The emergence of late time cosmological acceleration and other problems in f(R) gravity

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ACGC

Cosmic Concordance or the Boring Universe

Negative → acceleration

Dominates today!!

$$q = \frac{1}{2}\Omega_M - \Omega_\Lambda$$

Deceleration parameter: measures change in expansion rate

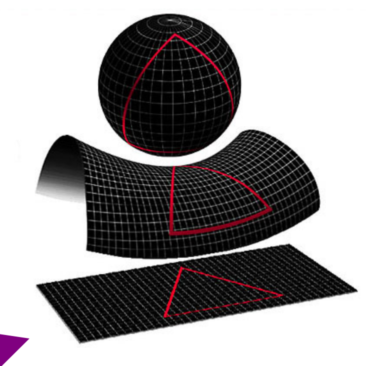
$$\Omega_M + \Omega_\Lambda - \Omega_K = 1$$

Matter

Cosmological Constant

Curvature

$\Omega_K = 0$



Because we do not know what DE is, it represents a large measure of ignorance in the standard model.

The simplest alternatives...

The Concordance model provides us with a great phenomenological description, but.........

.....geometry is RW, there is NO dark energy, gravity is modified and the universe is accelerating.

The simplest alternatives...

The Concordance model provides us with a great phenomenological description, but.........

.....geometry is NOT RW, there is NO dark energy, gravity is not modified and the universe is not accelerating.

A cautionary tale.....of Lambda

In a FRW model the complete dynamics of the universe is determined by a single function of time, the scale factor. Hence the key observables are functionals of the Hubble parameter H(z).

Critically: in a LCDM model, on small scales, the late-time growth of perturbations is also a function of H(z).

$$\Delta'' + \left(\frac{d\ln H}{da} + \frac{3}{a}\right) = \frac{3}{2}\frac{\Omega_{m0}}{a^5}\Delta$$

$$\Delta_+ = \frac{5}{2}\frac{H(z)}{H_0}\Omega_{m0}\int_z^\infty \frac{(1+z')dz'}{[H(z')/H_0]^3}$$

$$\frac{d\ln\Delta_+}{d\ln a} = \Omega_m(a)^\gamma$$

$$\gamma = 0.55 \ \text{for LCDM}$$

Rigidities between different sets of independent observables that can be used to test the underlying hypothesis of the model.

A cautionary tale.....of Lambda

However.......

It is possible to construct perturbed LTB models with the same background light-cone structure as a LCDM model that give a γ significantly different

...so it is important to make sure that the FRW geometry holds when applying tests of GR based on observations of LSS.

- PD, Goheer, Osano and Uzan (JCAP 2010)

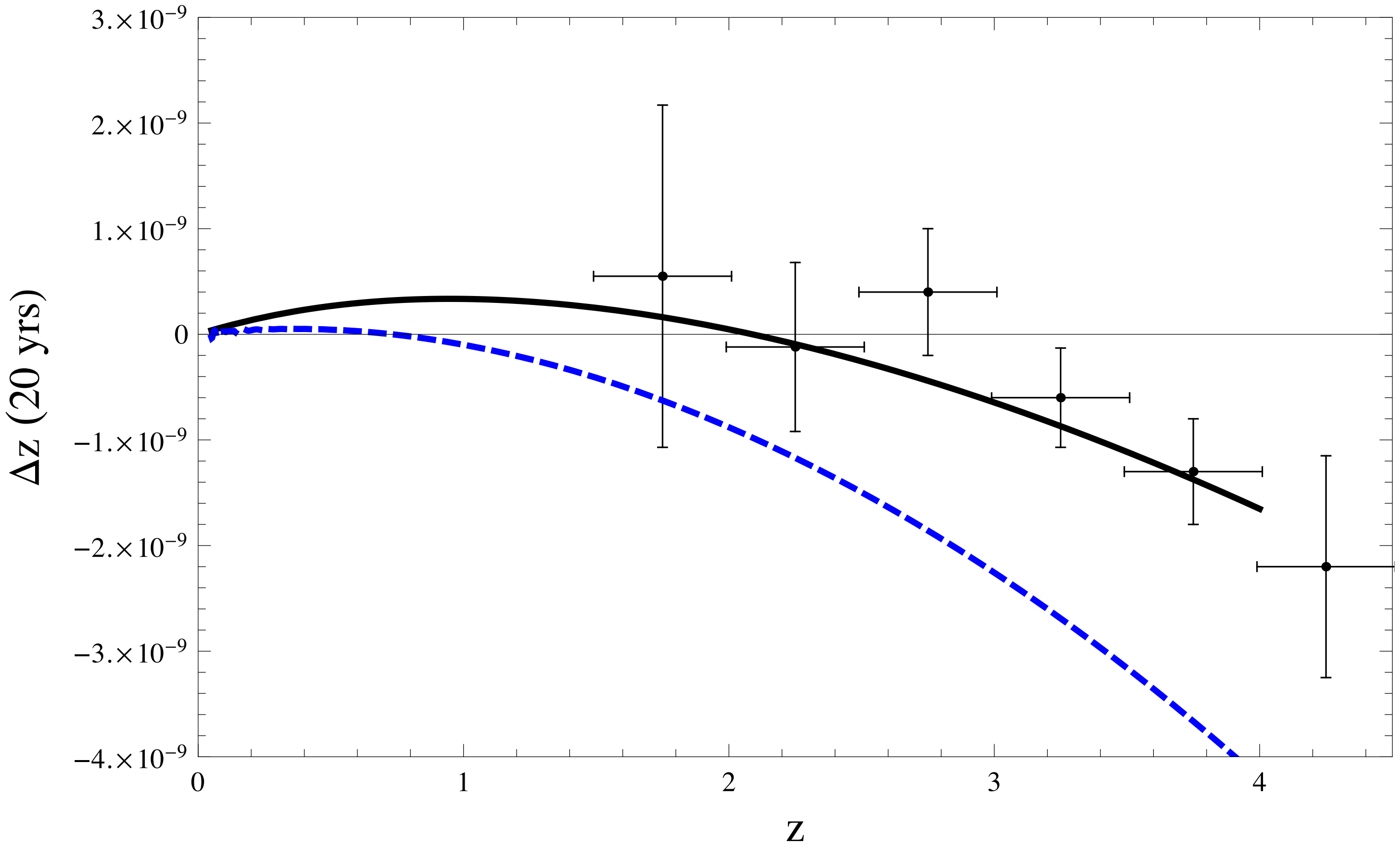
A couple of key observations

Redshift drift and other tests of the Copernican principle.

$$\dot{z} = (1+z)H_0 - H_\perp(z),$$

Determining the DE equation of state.

$$w = w(z)$$



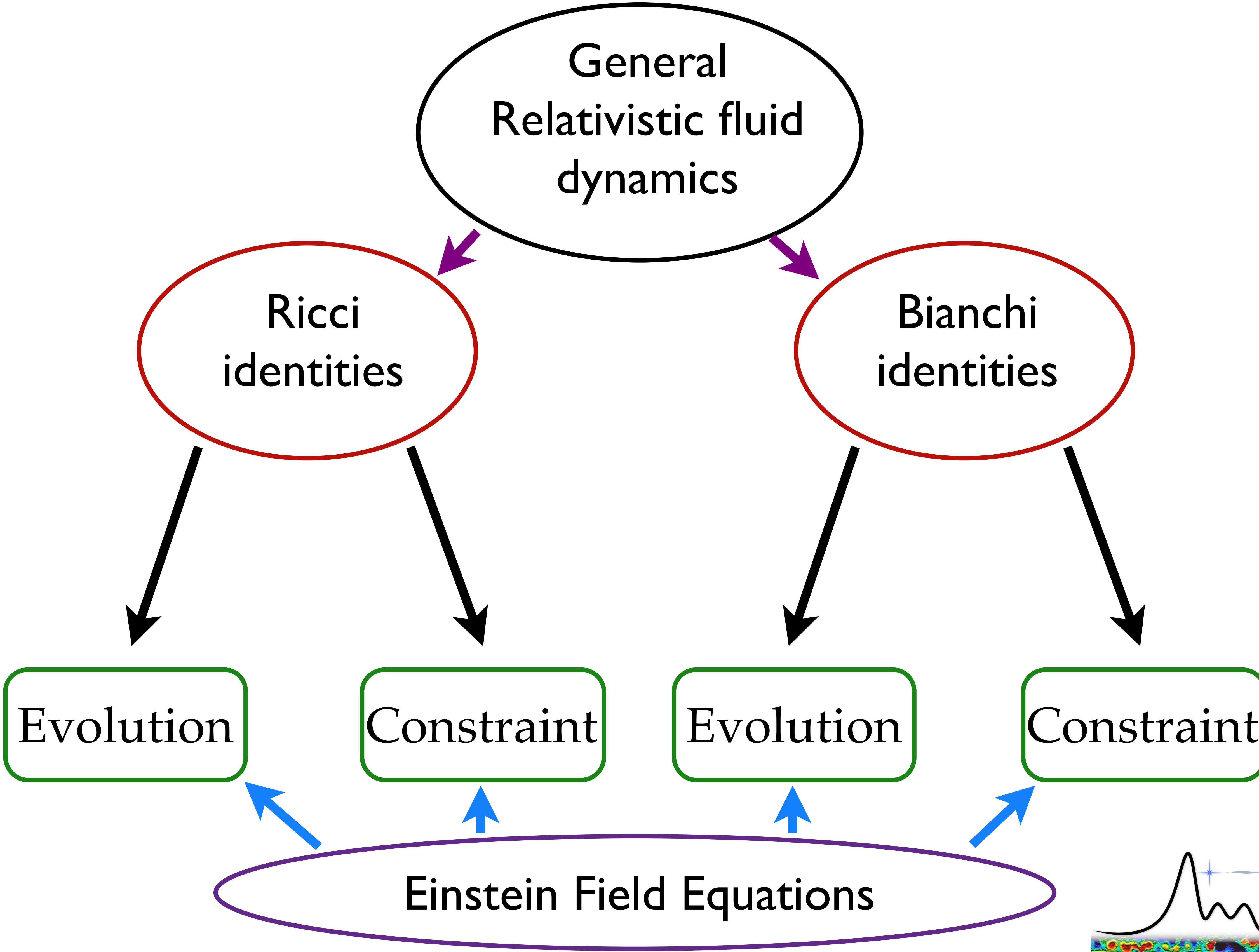
- PD, Goheer, Osano and Uzan (JCAP 2010)

Maybe the universe is not so boring after all.......

Outline of talk

- Top-down approaches to f(R) cosmological modeling
- Reconstruction methods.
- Dynamical systems approach.
- Structure formation in f(R) gravity.
- Tensor anisotropies.
- Some problems with viable f(R) theories.
- Bottom-up approaches.
- Building "Swiss Cheese" models by embedding spherically symmetric solutions in an expanding FLRW background.
- Patchwork universes and the emergence of cosmological acceleration
- A possible cure for sudden curvature singularities if viable f(R) models.

Our relativistic toolbox



f(R) theories of gravity

The class of models we will consider can be derived from the classical action:

$$\mathcal{A} = \int d^4x\sqrt{-g}\left[f(R) + \mathcal{L}_m\right],$$

Varying the action with respect to the metric gives the following field equations:

$$f' G_{ab} = f'\left(R_{ab} - \frac{1}{2}g_{ab}R\right) = T^m_{ab} + \frac{1}{2}g_{ab}\left(R - Rf'\right) + \nabla_b\nabla_a f' - g_{ab}\nabla_c\nabla^c f'\ ,$$

$$G_{ab} = \tilde{T}^m_{ab} + T^R_{ab} = T^{tot}_{ab}\ ,$$

This last step is extremely important as it allows us to treat 4th order gravity as standard GR in the presence of two effective fluids.

The energy-momentum tensor of the curvature "fluid" can be decomposed as follows:

$$\mu^R = \frac{1}{f'}\left[\frac{1}{2}(Rf'-f) - \Theta f''\dot{R} + f''\tilde{\nabla}^2 R + f''\,\dot{u}_b\tilde{\nabla}R\right] ,$$

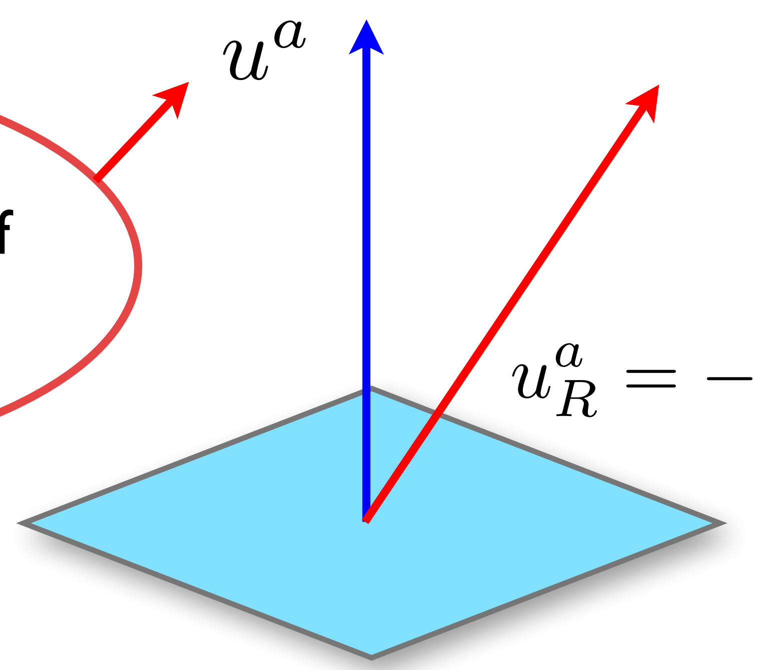
$$p^R = \frac{1}{f'}\left[\frac{1}{2}(f-Rf') + f''\ddot{R} + 3f'''\dot{R}^2 + \frac{2}{3}\Theta f''\dot{R} - \frac{2}{3}f''\tilde{\nabla}^2 R + \right.$$
$$\left. -\frac{2}{3}f'''\tilde{\nabla}^a R\tilde{\nabla}_a R - \frac{1}{3}f''\,\dot{u}_b\tilde{\nabla}R\right] ,$$

Note no background contribution.

$$q^R_a = -\frac{1}{f'}\left[f'''\dot{R}\tilde{\nabla}_a R + f''\tilde{\nabla}_a\dot{R} - \frac{1}{3}f''\tilde{\nabla}_a R\right] ,$$

$$\pi^R_{ab} = \frac{1}{f'}\left[f''\tilde{\nabla}_{\langle a}\tilde{\nabla}_{b\rangle}R + f'''\tilde{\nabla}_{\langle a}R\tilde{\nabla}_{b\rangle}R + \sigma_{ab}\dot{R}\right] .$$

Taken to be the motion of STANDARD matter



$$u^a_R = -\frac{\nabla_a R}{\dot{R}}$$

So one can think of this as a curvature "fluid" moving relative to u^a

Top down approach to f(R) cosmological modeling

This is the standard approach......

- Assume that the universe on large scale is described by a RW geometry.
- Describe large scale structure by perturbing away from this background model to the required order in perturbation theory assuming that perturbation theory converges.

Various methods can be used to construct f(R) background models.

Reconstruction methods

Field equations can be inverted to provide a way of reconstructing the theory from the expansion history. For example fixing a(R) for an exact LCDM expansion history:

$$
\begin{aligned}
& -3(R-3\Lambda)(R-4\Lambda)f''(R) \\
& +\left(\frac{R}{2}-3\Lambda\right)f'(R)+\frac{1}{2}f(R)-\rho(R) \;=\; 0\,.
\end{aligned}
$$

$$
\rho(a)=\frac{\rho_0}{a^3}\;\Rightarrow\;\rho(R)=R-4\Lambda\,.
$$

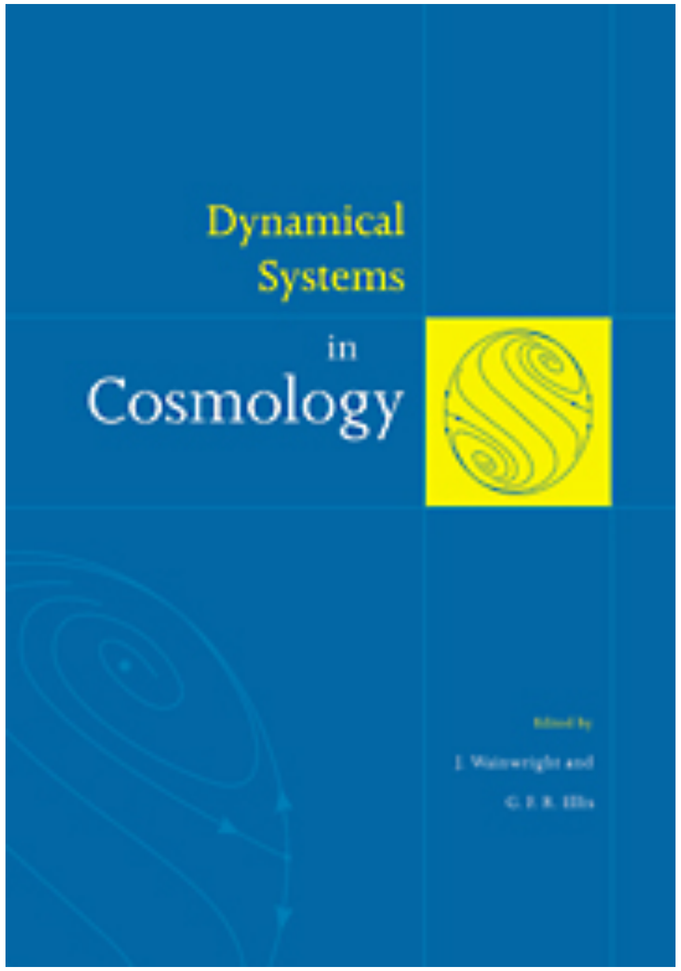
Other reconstructions possible based on t(R), q(R)........

The only real-valued solution of this equation is the Lagrangian of General Relativity with a positive cosmological constant.

- PD, Elizalde, Goswami, Odintsov, Saez-Gomez (PRD 2010)
- Carloni, Goswami, PD (CQG 2012).
- Bouhmadi-Lopez et al (arXiv: 1302.2038)

Dynamical systems approach

A very powerful way to study the complete cosmological dynamics for a given f(R) theory is to use the theory of dynamical systems. This provides an excellent way of generating cosmologically relevant exact solutions and how they relate to each other in phase space.



The approach we take is largely based on a paper by Goliath and Ellis (PRD, 1999) and the book edited by Wainwright and Ellis.

- Carloni, PD, Capozziello, Troisi (CQG, 2005)
- Amendola et. al. (PRD, 2007)
- Carloni, PD, Troisi (GRG 2009)
- Abdelwahab, Goswami, PD (PRD 2012)

A compact dynamical system

Friedmann constraint

$$\left(\Theta + \frac{3}{2}\frac{\dot{f}'}{f'}\right)^2 + \frac{3}{2}\frac{f}{f'} = \frac{3\rho}{f'} + \frac{3}{2}R + \left(\frac{3}{2}\frac{\dot{f}}{f'}\right)^2$$

$$D^2 = \left(\Theta + \frac{3}{2}\frac{\dot{f}'}{f'}\right)^2 + \frac{3}{2}\frac{f}{f'} \qquad \frac{d}{d\tau} \equiv \frac{1}{D}\frac{d}{dt}$$

$$x = \frac{3}{2}\frac{\dot{f}'}{f'}\frac{1}{D}, \quad z = \frac{3}{2}\frac{R}{D^2}, \quad y = \frac{3}{2}\frac{f}{f'}\frac{1}{D^2}, \quad \Omega_m = \frac{3\rho_m}{f'}\frac{1}{D^2}, \quad Q = \frac{\Theta}{D}$$

Constraints

$$1 = \Omega_m + x^2 + z \quad -1 \leq x \leq 1, \quad 0 \leq \Omega_m \leq 1, \quad -2 \leq Q \leq 2,$$

$$1 = (Q + x)^2 + y \quad 0 \leq z \leq 1, \quad 0 \leq y \leq 1$$

- Carloni, PD, Capozziello, Troisi (CQG, 2005)
- Carloni, PD, Troisi (GRG 2009)
- Abdelwahab, Goswami, PD (PRD 2012)

A compact dynamical system

$$\frac{dz}{d\tau} = -\frac{1}{3}z\left[(Q+x)\left(2\,z+4\,xQ-\left(1-z-x^2\right)(1+3\,w)\right)\right.$$
$$\left.-2\,Q-4\,x+2\,x\Gamma\;(z-1)\right]$$

$$\frac{dx}{d\tau} = \frac{1}{6}\left[-2\,x^2 z\Gamma+\left(1-z-x^2\right)(1-3\,w)\right.$$
$$+2z+4\,\left(x^2-1\right)\left(1-Q^2-xQ\right)$$
$$\left.+x\,(Q+x)\left(\left(1-z-x^2\right)(1+3\,w)-2\,z\right)\right]$$

$$\frac{dQ}{d\tau} = \frac{1}{6}\left[-4\,xQ^3+(5+3\,w)\,Qx\,(1-xQ)\right.$$
$$-Q^2\,(1-3\,w)-Qx^3\,(1+3\,w)$$
$$\left.-3\,zQ\,(1+w)\,(Q+x)+2\,z\,(1-\Gamma\,Qx)\right]$$

$$r = \frac{Rf'}{f} = \frac{y}{z}$$

$$\Gamma = \frac{f'}{Rf''}$$

Closure condition

Note that incorrect and misleading results are obtained if one takes a one parameter $m(r) \equiv \Gamma^{-1}$ approach to the analysis of this dynamical system.

A simple example

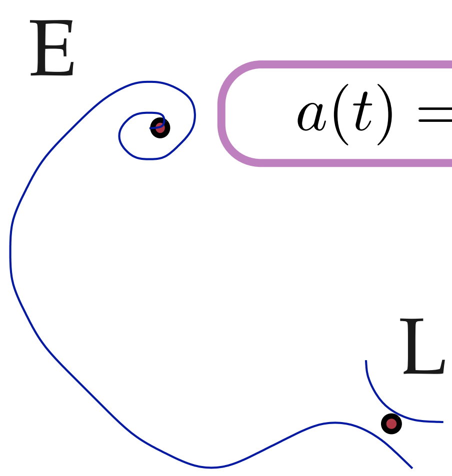
$$f(R) = R + \alpha R^n$$

Closure condition → $$\Gamma = \frac{z}{n(y-z)}$$

Fixed points	Coordinates (x, Ω, z, Q)	Solution $a(t)$
$A_\pm$	$(1, 0, 0, \pm 2)$	$a_0\sqrt{t-t_0}$
B	$(\pm 1, 0, 0, 0)$	a_0
C	$(-\frac{\sqrt{3+12\omega+9\omega^2}}{1+3\omega}, -\frac{2}{1+3\omega}, 0, 0)$	a_0
$D_\pm$	$(\frac{1-3\omega}{3(\omega-1)}, -\frac{4(3\omega-2)}{9(\omega-1)^2}, 0, \pm\frac{2}{3(\omega-1)})$	$a_0\sqrt{t-t_0}$
$E_\pm$	$(0, 0, 1, \pm\frac{1}{\sqrt{2}})$	$a_0 e^{Ct}$
$F_\pm$	$(f_1(n,\omega), g_1(n,\omega), l_1(n,\omega), n_1(n,\omega))$	$a_0\sqrt{t-t_0}$
$G_\pm$	$(f_2(n,\omega), g_2(n,\omega), l_2(n,\omega), n_2(n,\omega))$	$a_0(t-t_0)^{s(n,\omega)}$
$I_\pm$	$(f_3(n), g_3(n), l_3(n), n_3(n))$	$a_0((n-2)t-t_0)^{\frac{-1+3n-2n^2}{-2+n}}$
$L_\pm$	$(f_4(n,\omega), g_4(n,\omega), l_4(n,\omega), n_4(n,\omega))$	$a_0(3t(1+\omega)-t_0)^{\frac{2n}{3(1+\omega)}}$
$N_\pm$	$(0, \frac{2}{3}, \frac{1}{3}, \pm\frac{\sqrt{6}}{3})$	$a_0(2t-t_0)^{2/3}$

Power-law inflation

Same point present in R^n gravity



$$a(t) = a_0 e^{Ct}$$

de Sitter attractor

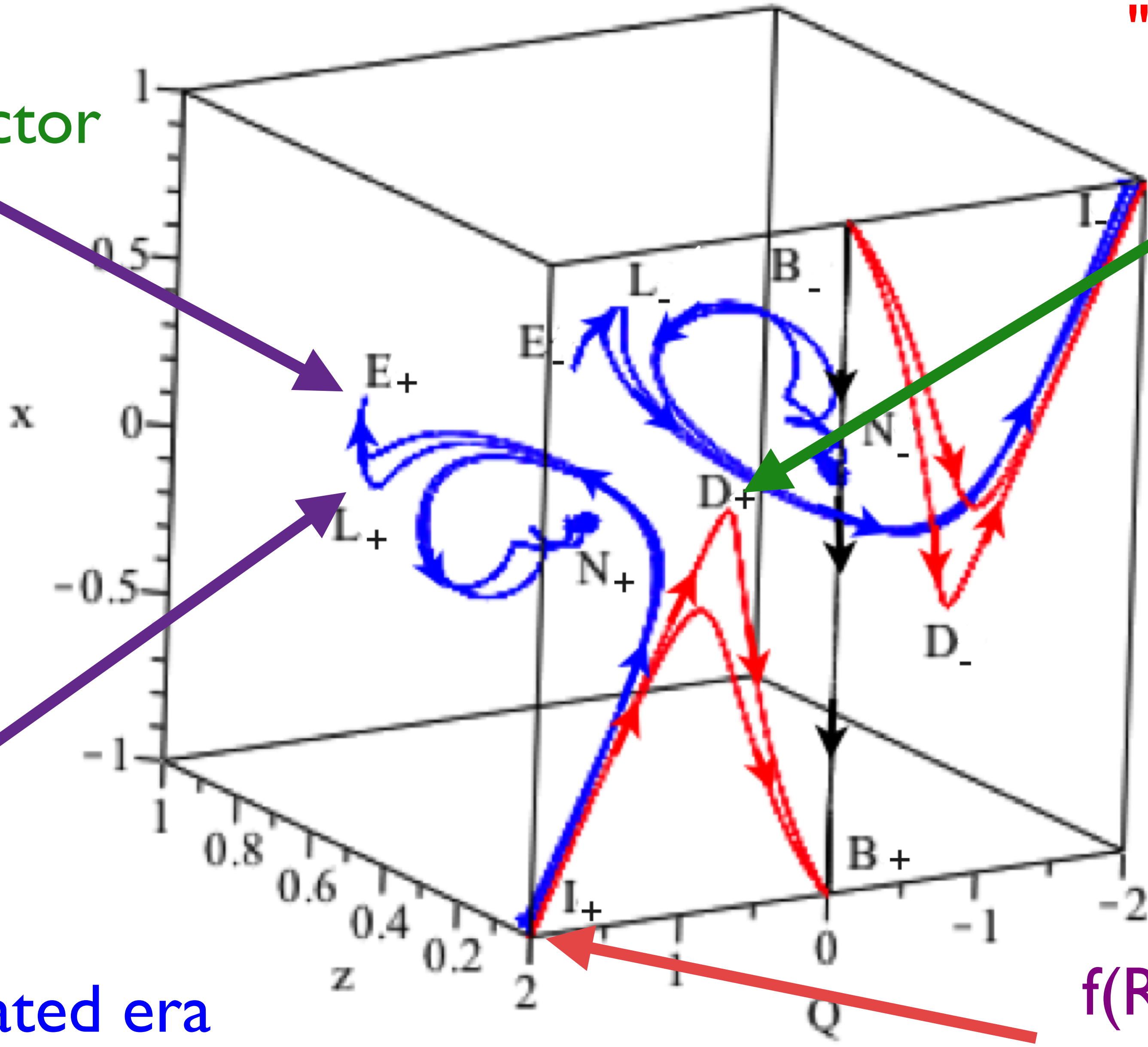
$$a(t) = a_0 t^{\frac{2n}{3(1+w)}}$$

Matter dominated Friedmann saddle point (structure growth)

Interesting cosmic histories

"Dark Radiation" phase

de Sitter attractor



$Q > 0$

Expanding

$Q < 0$

Contracting

Matter dominated era

f(R) driven power-law inflation

- Abdelwahab, Goswami, PD (PRD 2012)

Growth of large scale structure

Scalar perturbations governed by the 4th order system:

$$\dot{\Delta}_m = w\Theta\Delta_m - (1+w)Z\,,$$

$$\dot{Z} = \left(\frac{\dot{R}f''}{f'} - \frac{2\Theta}{3}\right)Z + \left[\frac{3(w-1)(3w+2)}{6(w+1)}\frac{\mu}{f'} + \frac{2w\Theta^2 + 3w(\mu^R + 3p^R)}{6(w+1)}\right]\Delta_m + \frac{\Theta f''}{f'}\Re$$
$$+ \left[\frac{1}{2} - \frac{1}{2}\frac{f}{f'}\frac{f''}{f'} - \frac{f''}{f'}\frac{\mu}{f'} + \dot{R}\Theta\left(\frac{f''}{f'}\right)^2 + \dot{R}\Theta\frac{f^{(3)}}{f'}\right]\mathcal{R} - \frac{w}{w+1}\tilde{\nabla}^2\Delta_m - \frac{f''}{f'}\tilde{\nabla}^2\mathcal{R}\,,$$

$$\dot{\mathcal{R}} = \Re - \frac{w}{w+1}\dot{R}\,\Delta_m\,,$$

$$\dot{\Re} = -\left(\Theta + 2\dot{R}\frac{f^{(3)}}{f''}\right)\Re - \dot{R}Z - \left[\frac{(3w-1)}{3}\frac{\mu}{f''} + 3\frac{w}{w+1}(p^R + \mu^R)\frac{f'}{f''} + \frac{w}{3(w+1)}\dot{R}\left(\Theta - 3\dot{R}\frac{f^{(3)}}{f''}\right)\right]\Delta_m$$
$$+ \left[2\frac{K}{S^2} - \left(\frac{1}{3}\frac{f'}{f''} + \frac{f^{(4)}}{f'}\dot{R}^2 + \Theta\frac{f^{(3)}}{f'}\dot{R} - \frac{2}{9}\Theta^2 + \frac{1}{3}(\mu^R + 3p^R) + \ddot{R}\frac{f^{(3)}}{f''} - \frac{1}{6}\frac{f}{f'} + \frac{1}{2}(w+1)\frac{\mu}{f'} - \frac{1}{3}\dot{R}\Theta\frac{f''}{f'}\right)\right]\mathcal{R} + \tilde{\nabla}^2\mathcal{R}\,,$$

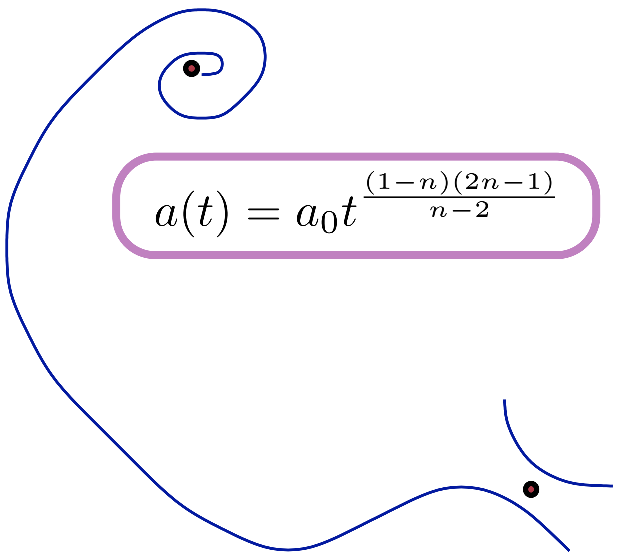
$$\frac{C}{S^2} + \left(\frac{4}{3}\Theta + \frac{2\dot{R}f''}{f'}\right)Z - 2\frac{\mu}{f'}\Delta_m + \left[2\dot{R}\Theta\frac{f^{(3)}}{f'} - \frac{f''}{f'}\left(f - 2\mu + 2\dot{R}\Theta f''\right)\right]\mathcal{R} + \frac{2\Theta f''}{f'}\Re - \frac{2f''}{f'}\tilde{\nabla}^2\mathcal{R} = 0\,.$$

- Carloni, PD, Troisi (PRD, 2008)
- Ananda, Carloni, PD (CQG, 2009)

General evolution of the background

A single fluid description is not enough to make a meaningful comparison to the standard LCDM model [in this case $f(R) = \alpha R^n$]

Expansion history obtained by integrating dynamical systems equations along the best fit orbit

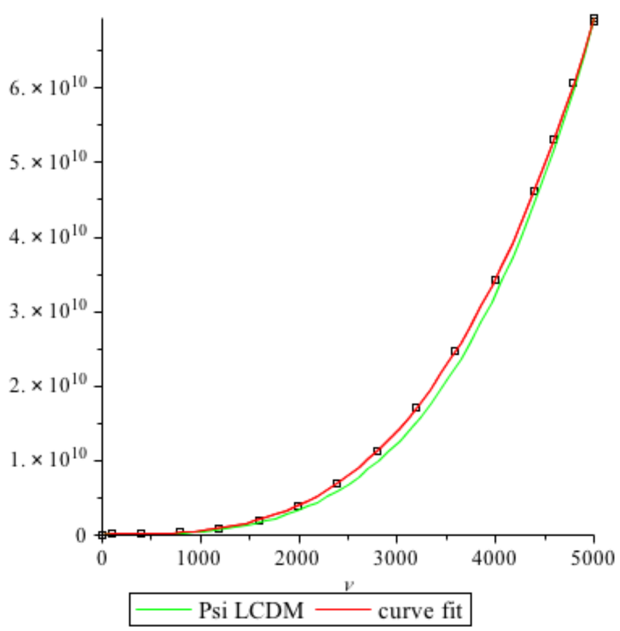


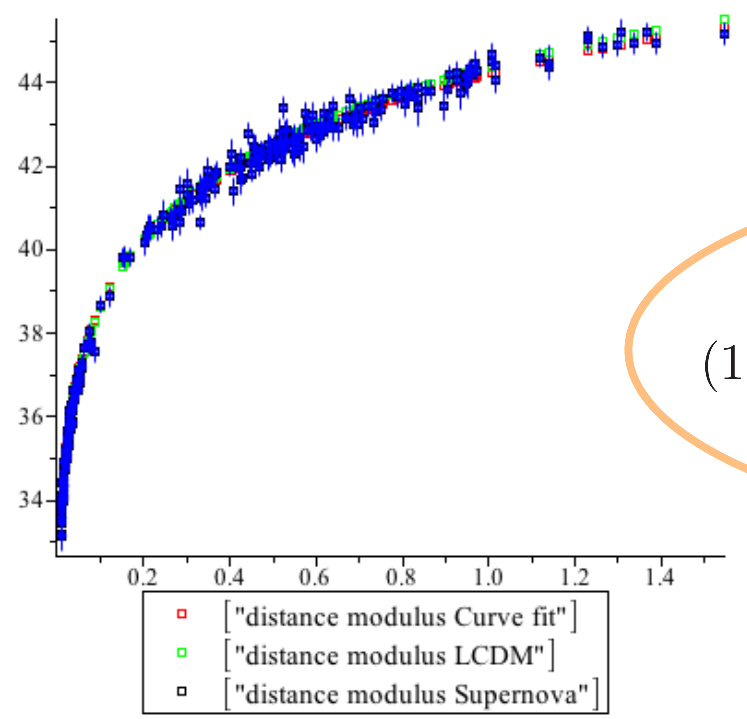
$$a(t) = a_0 t^{\frac{(1-n)(2n-1)}{n-2}}$$

$$a(t) = a_0 t^{\frac{2n}{3(1+w)}}$$

$$
\begin{aligned}
-(z+1)\frac{\mathrm{d}x}{\mathrm{d}z} &= -x - x^2 + \frac{(4-2n+nx)y}{n-1} + \Omega_d\,, \\
-(z+1)\frac{\mathrm{d}y}{\mathrm{d}z} &= 4y + \frac{(x+2ny)y}{n-1}\,, \\
-(z+1)\frac{\mathrm{d}\Omega_d}{\mathrm{d}z} &= \left(1 - x + \frac{2ny}{n-1}\right)\Omega_d\,.
\end{aligned}
$$

$$(1+z)\frac{\mathrm{d}h}{\mathrm{d}z} = \frac{h(2+ny)}{n-1}\,, \qquad q = \frac{ny}{(n-1)} + 1\,.$$





- Abebe, Abdelwahab, de la Cruz-Dombriz, PD (CQG, 2012)

The matter power spectrum

Obtained by solving the exact linear structure growth equations along the best fit orbit for a given f(R) [in this case $f(R) = \alpha R^n$] to get T(k)

$$\frac{(-1+n)^2(1+z)}{2ny}\hat{\mathcal{R}}^{k\prime} - \frac{3h^2\left[(-1+n)(1+(-2+n)\Omega_d)+(-2+n)y\right]+\hat{k}^2(-1+n)^2(1+z)^2}{6h^2ny}\hat{\mathcal{R}}^{k}$$

$$+h^2(1+z)^2\Delta_m^{k\prime\prime}+h^2\frac{\left[(-1+n)\Omega_d+y\right](1+z)}{n-1}\Delta_m^{k\prime}-3h^2\Omega_d\Delta_m^k=0,$$

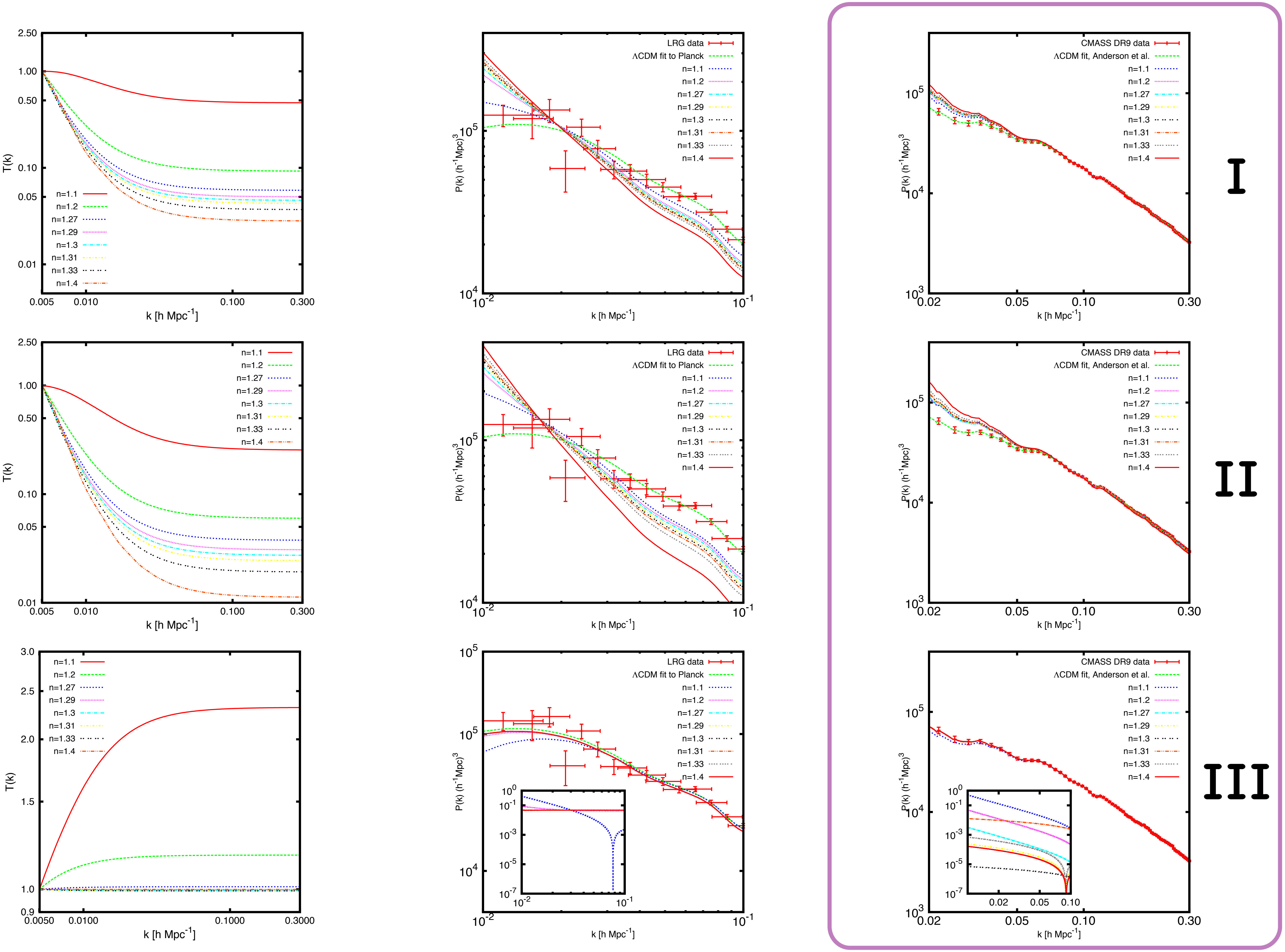
$$\hat{\mathcal{R}}^{k\prime\prime}-\frac{\left[4-4\Omega_d+4y+n\left(-2+2\Omega_d-3y\right)\right]}{(-1+n)(1+z)}\hat{\mathcal{R}}^{k\prime}+\left\{\frac{\hat{k}^2}{h^2}+\frac{(-2+n)\left[-\Omega_d^2-(-1+y)^2+\Omega_d(1+n+2y)\right]}{(-1+n)^2(1+z)^2}\right\}\hat{\mathcal{R}}^{k}$$

$$+\frac{6h^2ny\left(1-\Omega_d+y\right)}{(-1+n)^2(1+z)}\Delta_m^{k\prime}+\frac{6h^2n\Omega_d y}{(-1+n)^2(1+z)^2}\Delta_m^k=0\,,$$

Initial conditions

- **I**: $\Delta_m^k|_0=\hat{\mathcal{R}}^k|_0=10^{-5},\ \Delta_m^{k\prime}|_0=\hat{\mathcal{R}}^{k\prime}|_0=10^{-5},$
- **II**: $\Delta_m^k|_0=\hat{\mathcal{R}}^k|_0=10^{-5},\ \Delta_m^{k\prime}|_0=\hat{\mathcal{R}}^{k\prime}|_0=10^{-8},$
- **III**: $\Delta_m^k|_0=\hat{\mathcal{R}}^k|_0=10^{-5},\ \Delta_m^{k\prime}|_0=\hat{\mathcal{R}}^{k\prime}|_0=0,$

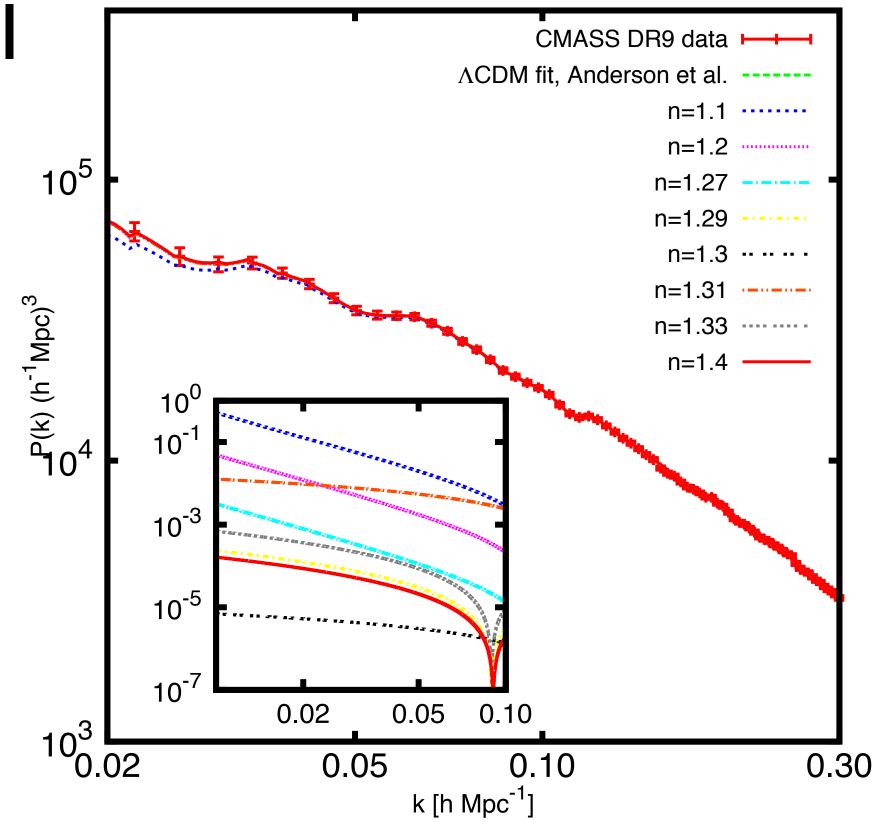
$$P_k^{f(R)} = T(k)P_k^{\Lambda\mathrm{CDM}}|_{eq}$$


T(k)
k [h Mpc^{-1}]
n=1.1
n=1.2
n=1.27
n=1.29
n=1.3
n=1.31
n=1.33
n=1.4
P(k) (h^{-1}Mpc)^3
LRG data
ΛCDM fit to Planck
CMASS DR9 data
ΛCDM fit, Anderson et al.
I
II
III

DR9 CMASS galaxy sample observed by SDSS-III

n	1.1	1.2	1.27	1.29	1.3	1.31	1.33	1.4
h_0	0.65	0.75	0.94	0.99	1.44	2.43	7.34	159.67
q_0	0.39	0.20	0.10	0.25	0.36	0.35	0.22	-0.17

Table I: Present-day values of the Hubble $h_0 \equiv H(\text{today})/H_0$ and deceleration (q_0) parameters for the R^n models under consideration. H_0 corresponds to the ΛCDM Hubble parameter value today. Only $n = 1.4$ provides acceleration at the present time, whereas $n = 1.29$ gives the closest value for h_0 to ΛCDM.



n exponent	1.1	1.2	1.27	1.29	1.3	1.31	1.33	1.4
χ^2	4.5463	1.0507	1.0366	1.0357	1.0355	1.0458	1.0360	1.0357
σ exclusion	1.874	0.123	0.0316	0.012	0.002	0.101	0.020	0.001
% suppression	13	1.5	0.1	0.01	0.001	1	0.04	0.009

Table III: Fits to the SDSS CMASS DR9 data for R^n cosmology by using set of initial conditions **III**: eight different values of exponent n were investigated from $n = 1.1$ to 1.4. Values for χ^2 and the confidence region σ are presented in the second and third rows respectively. The data to be fitted by the theoretical spectra are taken from [38]. The fit provided by ΛCDM ($\chi^2 = 61.1/59 \approx 1.03559$) is slightly improved by the $n = 1.3$ parameter value. The final row gives the suppression in the overall initial amplitude required to get the best fits. For all the values, this suppression turns out to be smaller than 15% and is therefore in the experimental uncertainty interval for this quantity. For the best fit $n = 1.3$ the corresponding suppression is 10^{-3}% and very good fits are also obtained for $n = 1.27$, 1.29, 1.33 and 1.4 with similar suppressions.

- Abebe, de la Cruz-Dombriz, PD (PRD, 2014)

CMB Tensor anisotropies

The k-modes for tensor perturbations for a general f(R) theory satisfy:

$$\ddot{\sigma}_k + A\dot{\sigma}_k + B\sigma_k = \frac{a}{k}\frac{\mu}{f'}\left[\dot{\pi}_k^{\gamma} - \left(H + 3\,H\omega + \frac{f''}{f'}\right)\pi_k^{\gamma}\right]\,,$$

$$A \equiv 3H + \frac{f''}{f'}\dot{R}, \qquad B \equiv \frac{k^2}{a^2} + 2\frac{\ddot{a}}{a} + \dot{R}^2\left[\frac{f'''}{f'} - \left(\frac{f''}{f'}\right)^2\right] + \frac{f''}{f'}\ddot{R} + H\frac{f''}{f'}\dot{R}\,.$$

$$\frac{\mathrm{d}^2 u_k}{\mathrm{d}\tau^2} + \left(-\frac{1}{2}m\mathcal{H}\frac{f''}{f'} - \frac{m}{a}\frac{\mathrm{d}^2 a}{\mathrm{d}\tau^2} + a^2 B\right)u_k = 0\,.$$

$$u_k = a^m \sigma_k$$

} Initial conditions for CAMB

Obtained deep in the radiation dominated era.

Background evolution

To illustrate how this works, again take $f(R) = R^n$

$$a\frac{\mathrm{d}x}{\mathrm{d}a} = -x - x^2 + \frac{(4-2n+nx)y}{n-1} + \Omega_d\,,$$

$$a\frac{\mathrm{d}y}{\mathrm{d}a} = \left[4 + \frac{(x+2ny)}{n-1}\right] y\,,$$

$$a\frac{\mathrm{d}\Omega_d}{\mathrm{d}a} = \left[1 - x + \frac{2ny}{n-1}\right]\Omega_d\,,$$

$$a\frac{\mathrm{d}\Omega_r}{\mathrm{d}a} = \left[-x + \frac{2ny}{n-1}\right]\Omega_r\,,$$

$$a\frac{\mathrm{d}H}{\mathrm{d}a} = -H\left(2 + \frac{ny}{n-1}\right)\,.$$

Point	Coordinates $[x,\, y,\, \Omega^d,\, \Omega^r]$
A	$[-1, 0, 0, 0]$
B	$\left[\frac{4-2n}{1-2n}, \frac{5-4n}{2n-1}, 0, 0\right]$
C	$[-2, 0, 0, -2]$
D	$[0, 0, 0, 0]$
E	$[2-2n, -2(-1+n)^2, 0, 0]$
F	$[0, 0, 0, 1]$
G	$\left[-3+\frac{3}{n}, \frac{-3+(7-4n)n}{2n^2}, \frac{-3+(13-8n)n}{2n^2}, 0\right]$
H	$[-1, 0, -1, 0]$
I	$[1, 0, 2, 0]$
J	$\left[-4+\frac{4}{n}, -\frac{2(n-1)^2}{n^2}, 0, -5+\frac{8n-2}{n^2}\right]$

$$\omega_E \equiv \frac{p_R}{\mu_R} - \frac{\dot{R} f'' \mu_m}{\mu_R f'^2}$$

Initial conditions close to J

Deep in the radiation dominated epoch we can assume that the expansion history is well described by the solution described by the equilibrium point J.....

In this case we have for $n \neq 2$ .....

$$\frac{\mathrm{d}^2 u_k}{\mathrm{d}\tau^2} + \left(k^2 - 2\,\tau^{-2}\right) u_k = 0\,,$$

.....so initial conditions the same as GR!!

Power spectra: the various combinations

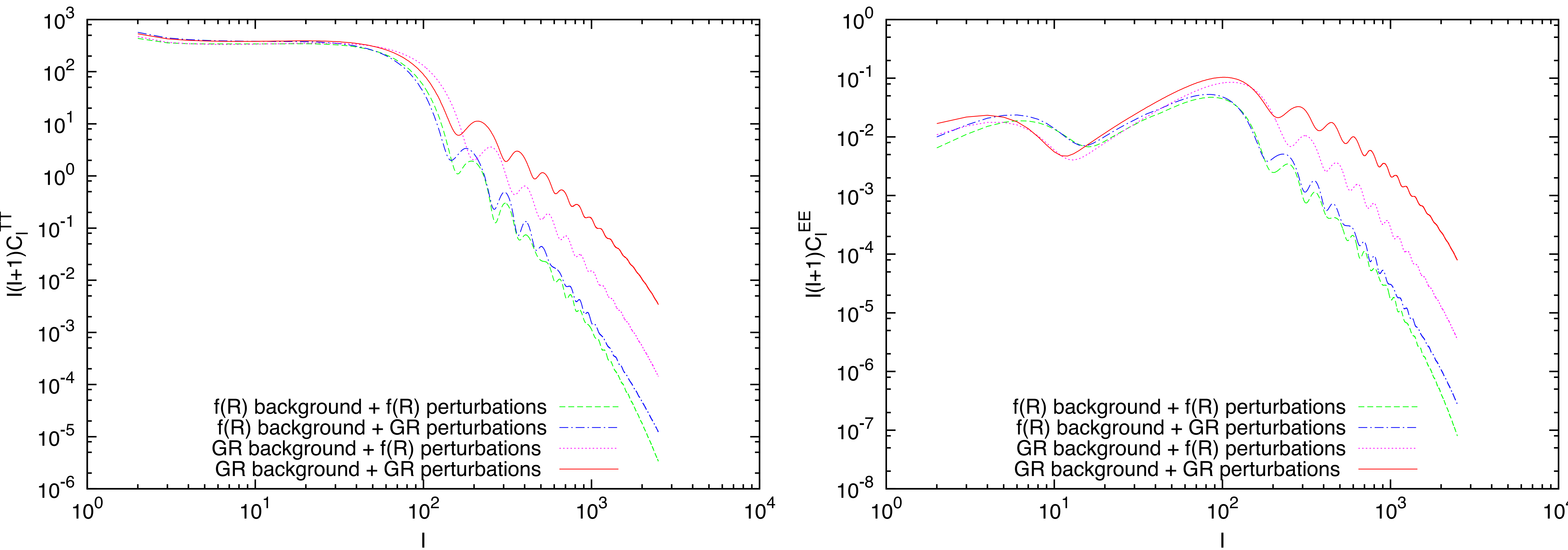


Figure 4: The temperature (left panel) and electrical (right panel) power spectra for tensor perturbations in all the possible background and perturbations scenarios. R^n model for $n = 1.28$: Power spectra for GR background and GR perturbations are depicted in red continuous, with no dependence on the R^n model and shown just for comparison ; GR background and f(R) perturbations pink dotted line; $f(R)$ background and GR perturbations in dotted-dashed blue line; $f(R)$ both background and perturbations in dashed green line.

The effect on the power spectrum is affected most by changes to the background model.

- Abdelwahab, Bishop, de la Cruz-Dombriz, PD (arXiv: 1412.6350)

Viable f(R) theories of gravity

Probably the best known f(R) theories that fall into this class are given by: $f = aR + F,$ with:

$$F = \lambda R_0 \left(\left(1 + \frac{R^2}{R_0^2} \right)^{-n} - 1 \right)$$

Starobinskiy arXiv: 0706.2041

$$F = -m^2 \frac{c_1 (R/m^2)^n}{c_2 (R/m^2)^n + 1}$$

Hu & Sawicki, arXiv: 0705.1158

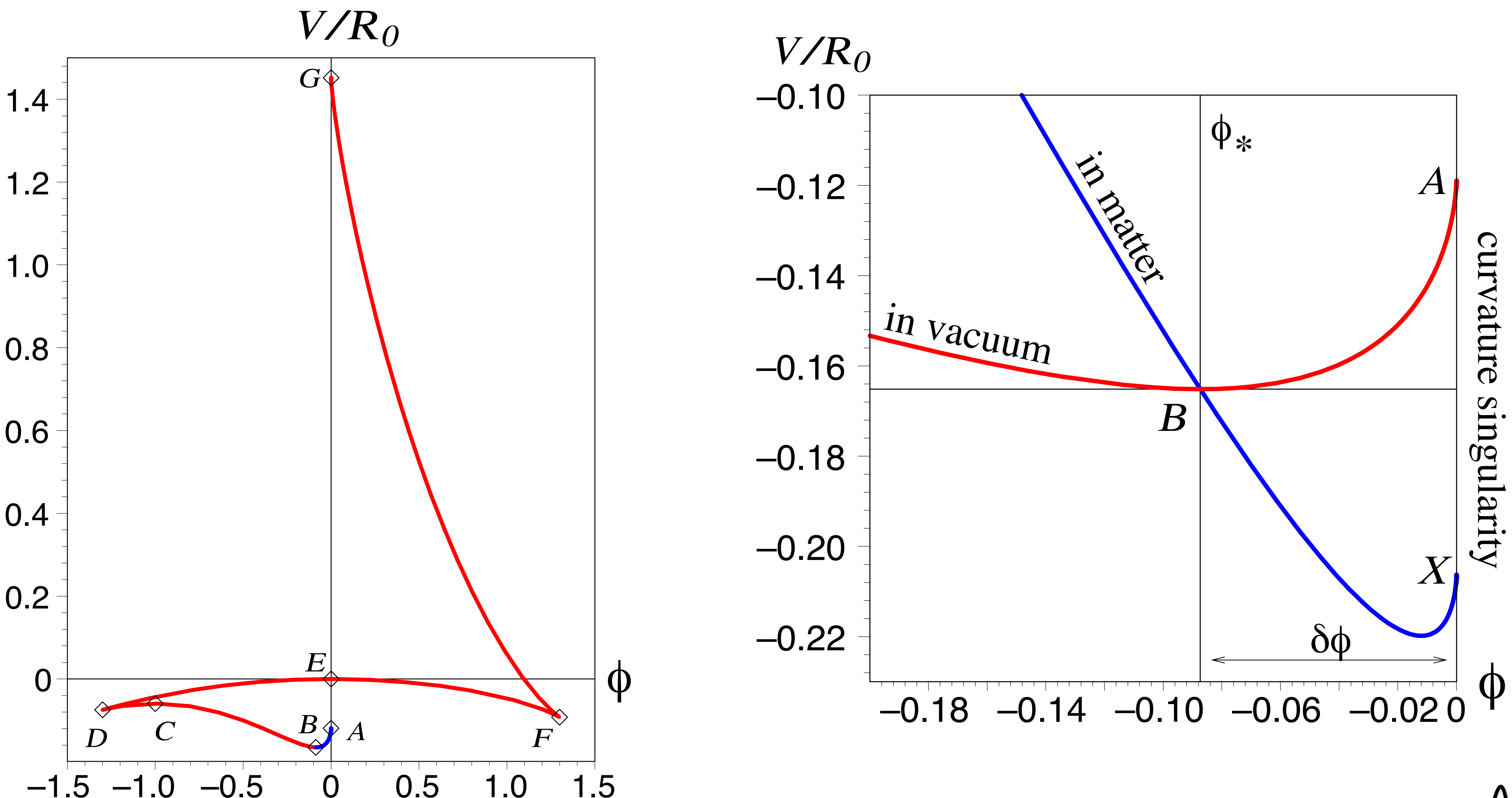
$$F = \frac{1}{a} \log\left[\cosh(aR) - \tanh(b)\sinh(aR)\right]$$

Appleby & Battye, arXiv: 0705.3199

- These theories require a screening mechanism, to hide the extra dynamical degree of freedom on small scales
- They provide expansion histories that are in excellent agreement with LCDM…. BUT

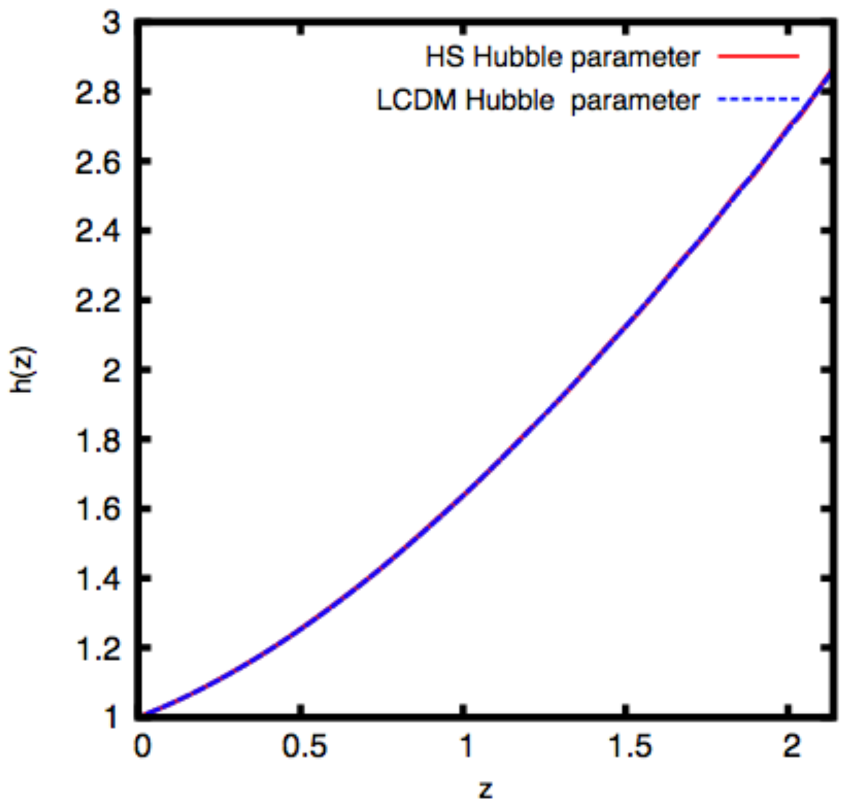
Some problems with viable f(R) theories

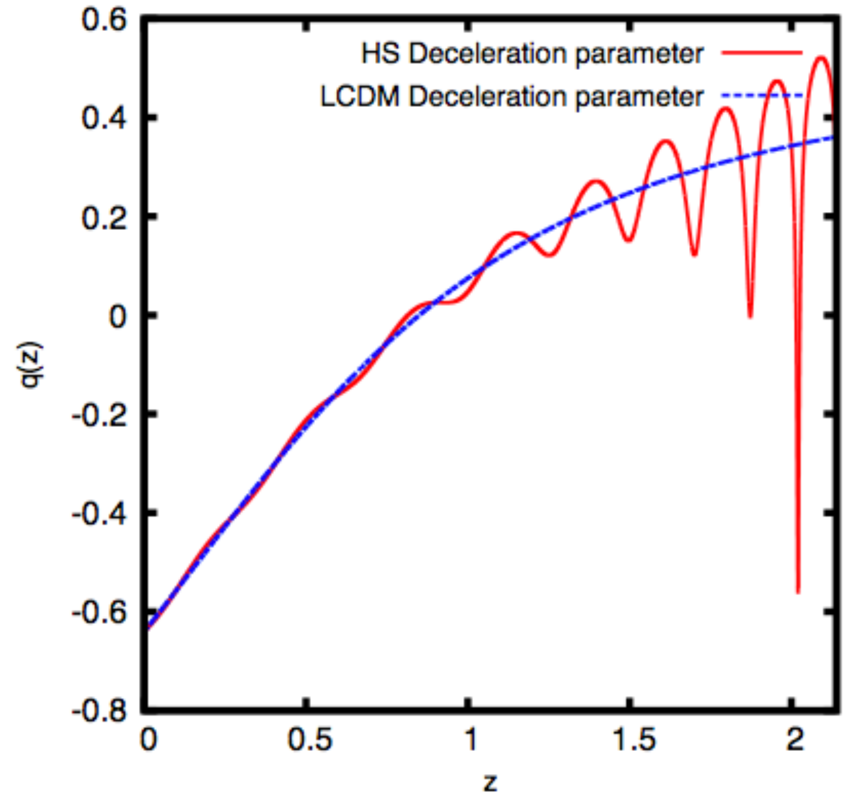
- Energetically accessible curvature singularity (A) exists at finite redshifts if initial conditions are taken to be the same as LCDM today.



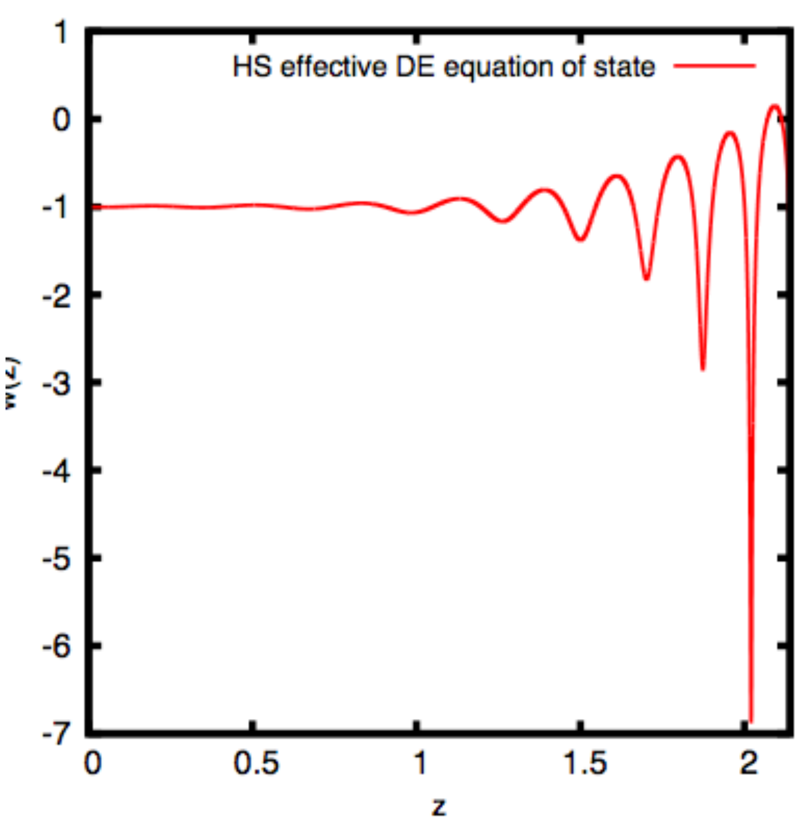
Frolov, arXiv: 0803.2500

- So while H(z) seems to be well behaved, oscillations develop in the second derivative of the scale factor, leading to singular deceleration and EOS parameters.





In this case we have a Hu and Sawicki model with n=3, with LCDM initial conditions taken at z=0. The curvature singularity occurs at z~2.1.



- Problem can be circumvented, by choosing initial conditions at high redshifts (R>>m^2), leading to a singularity-free region of parameter space (see Kandhai's talk).

Bottom up approach to cosmological modeling

- Very little has been done on trying to understand how weak field systems can be embedded into expanding cosmological backgrounds.

- Look at two cases

- The Einstein-Strauss like constructions used for example to build "Swiss Cheese" inhomogeneous cosmological models.

- Solve for the geometry of spacetime in the vicinity of astrophysical objects and then patch together a large number of such regions.

Israel junction conditions

$$
\begin{aligned}
[\gamma_{ab}]^+_- &= 0 \,, \qquad [R]^+_- = 0 \,, \\
[K_{ab}]^+_- &= 0 \,, \qquad [\partial_y f_R]^+_- = 0 \,.
\end{aligned}
$$

Einstein-Strauss like constructions

- The FLRW region can only be Minkowski spacetime (in Milne coordinates if (k=-1)

- This is because the conditions that the Ricci scalar and its first derivative should match across the boundary make the non-trivial f(R) theories qualitatively different from general relativity, where R can be discontinuous

- If a spherically symmetric object is joined to a FLRW geometry in f(R) theories, then one must expect an evolution of the boundary values of R and R' , which is something that pure Schwarzschild or Schwarzschild-de Sitter solutions cannot satisfy.

- Worse still.... other well know spherically symmetric solutions of f(R) gravity also can't be embedded in FLRW spacetimes.

Clifton, PD, Goswami, Nzioki , PRD 2013

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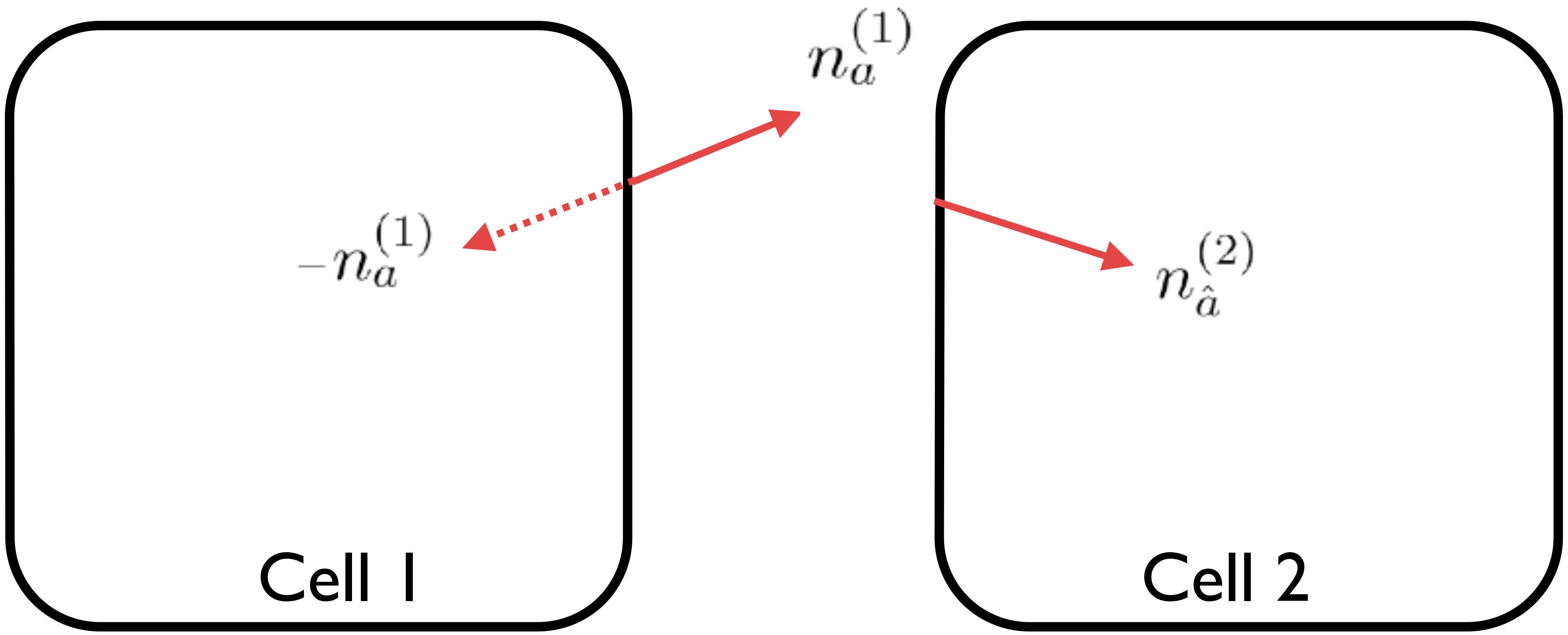
Patchwork universes

- Divide the universe up into a patchwork of regions, each of which can be described using post-Newtonian gravity:



- Match these regions together using junction conditions applied at their boundaries:

Clifton, PD, arXiv: 1501.04004

Patchwork universes

- Cosmological expansion can be seen as an emergent phenomenon

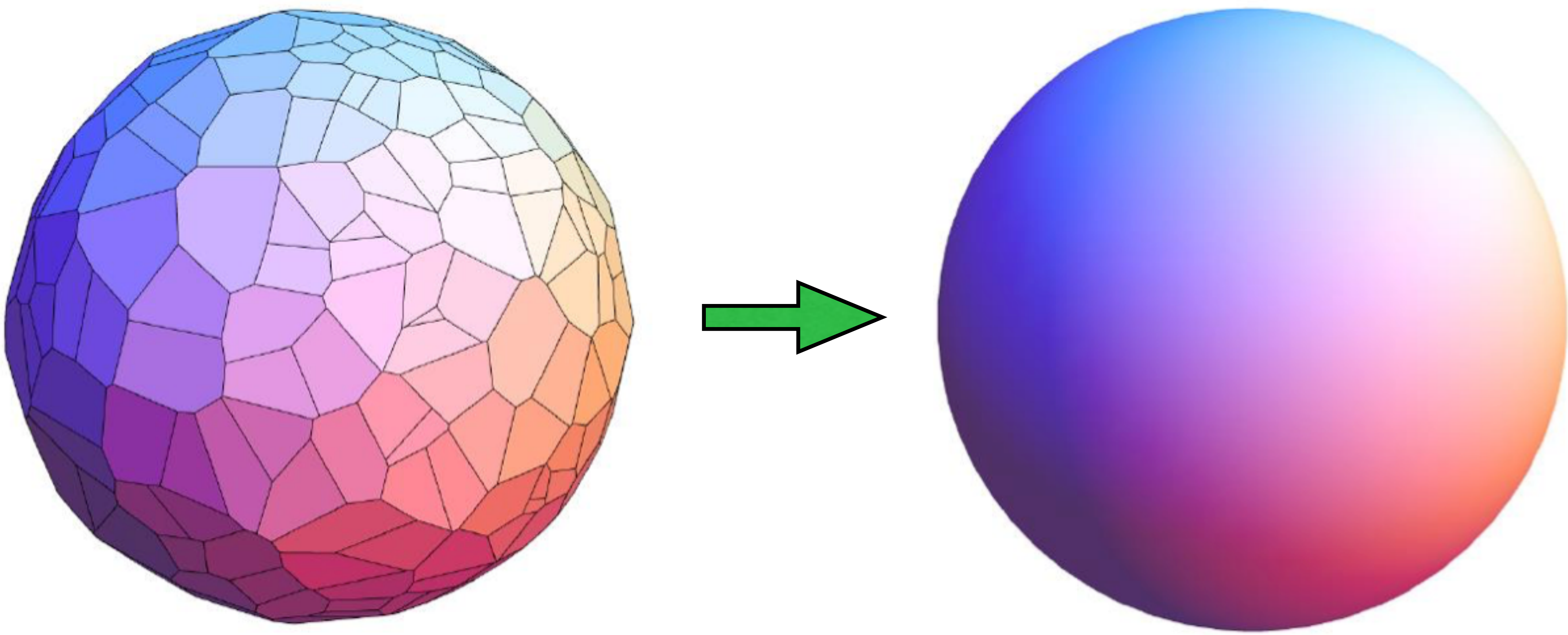


Image from Green and Wald, arXiv: 1407.8084

Construction of a patchwork universe in f(R) gravity

- Consider the geometry inside a single cell:

- Each cell contains a galaxy sized amount of mass and a spatial extent similar to intergalactic separation.

- Assume that the rate of expansion is of similar order of magnitude to the real universe - H_0

$$v \sim Hd \sim 10^{-4} c \qquad \phi \sim \frac{Gm}{c^2 d} \sim 5 \times 10^{-8} \qquad \phi \sim \frac{v^2}{c^2} \sim O(\epsilon^2)$$

$$ds^2 = -(1-2\phi)dt^2 + (1+2\psi)(dx^2 + dy^2 + dz^2)$$

$$\psi \sim \phi \sim O(\epsilon^2)$$

Construction of a patchwork universe in f(R) gravity

- For general f(R) theories, can't make any assumptions about smallness of f and f_R but assume that we can expand the function the Lagrangian in terms of our smallness parameter:

$$f = f^{(0)} + f^{(2)} + O(\epsilon^4) \qquad \text{and} \qquad f_R = f_R^{(0)} + f_R^{(2)} + O(\epsilon^4)$$

- The field equations and junction conditions give

$$f^{(0)} = 0$$
$$f_R^{(0)} = a(t)$$

- Class I: $a = \text{constant}$, $f(R) = aR + F(R)$
- Class II: $a = a(R)$, $R = R_0 + O(\epsilon^4)$

where $F_R \sim O(\epsilon^2)$ and $R_0(t) \sim O(\epsilon^2)$.

Class I theories

- The field equations and junction conditions give:

$$\psi = \frac{1}{a}\left(U - \frac{1}{2}F_R\right) \quad \text{and} \quad \phi = \frac{1}{a}\left(\hat{U} + \frac{1}{2}F_R\right),$$

where
$$\Delta U = -4\pi\rho + \frac{1}{4}F_0$$
$$\Delta \hat{U} = -4\pi\rho - \frac{1}{2}F_0$$
and $F_0 =$constant.

- The new scalar degree of freedom must also obey

$$\Delta F_R = \frac{1}{3}R + \frac{2}{3}F_0 - \frac{8\pi}{3}\rho.$$

- These equations, together with the junction condition $[\partial_y f_R]^+_- = 0$, mean that the large-scale expansion must proceed in the same way as in the LCDM model of GR.

Class II theories

- In this case $f = f^{(2)} + O(\epsilon^4)$ where $a = a(t) \sim O(1)$

$$f_R = a + f_R^{(2)} + O(\epsilon^4)$$

$$R^{(2)} = R^{(2)}(t)$$
$$f^{(2)} = f^{(2)}(t)$$

- The field equations then show that inhomogeneous cosmological solutions in this class cannot obey the chameleon mechanism and must have $\gamma_{\text{PPN}} = 1/2$.

- Solutions of this class are therefore are unable to both reproduce the correct cosmological expansion history and give the correct weak field limit.

Some consequences

- When non-linear structure forms, the large scale evolution of all viable models approaches that of the LCDM model of GR. Should be possible to also show this by doing an order by order perturbative approach starting with a FLRW background.
- If this is the case, the sudden curvature singularities that exist in the FLRW solutions of many f(R) theories are suppressed after the formation of Large Scale Structure.
- In Class I theories the effective cosmological constant must be constructed from the parameters of the theory. In the case of the Hu & Sawicki model, this is related to the height of the plateau of F(R).

Optomistic

- Provided viable f(R) phenomenology can be motivated from a more fundamental theory, late time acceleration can be seen as an emergent phenomenon determined by the parameters of this theory.
- Curvature singularities removed with the onset of structure formation.

Pessimistic

- It appears therefore that these "designer" theories do not solve any of the problems associated with the cosmological constant.
- The difficulties created by introducing a new light degree of freedom on small scales seem to have any obvious benefit.

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Conclusions and future work

- f(R) theories give rise to rich cosmological dynamics able to explain the late time accelerated expansion of the universe.
- Simple extensions to GR such as $f(R) = R + \alpha R^n$ while able to provide a good fit to LSS data, appear unable to simultaneously provide the correct values of the Hubble and deceleration parameters.
- Viable f(R) theories suffer from a curvature singularity at finite redshifts when initial conditions are taken to the the same as LCDM today.
- The standard Einstein-Strauss construction appears not to be possible in f(R) gravity and other known spherically symmetric solutions appear not to be embeddable in an expanding FLRW background.
- A bottom up approach to cosmological modeling seems to remove problematic curvature singularities, but suggest that designer f(R) gravity is unable to provide a more attractive alternative to LCDM
- Future work should focus on trying to reconstruct the theory of gravity from observational data.