Present status of inflation and of phantom forms of dark energy

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Present status of inflation

The simplest one-parametric inflationary models

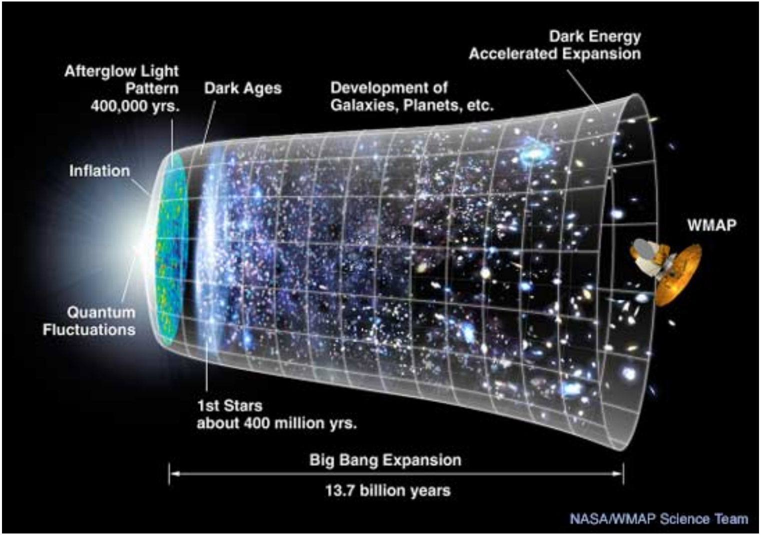
Inflation and its smooth reconstruction in GR

Inflation and its smooth reconstruction in $f(R)$ gravity

Quantum corrections to the simplest model

May primordial and/or present dark energy be phantom?

Conclusions


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years
NASA/WMAP Science Team

Inflation

The inflationary scenario is based on the two cornerstone independent ideas (hypothesis):

1. Existence of inflation (or, quasi-de Sitter stage) – a stage of accelerated, close to exponential expansion of our Universe in the past preceding the hot Big Bang with decelerated, power-law expansion.

2. The origin of all inhomogeneities in the present Universe is the effect of gravitational creation of particles and field fluctuations during inflation from the adiabatic vacuum (no-particle) state for Fourier modes covering all observable range of scales (and possibly somewhat beyond).

Existing analogies in other areas of physics.
1. The present dark energy.
2. Creation of electrons and positrons in an external electric field.

NB. 1. This effect is similar to particle creation by black holes, but no problems with the loss of information, 'firewalls', trans-Planckian energy etc. in cosmology, as far as observational predictions are calculated.

2. The observational argument for the choice of the adiabatic vacuum as the initial condition for all Fourier modes in the WKB-regime: no cosmic rays with energies beyond the GZK-cutoff at present that, in particular, means the absence of trans-Planckian particle creation during the recent epoch of the Universe expansion (A. A. Starobinsky, JETP Lett. **73**, 371 (2001); A. A. Starobinsky and I. I. Tkachev, JETP Lett. **76**, 235 (2002)).

3. Present dark energy can originate thorough this effect, too, see e.g. the recent paper D. Glavan, T. Prokopec, A. A. Starobinsky, Eur. Phys. J. C **78**, 371 (2018); arXiv:1710.07824.

Outcome of inflation

In the super-Hubble regime ($k \ll aH$) in the coordinate representation:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dx^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\mathcal{R}(\mathbf{r})\delta_{lm} + \sum_{a=1}^{2} g^{(a)}(\mathbf{r})\, e_{lm}^{(a)}$$

$$e_l^{l(a)} = 0, \;\; g^{(a)}_{,l}\, e_m^{l(a)} = 0, \;\; e_{lm}^{(a)}\, e^{lm(a)} = 1$$

$\mathcal{R}$ describes primordial scalar perturbations, g – primordial tensor perturbations (primordial gravitational waves (GW)).

The most important quantities:

$$n_s(k) - 1 \equiv \frac{d \ln P_{\mathcal{R}}(k)}{d \ln k}, \quad r(k) \equiv \frac{P_g}{P_{\mathcal{R}}}$$

In fact, metric perturbations h_{lm} are quantum (operators in the Heisenberg representation) and remain quantum up to the present time. But, after omitting of a very small part, decaying with time, they become commuting and, thus, equivalent to classical (c-number) stochastic quantities with the Gaussian statistics (up to small terms quadratic in $\mathcal{R}, g$).

In particular:

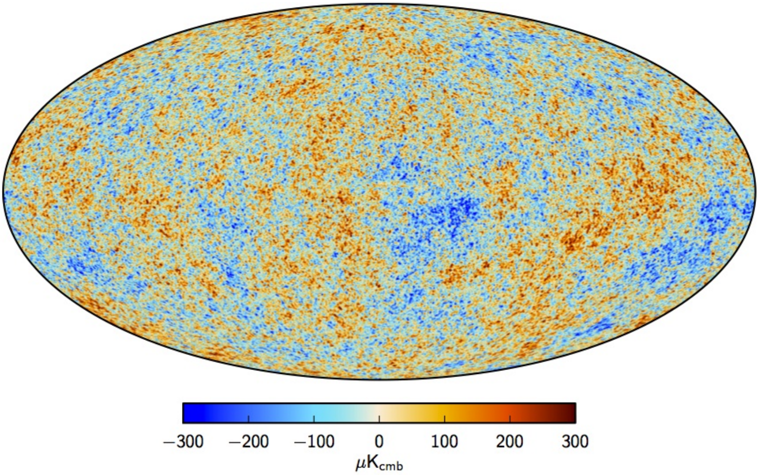
$$\hat{\mathcal{R}}_k = \mathcal{R}_k \, i(\hat{a}_{\mathbf{k}} - \hat{a}^\dagger_{\mathbf{k}}) + \mathcal{O}\left((\hat{a}_{\mathbf{k}} - \hat{a}^\dagger_{\mathbf{k}})^2\right) + ... + \mathcal{O}(10^{-100})(\hat{a}_{\mathbf{k}} + \hat{a}^\dagger_{\mathbf{k}}) + ,,,$$

The last term is time dependent, it is affected by physical decoherence and may become larger, but not as large as the second term.

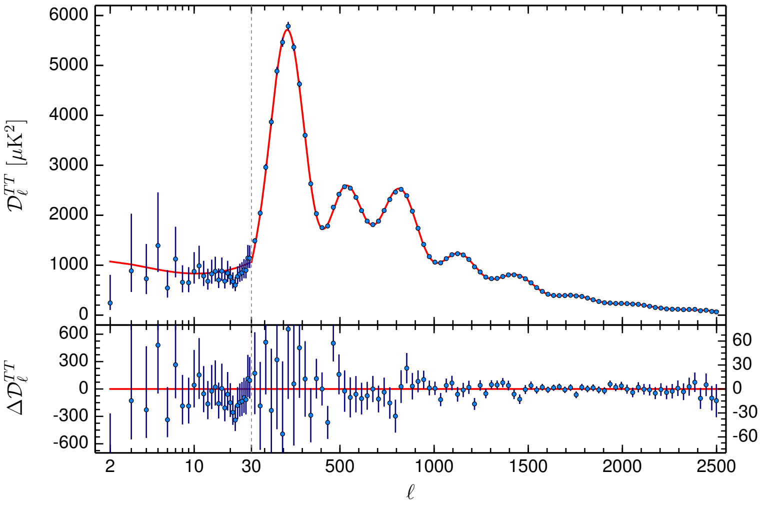
Remaining quantum coherence: deterministic correlation between $\mathbf{k}$ and $-\mathbf{k}$ modes - shows itself in the appearance of acoustic oscillations (primordial oscillations in case of GW).

CMB temperature anisotropy

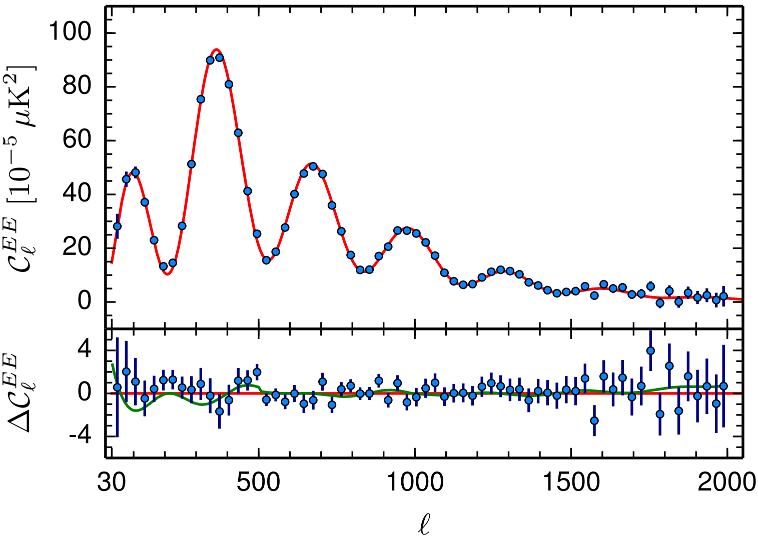
Planck-2015: P. A. R. Ade et al., arXiv:1502.01589



CMB temperature anisotropy multipoles



CMB E-mode polarization multipoles



Present status of inflation

Now we have numbers: N. Agranim et al., arXiv:1807.06209

The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ in the first order in $|n_s - 1| \sim N_H^{-1}$ has been discovered (using the multipole range $\ell > 40$):

$$< \mathcal{R}^2(\mathbf{r}) >= \int \frac{P_{\mathcal{R}}(k)}{k}\, dk, \;\; P_{\mathcal{R}}(k) = (2.10 \pm 0.03) \cdot 10^{-9} \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$k_0 = 0.05 \ \mathrm{Mpc}^{-1}, \quad n_s - 1 = -0.035 \pm 0.004$$

Two fundamental observational constants of cosmology in addition to the three known ones (baryon-to-photon ratio, baryon-to-matter density and the cosmological constant). Existing inflationary models can predict (and predicted, in fact) one of them, namely $n_s - 1$, relating it finally to $N_H = \ln \frac{k_B T_\gamma}{\hbar H_0} \approx 67.2$. (note that $(1 - n_s)N_H \sim 2$).

Physical scales related to inflation

"Naive" estimate where I use the reduced Planck mass $\tilde{M}_{Pl} = (8\pi G)^{-1}$.

I. Curvature scale

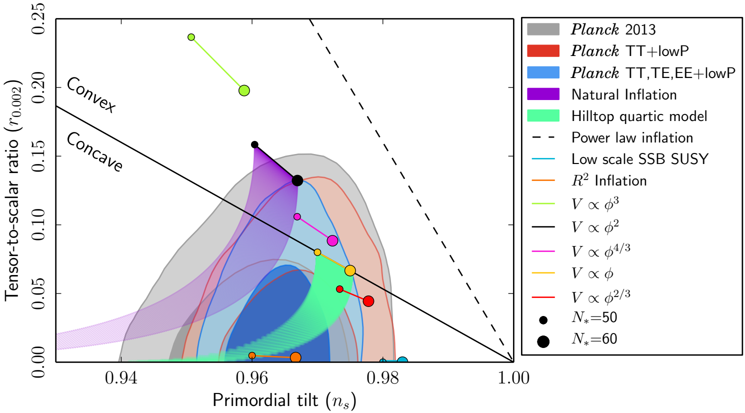
$$H \sim \sqrt{P_{\mathcal{R}}}\tilde{M}_{Pl} \sim 10^{14}\mathrm{GeV}$$

II. Inflaton mass scale

$$|m_{infl}| \sim H\sqrt{|1 - n_s|} \sim 10^{13}\mathrm{GeV}$$

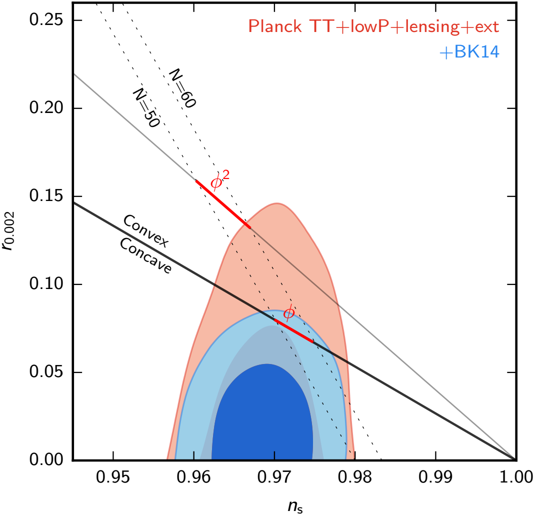
New range of mass scales significantly less than the GUT scale.

Direct approach: comparison with simple smooth models



Combined BICEP2/Keck Array/Planck results

P. A. R. Ade et al., Phys. Rev. Lett. 116, 031302 (2016)



The simplest models producing the observed scalar slope

1. The $R + R^2$ model (Starobinsky, 1980):

$$\mathcal{L} = \frac{f(R)}{16\pi G}, \quad f(R) = R + \frac{R^2}{6M^2}$$

$$M = 2.6 \times 10^{-6} \left(\frac{55}{N}\right) M_{\rm Pl} \approx 3.1 \times 10^{13}\,\mathrm{GeV}$$

$$n_s - 1 = -\frac{2}{N} \approx -0.036, \quad r = \frac{12}{N^2} \approx 0.004$$

$$N = \ln\frac{k_f}{k} = \ln\frac{a_0 T_\gamma}{k} - \mathcal{O}(10), \quad H_{dS}(N = 55) = 1.3 \times 10^{14}\,\mathrm{GeV}$$

2. The same prediction from a scalar field model with $V(\phi) = \frac{\lambda\phi^4}{4}$ at large ϕ and strong non-minimal coupling to gravity $\xi R\phi^2$ with $\xi < 0, \ |\xi| \gg 1$, including the Brout-Englert-Higgs inflationary model.

The simplest purely geometrical inflationary model

$$\mathcal{L} = \frac{R}{16\pi G} + \frac{N^2}{288\pi^2 P_{\mathcal{R}}(k)} R^2 + (\text{small rad. corr.})$$

$$= \frac{R}{16\pi G} + 5.1 \times 10^8 \, R^2 + (\text{small rad. corr.})$$

The quantum effect of creation of particles and field fluctuations works twice in this model:
a) at super-Hubble scales during inflation, to generate space-time metric fluctuations;
b) at small scales after inflation, to provide scalaron decay into pairs of matter particles and antiparticles (AS, 1980, 1981).

Weak dependence of the time t_r when the radiation dominated stage begins:

$$N(k) \approx N_H + \ln \frac{a_0 H_0}{k} - \frac{1}{3} \ln \frac{M_{\rm Pl}}{M} - \frac{1}{6} \ln(M_{\rm Pl} t_r)$$

The most effective decay channel: into minimally coupled scalars with $m \ll M$. Then the formula

$$\frac{1}{\sqrt{-g}}\frac{d}{dt}(\sqrt{-g}\,n_s) = \frac{R^2}{576\pi}$$

(Ya. B. Zeldovich and A. A. Starobinsky, JETP Lett. 26, 252 (1977)) can be used for simplicity, but the full integral-differential system of equations for the Bogoliubov α_k, β_k coefficients and the average EMT was in fact solved in AS (1981). Scalaron decay into graviton pairs is suppressed (A. A. Starobinsky, JETP Lett. 34, 438 (1981)).

For this channel of the scalaron decay:

$$N(k) \approx N_H + \ln\frac{a_0 H_0}{k} - \frac{5}{6}\ln\frac{M_{\rm Pl}}{M}$$

Possible microscopic origins of this phenomenological model.

1. Follow the purely geometrical approach and consider it as the specific case of the fourth order gravity in 4D

$$\mathcal{L} = \frac{R}{16\pi G} + AR^2 + BC_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta} + (\text{small rad. corr.})$$

for which $A \gg 1, \ A \gg |B|$. Approximate scale (dilaton) invariance and absence of ghosts in the curvature regime $A^{-2} \ll (RR)/M_P^4 \ll B^{-2}$.

One-loop quantum-gravitational corrections are small (their imaginary parts are just the predicted spectra of scalar and tensor perturbations), non-local and qualitatively have the same structure modulo logarithmic dependence on curvature.

2. Another, completely different way:

consider the $R + R^2$ model as an approximate description of GR + a non-minimally coupled scalar field with a large negative coupling ξ ($\xi_{conf} = \frac{1}{6}$) in the gravity sector::

$$L = \frac{R}{16\pi G} - \frac{\xi R \phi^2}{2} + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi), \quad \xi < 0, \quad |\xi| \gg 1 \ .$$

Geometrization of the scalar:

for a generic family of solutions during inflation and even for some period of non-linear scalar field oscillations after it, the scalar kinetic term can be neglected, so

$$\xi R \phi = -V'(\phi) + \mathcal{O}(|\xi|^{-1}) \ .$$

No conformal transformation, we remain in the the physical (Jordan) frame!

These solutions are the same as for $f(R)$ gravity with

$$L = \frac{f(R)}{16\pi G}, \ f(R) = R - \frac{\xi R \phi^2(R)}{2} - V(\phi(R)).$$

For $V(\phi) = \frac{\lambda(\phi^2 - \phi_0^2)^2}{4}$, this just produces $f(R) = \frac{1}{16\pi G}\left(R + \frac{R^2}{6M^2}\right)$ with $M^2 = \lambda/24\pi\xi^2 G$ and $\phi^2 = |\xi|R/\lambda$.

The same theorem is valid for a multi-component scalar field, as well as for the mixed Higgs-R^2 model.

Inflation in the mixed Higgs-R^2 Model

M. He, A. A. Starobinsky and J. Yokoyama, JCAP **1805** (2018) 064; arXiv:1804.00409.

$$\mathcal{L} = \frac{1}{16\pi G}\left(R + \frac{R^2}{6M^2}\right) - \frac{\xi R\phi^2}{2} + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - \frac{\lambda\phi^4}{4}, \quad \xi < 0, \quad |\xi| \gg 1$$

In the attractor regime during inflation (and even for some period after it), we return to the $f(R) = R + \frac{R^2}{6M^2}$ model with the renormalized scalaron mass $M \to \tilde{M}$:

$$\frac{1}{\tilde{M}^2} = \frac{1}{M^2} + \frac{24\pi\xi^2 G}{\lambda}$$

More generally, R^2 inflation (with an arbitrary n_s, r) serves as an intermediate dynamical attractor for a large class of scalar-tensor gravity models.

Inflation in GR

Inflation in GR with a minimally coupled scalar field with some potential.

In the absence of spatial curvature and other matter:

$$H^2 = \frac{\kappa^2}{3}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right)$$

$$\dot{H} = -\frac{\kappa^2}{2}\dot{\phi}^2$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

where $\kappa^2 = 8\pi G$ ($\hbar = c = 1$).

Reduction to the first order equation

It can be reduced to the first order Hamilton-Jacobi-like equation for $H(\phi)$. From the equation for $\dot{H}$, $\frac{dH}{d\phi} = -\frac{\kappa^2}{2}\dot{\phi}$. Inserting this into the equation for H^2, we get

$$\frac{2}{3\kappa^2}\left(\frac{dH}{d\phi}\right)^2 = H^2 - \frac{\kappa^2}{3}V(\phi)$$

Time dependence is determined using the relation

$$t = -\frac{\kappa^2}{2}\int\left(\frac{dH}{d\phi}\right)^{-1} d\phi$$

However, during oscillations of ϕ, $H(\phi)$ acquires non-analytic behaviour of the type $const + \mathcal{O}(|\phi - \phi_1|^{3/2})$ at the points where $\dot{\phi} = 0$, and then the correct matching with another solution is needed.

Inflationary slow-roll dynamics

Slow-roll occurs if: $|\ddot{\phi}| \ll H|\dot{\phi}|, \ \ \dot{\phi}^2 \ll V$, and then $|\dot{H}| \ll H^2$.

Necessary conditions: $|V'| \ll \kappa V, \ \ |V''| \ll \kappa^2 V$. Then

$$H^2 \approx \frac{\kappa^2 V}{3}, \ \ \dot{\phi} \approx -\frac{V'}{3H}, \ \ N \equiv \ln \frac{a_f}{a} \approx \kappa^2 \int_{\phi_f}^{\phi} \frac{V}{V'}\, d\phi$$

First obtained in A. A. Starobinsky, Sov. Astron. Lett. 4, 82 (1978) in the $V = \frac{m^2\phi^2}{2}$ case and for a bouncing model.

Spectral predictions of the one-field inflationary scenario in GR

Scalar (adiabatic) perturbations:

$$P_{\mathcal{R}}(k) = \frac{H_k^4}{4\pi^2\dot{\phi}^2} = \frac{GH_k^4}{\pi|\dot{H}|_k} = \frac{128\pi G^3 V_k^3}{3V_k'^2}$$

where the index k means that the quantity is taken at the moment $t = t_k$ of the Hubble radius crossing during inflation for each spatial Fourier mode $k = a(t_k)H(t_k)$. Through this relation, the number of e-folds from the end of inflation back in time $N(t)$ transforms to $N(k) = \ln\frac{k_f}{k}$ where $k_f = a(t_f)H(t_f)$, t_f denotes the end of inflation.

The spectral slope

$$n_s(k) - 1 \equiv \frac{d\ln P_{\mathcal{R}}(k)}{d\ln k} = \frac{1}{\kappa^2}\left(2\frac{V_k''}{V_k} - 3\left(\frac{V_k'}{V_k}\right)^2\right)$$

is small by modulus – confirmed by observations!

Tensor perturbations (A. A. Starobinsky, JETP Lett. 50, 844 (1979)):

$$P_g(k) = \frac{16GH_k^2}{\pi}; \quad n_g(k) \equiv \frac{d\ln P_g(k)}{d\ln k} = -\frac{1}{\kappa^2}\left(\frac{V_k'}{V_k}\right)^2$$

The consistency relation:

$$r(k) \equiv \frac{P_g}{P_{\mathcal{R}}} = \frac{16|\dot{H}_k|}{H_k^2} = 8|n_g(k)|$$

Tensor perturbations are always suppressed by at least the factor $\sim 8/N(k)$ compared to scalar ones. For the present Hubble scale, $N(k_H) = (50-60)$. Typically, $|n_g| \le |n_s - 1|$, so $r \le 8(1-n_s) \sim 0.3$ – confirmed by observations!

Inverse reconstruction of inflationary models in GR

In the slow-roll approximation:

$$\frac{V^3}{V'^2} = CP_{\mathcal{R}}(k(t(\phi))), \quad C = \frac{12\pi^2}{\kappa^6}$$

Changing variables for ϕ to $N(\phi)$ and integrating, we get:

$$\frac{1}{V(N)} = -\frac{\kappa^4}{12\pi^2}\int \frac{dN}{P_{\mathcal{R}}(N)}$$

$$\kappa\phi = \int dN \sqrt{\frac{d\ln V}{dN}}$$

Here, $N \gg 1$ stands both for $\ln(k_f/k)$ at the present time and the number of e-folds back in time from the end of inflation. First derived in H. M. Hodges and G. R. Blumenthal, Phys. Rev. D 42, 3329 (1990).

The two-parameter family of isospectral slow-roll inflationary models, but the second parameter shifts the field ϕ only.

Minimal "scale-free" reconstruction

Minimal inflationary model reconstruction avoiding introduction of any new physical scale both during and after inflation and producing the best fit to the Planck data.

Assumption: the numerical coincidence between $2/N_H \sim 0.04$ and $1 - n_s$ is not accidental but happens for all $1 \ll N \lesssim 60$: $P_{\mathcal{R}} = P_0 N^2$. Then:

$$V = V_0 \frac{N}{N + N_0} = V_0 \tanh^2 \frac{\kappa\phi}{2\sqrt{N_0}}$$

$$r = \frac{8N_0}{N(N + N_0)}$$

$r \sim 0.003$ for $N_0 \sim 1$. From the upper limit on r:

$$N_0 < \frac{0.07N^2}{8 - 0.07N}$$

$N_0 < 57$ for $N = 57$.

Another example: $P_{\mathcal{R}} = P_0 N^{3/2}$.

$$V(\phi) = V_0 \frac{\phi^2 + 2\phi\phi_0}{(\phi + \phi_0)^2}$$

Not bounded from below (of course, in the region where the slow-roll approximation is not valid anymore). Crosses zero linearly.

More generally, the two "aesthetic" assumptions – "no-scale" scalar power spectrum and $V \propto \phi^{2n}, \; n = 1, 2...$ at the minimum of the potential – lead to $P_{\mathcal{R}} = P_0 N^{n+1}, \; n_s - 1 = -\frac{n+1}{N}$ unambiguously. From this, only $n = 1$ is permitted by observations. Still an additional parameter appears due to tensor power spectrum – no preferred one-parameter model (if the $V(\phi) \propto \phi^2$ model is excluded).

Inflation in $f(R)$ gravity

The simplest model of modified gravity (= geometrical dark energy) considered as a phenomenological macroscopic theory in the fully non-linear regime and non-perturbative regime.

$$S = \frac{1}{16\pi G}\int f(R)\sqrt{-g}\,d^4x + S_m$$

$$f(R) = R + F(R), \quad R \equiv R_\mu^\mu$$

Here $f''(R)$ is not identically zero. Usual matter described by the action S_m is minimally coupled to gravity.

Vacuum one-loop corrections depending on R only (not on its derivatives) are assumed to be included into $f(R)$. The normalization point: at laboratory values of R where the scalaron mass (see below) $m_s \approx \mathrm{const}$.

Metric variation is assumed everywhere. Palatini variation leads to a different theory with a different number of degrees of freedom.

Field equations

$$\frac{1}{8\pi G}\left(R_\mu^\nu - \frac{1}{2}\,\delta_\mu^\nu R\right) = -\left(T^\nu_{\mu\,(vis)} + T^\nu_{\mu\,(DM)} + T^\nu_{\mu\,(DE)}\right)\ ,$$

where $G = G_0 = const$ is the Newton gravitational constant measured in laboratory and the effective energy-momentum tensor of DE is

$$8\pi G T^\nu_{\mu\,(DE)} = F'(R)\,R_\mu^\nu - \frac{1}{2}\,F(R)\delta_\mu^\nu + \left(\nabla_\mu\nabla^\nu - \delta_\mu^\nu\nabla_\gamma\nabla^\gamma\right)F'(R)\,.$$

Because of the need to describe DE, de Sitter solutions in the absence of matter are of special interest. They are given by the roots $R = R_{dS}$ of the algebraic equation

$$Rf'(R) = 2f(R)\ .$$

The special role of $f(R) \propto R^2$ gravity: admits de Sitter solutions with any curvature.

Reduction to the first order equation

In the absence of spatial curvature and $\rho_m = 0$, it is always possible to reduce these equations to a first order one using either the transformation to the Einstein frame and the Hamilton-Jacobi-like equation for a minimally coupled scalar field in a spatially flat FLRW metric, or by directly transforming the 0-0 equation to the equation for $R(H)$:

$$\frac{dR}{dH} = \frac{(R - 6H^2)f'(R) - f(R)}{H(R - 12H^2)f''(R)}$$

See, e.g. H. Motohashi amd A. A. Starobinsky, Eur. Phys. J C **77**, 538 (2017), but in the special case of the $R + R^2$ gravity this was found and used already in the original AS (1980) paper.

Analogues of large-field (chaotic) inflation: $F(R) \approx R^2 A(R)$ for $R \to \infty$ with $A(R)$ being a slowly varying function of R, namely

$$|A'(R)| \ll \frac{A(R)}{R}\ , \ |A''(R)| \ll \frac{A(R)}{R^2}\ .$$

Analogues of small-field (new) inflation, $R \approx R_1$:

$$F'(R_1) = \frac{2F(R_1)}{R_1}\ , \ F''(R_1) \approx \frac{2F(R_1)}{R_1^2}\ .$$

Thus, all inflationary models in $f(R)$ gravity are close to the simplest one over some range of R.

Perturbation spectra in slow-roll $f(R)$ inflationary models

Let $f(R) = R^2 A(R)$. In the slow-roll approximation $|\ddot{R}| \ll H|\dot{R}|$:

$$P_{\mathcal{R}}(k) = \frac{\kappa^2 A_k}{64\pi^2 A_k'^2 R_k^2}, \quad P_g(k) = \frac{\kappa^2}{12 A_k \pi^2}$$

$$N(k) = -\frac{3}{2}\int_{R_f}^{R_k} dR\, \frac{A}{A' R^2}$$

where the index k means that the quantity is taken at the moment $t = t_k$ of the Hubble radius crossing during inflation for each spatial Fourier mode $k = a(t_k)H(t_k)$.

Smooth reconstruction of inflation in $f(R)$ gravity

$$f(R) = R^2 A(R)$$

$$A = const - \frac{\kappa^2}{96\pi^2} \int \frac{dN}{P_{\mathcal{R}}(N)}$$

$$\ln R = const + \int dN \sqrt{-\frac{2\, d \ln A}{3\, dN}}$$

Here, the additional assumptions that $P_{\mathcal{R}} \propto N^{\beta}$ and that the resulting $f(R)$ can be analytically continued to the region of small R without introducing a new scale, and it has the linear (Einstein) behaviour there, leads to $\beta = 2$ and the $R + R^2$ inflationary model with $r = \frac{12}{N^2} = 3(n_s - 1)^2$ unambiguously.

Quantum corrections to the simplest model

Due to the scale-invariance of the $R + R^2$ model for $R \gg M^2$, one may expect logarithmic running of the dimensionless coefficient in front of the R^2 term for large energies and curvatures. The concrete 'asymptotically safe' model with

$$f(R) = R + \frac{R^2}{6M^2\left[1 + b\ln\left(\frac{R}{\mu^2}\right)\right]}$$

was recently considered in L.-H. Liu, T. Prokopec, A. A. Starobinsky, Phys. Rev. D **98**, 043505 (2018); arXiv:1806.05407.

However, comparison with CMB observational data shows that b is small by modulus: $|b| \lesssim 10^{-2}$. Thus, from the observational point of view this model can be simplified to

$$f(R) = R + \frac{R^2}{6M^2}\left[1 - b\ln\left(\frac{R}{\mu^2}\right)\right],$$

for which the analytic solution exists:

$$n_s - 1 = -\frac{4b}{3}\left(e^{\frac{2bN}{3}} - 1\right)^{-1}$$

$$r = \frac{16b^2}{3}\frac{e^{\frac{4bN}{3}}}{\left(e^{\frac{2bN}{3}} - 1\right)^2}$$

For $|b|N \ll 1$, these expressions reduce to those for the $R + R^2$ model.

Second type: terms arising from the conformal (trace) anomaly

The tensor producing the $\propto \left(R_{\mu\nu}R^{\mu\nu} - \frac{R^2}{3}\right)$ term in the trace anomaly:

$$T_\mu^\nu = \frac{k_2}{2880\pi^2}\left(R_\mu^\alpha R_\alpha^\nu - \frac{2}{3}RR_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R_{\alpha\beta}R^{\alpha\beta} + \frac{1}{4}\delta_\mu^\nu R^2\right)$$

It is covariantly conserved in the isotropic case only! Can be generalized to the weakly anisotropic case by adding a term proportional to the first power of the Weyl tensor.

$$T_0^0 = \frac{3H^4}{\kappa^2 H_1^2}, \quad T = -\frac{1}{\kappa^2 H_1^2}\left(R_{\mu\nu}R^{\mu\nu} - \frac{R^2}{3}\right), \quad H_1^2 = \frac{2880\pi^2}{\kappa^2 k_2}$$

The spectrum of scalar and tensor perturbations in this case was calculated already in A. A. Starobinsky, Sov. Astron. Lett. **9**, 302 (1983).

$$n_s - 1 = -2\beta \frac{e^{\beta N}}{e^{\beta N} - 1}, \quad \beta = \frac{M^2}{3H_1^2}$$

If $n_s > 0.957$ and $N = 55$, then $H_1 > 7.2M$.

Perspectives of future discoveries

- Primordial gravitational waves from inflation: r.
  $r \lesssim 8(1 - n_s) \approx 0.3$ (confirmed!) but may be much less. However, under reasonable assumptions one may expect that $r \gtrsim (n_s - 1)^2 \approx 10^{-3}$.
- A more precise measurement of $n_s - 1 \Longrightarrow$ duration of transition from inflation to the radiation dominated stage $\Longrightarrow$ information on inflaton (scalaron) couplings to known elementary particles at superhigh energies $E \lesssim 10^{13}$ Gev.
- Local non-smooth features in the scalar power spectrum at cosmological scales (?).
- Local enhancement of the power spectrum at small scales leading to a significant amount of primordial black holes (?).

The most well-known and influential papers of Prof. González-Díaz

The 4 most cited (135-225 citations according to INSPIRE) and influential papers by Prof. Pedro González-Díaz : Phys. Rev. D (2003), Phys. Lett. B (2004), Nucl. Phys. B (with C. L. Siguenza) and Phys. Lett. B (2008) (with M. Bouhmadi-Lopez and P. Martin-Moruno) – were related to different aspects of phantom present dark energy. One more paper (Phys. Lett. B (2004), with J. A. Jimenez-Madrid) was devoted to phantom primordial dark energy (phantom inflation) where the hypothesis of the "big trip" was introduced.

Possible forms of DE

- Physical DE.
  New non-gravitational field of matter. DE proper place – in the rhs of gravity equations.
- Geometrical DE.
  Modified gravity. DE proper place – in the lhs of gravity equations.
- Λ - intermediate case.

Observations: $T^{\nu}_{\mu\,(DE)}$ is very close to $\Lambda\delta^{\nu}_{\mu}$ for the concrete solution describing our Universe;

$$< w_{DE} >= -1.03 \pm 0.03$$

where $w_{DE} \equiv p_{DE}/\epsilon_{DE}$.
$w_{DE} > -1$ – normal case,
$w_{DE} < -1$ – phantom case,
$w_{DE} \equiv -1$ – the exact cosmological constant ("vacuum energy").

DE phantom behavior in modified classical gravity

1. Models of present DE in scalar-tensor gravity may generically have phantom behaviour due to change of G_{eff} (B. Boisseau et al., Phys. Rev. Lett. **85**, 2236 (2000)).
2. Viable models of present DE in $f(R)$ gravity typically exhibit phantom behaviour of dark energy during the matter-dominated stage and recent crossing of the phantom boundary $w_{\mathrm{DE}} = -1$ ((H. Motohashi, A. A. Starobinsky and J. Yokoyama, Progr. Theor. Phys. **123**, 887 (2010)).
3. Moreover, if the present mass of the scalaron is sufficiently large, there will be an infinite number of phantom boundary crossings during the future evolution of such cosmological models (H. Motohashi, A. A. Starobinsky and J. Yokoyama, JCAP **1106**, 006 (2011)).

However, in all cases $|1 + w|$ is small, less than a few percent.

Phantom behaviour in quantum regime

1. The weak and null energy conditions are always temporarily violated in the process of particle creation from the adiabatic vacuum as far as the effective average number of particles is small (Ya. B. Zeldovich and L. P. Pitaevsky, 1971, Ya. B. Zeldovich and A. A. Starobinsky, 1971). Otherwise, particle creation from vacuum would not be possible (S. W. Hawking, 1970).
2. Smallness and Gaussian statistics of primordial scalar perturbations require $\dot{H} < 0$, i. e. non-phantom behaviour, during the observable part of inflation. Still primordial DE may be temporary weakly phantom during its early, stochastic stage which is not directly observable.
3. Present 'quantum' DE consisting of quantum fluctuations of a non-minimally coupled scalar field which was a spectator one during inflation may be phantom, in principle, though this possibility has not been investigated in detail.

Conclusions

- The typical inflationary predictions that $|n_s - 1|$ is small and of the order of N_H^{-1}, and that r does not exceed $\sim 8(1 - n_s)$ are confirmed. Typical consequences following without assuming additional small parameters: $H_{55} \sim 10^{14}\,\mathrm{GeV}, \;\; m_{infl} \sim 10^{13}\,\mathrm{GeV}$.
- Though the Einstein gravity plus a minimally coupled inflaton remains sufficient for description of inflation with existing observational data, modified (in particular, scalar-tensor or $f(R)$) gravity can do it as well.
- Inflation in $f(R)$ gravity represents a dynamical attractor for slow-rolling scalar fields strongly coupled to gravity.
- From the scalar power spectrum $P_{\mathcal{R}}(k)$, it is possible to reconstruct an inflationary model both in the Einstein and $f(R)$ gravity up to one arbitrary physical constant of integration.

- In the Einstein gravity, the simplest inflationary models permitted by observational data are two-parametric, no preferred quantitative prediction for r, apart from its parametric dependence on $n_s - 1$, namely, $\sim (n_s - 1)^2$ or larger.
- In the $f(R)$ gravity, the simplest model is one-parametric and has the preferred value $r = \frac{12}{N^2} = 3(n_s - 1)^2$.
- Thus, it has sense to search for primordial GW from inflation at the level $r > 10^{-3}$!

- Comparison with observational data shows that logarithmic high-curvature quantum corrections to the $R + R^2$ model in the observable part of inflation are small, no more than a few percents. This smallness has been expected since it caused by the anomalously large value of the dimensionless coefficient in front of the R^2 term which finally follows from actual smallness of present large-scale inhomogeneity of the Universe.
- Present dark energy may by phantom due to modified gravity (G_{eff} changing) or quantum gravitational effects, but the expected amount of its phantomness is very small that agrees with observations. Primordial dark energy may not be phantom at the observable (final) part of inflation, but may be temporary weakly phantom during its early, stochastic stage when generated scalar perturbations are large (though not locally observable).