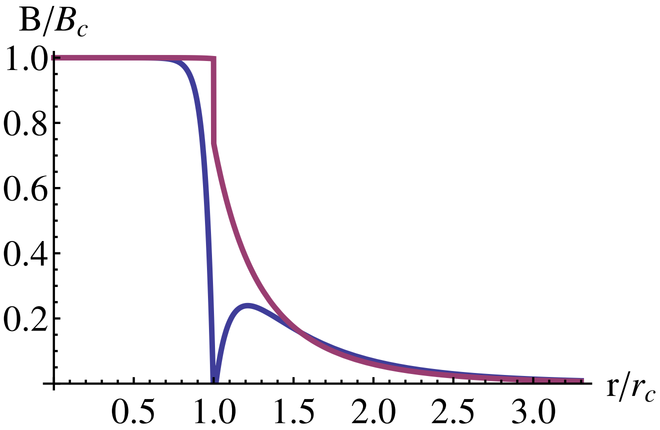
Conformal coupling $\tilde{g}^{\mu\nu} = \Omega^2(A^2)g^{\mu\nu}$

Going to the vector field in the gauge fixing description

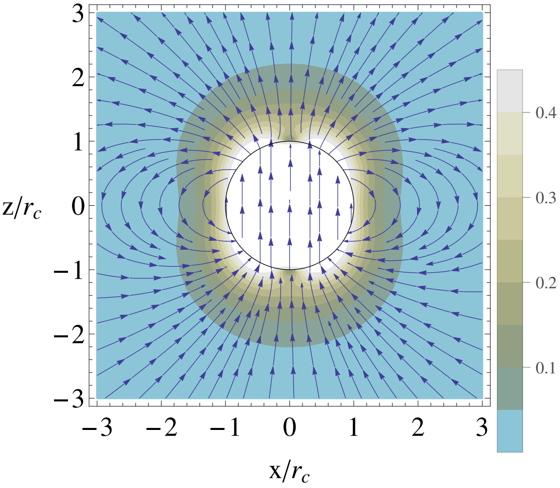
$$\nabla_\alpha(\nabla^\mu B_\mu) = -2\left(\frac{\partial V}{\partial B^2} + \rho\frac{\partial \Omega}{\partial B^2}\right)B_\alpha$$

(cf. for vectors we had, $\Box B_\mu = \ldots$)

Screening with three-form fields



Screening with three-form fields



Summary

- Three-forms possess accelerating attractors and saddle points which can describe three-form driven inflation or dark energy;
- Scalar spectral index predicted to be $n_s \approx 0.97$ for $N = 60$.
- Some models are within non-Gaussianity, and ratio of tensor to scalar perturbations bounds;
- Efficient reheating/preheating;
- Efficient generation of magnetic fields;
- In the presence of a coupling to dark matter, growth of structure is enhanced for small scales.
- New fifth-force screening solution.