Conceptual Issues in Quantum Cosmology

Claus Kiefer

Institut für Theoretische Physik
Universität zu Köln

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COBE DMR observations of the early Universe

George F Smoot†

Lawrence Berkeley Laboratory, Space Sciences Laboratory and Center for Particle Astrophysics, University of California, Berkeley, CA 94720 USA

Abstract. The Cosmic Background Explorer (COBE) satellite Differential Microwave Radiometer (DMR) instrument has recently reported the observation of very large scale structure in the microwave sky. The microwave sky as a whole is remarkably uniform in the millimeter to centimeter wavelength range. However, there are variations at a level $\leqslant 10^{-5}$. The natural interpretation of these variations is as the imprint of spatial curvature fluctuations in the early universe. The implications for gravitation and gravitational instability theories of structure formation are discussed along with a review of the experiment. The results are generally supportive of gravitational instability theories and inflationary/quantum cosmology models. Failure to find fluctuations within a factor of two of the COBE DMR level would have contradicted gravitational instability models with a scale-free approximately scale-invariant spectra.

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Gravitationally induced non-classical effects

Pedro F González-Díaz

Instituto de Optica, CSIC, Serrano 121, 28006 Madrid, Spain

The consistent Minkowskian Hamiltonian density describing the effective interaction between ordinary particles and baby universes which are branched off from the asymptotic regions through a wormhole with nonsimply connected inner topology can be generally expressed as [1].

$$H_{mb}(\beta) = H_m H_b(\beta) = -H_m(b^{\dagger 2} + b^2 - 2\beta b^\dagger b), \tag{1}$$

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Path integrals and wormholes in quantum gravity

Claus Kiefer

Institut für Theoretische Physik, der Universität Zürich, Schönberggasse 9, CH-8001, Zürich, Switzerland

Path integrals play a prominent part in many attempts to quantize gravity. While there is still no agreement on the general functional formulation, quantum gravitational path integrals have been applied, heuristically, in semiclassical situations as well as in simple models with finitely many degrees of freedom (minisuperspace models).

The discussion has mainly focused on two applications. The first one is concerned

Contents

Why Quantum Cosmology?

Gell-Mann and Hartle 1990:
Quantum mechanics is best and most fundamentally understood in the framework of quantum cosmology.

- Quantum theory is universally valid:
  Application to the Universe as a whole as the only closed quantum system in the strict sense
- Need quantum theory of gravity, since gravity dominates on large scales

Georges Lemaître 1931:

If the world has begun with a single quantum, the notions of space and time would altogether fail to have any meaning at the beginning . . . If this suggestion is correct, the beginning of the world happened a little before the beginning of space and time.

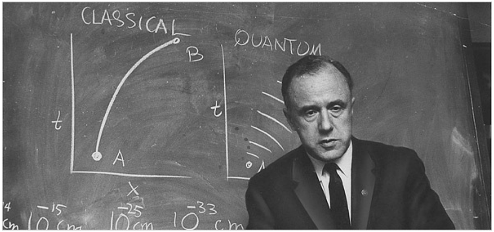
Main approaches to quantum gravity

No question about quantum gravity is more difficult than the question, "What is the question?"
(John Wheeler 1984)

- Quantum general relativity
  - Covariant approaches (perturbation theory, path integrals including spin foams, asymptotic safety, ...)
  - Canonical approaches (geometrodynamics, connection dynamics, loop dynamics, ...)
- String theory
- Fundamental discrete approaches
  (quantum topology, causal sets, group field theory, ...);
  have partially grown out of the other approaches

See e.g. C. Kiefer, *Quantum Gravity*, 3rd edition, Oxford 2012.

Quantum geometrodynamics



(a) John Archibald Wheeler

(b) Bryce DeWitt

Application of Schrödinger's procedure to general relativity leads to

$$\hat{H}\Psi \equiv \left(-16\pi G\hbar^2 G_{abcd}\frac{\delta^2}{\delta h_{ab}\delta h_{cd}} - (16\pi G)^{-1}\sqrt{h}\left({}^{(3)}R - 2\Lambda\right)\right)\Psi = 0$$

Wheeler–DeWitt equation

$$\hat{D}^a\Psi \equiv -2\nabla_b\frac{\hbar}{\mathrm{i}}\frac{\delta\Psi}{\delta h_{ab}} = 0$$

quantum diffeomorphism (momentum) constraint NO TIME!

Quantization of a Friedmann Universe

Closed Friedmann–Lemaître universe with scale factor a, containing a homogeneous massive scalar field ϕ (two-dimensional *minisuperspace*)

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\mathrm{d}\Omega_3^2$$

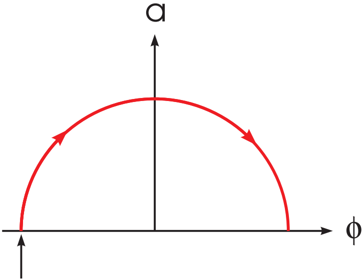
The Wheeler–DeWitt equation reads (with units $2G/3\pi = 1$)

$$\frac{1}{2}\left(\frac{\hbar^2}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{\hbar^2}{a^3}\frac{\partial^2}{\partial\phi^2} - a + \frac{\Lambda a^3}{3} + m^2a^3\phi^2\right)\psi(a,\phi) = 0$$

Factor ordering chosen in order to achieve covariance in minisuperspace

Determinism in classical and quantum theory

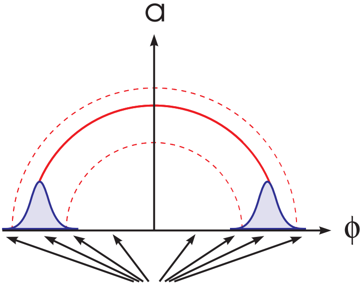
Classical theory



give e. g. here
initial conditions

Recollapsing part is deterministic successor of expanding part

Quantum theory



give initial conditions
on a=constant

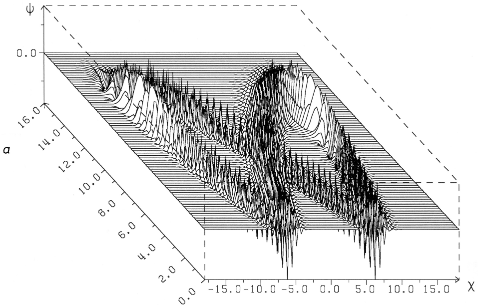
‘Recollapsing’ wave packet must be present ‘initially’

No intrinsic difference between ‘big bang’ and ‘big crunch’!

Example

Indefinite Oscillator

$$\hat{H}\psi(a,\chi) \equiv (-H_a + H_\chi)\psi \equiv \left(\frac{\partial^2}{\partial a^2} - \frac{\partial^2}{\partial \chi^2} - a^2 + \chi^2 \right) \psi = 0$$



C. K. (1990)

Singularity avoidance

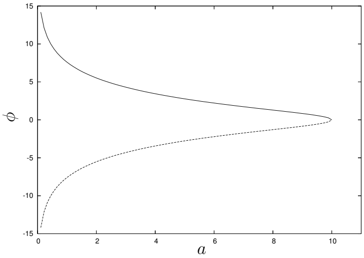
No general agreement on the criteria!

Sufficient criteria in quantum geometrodynamics:

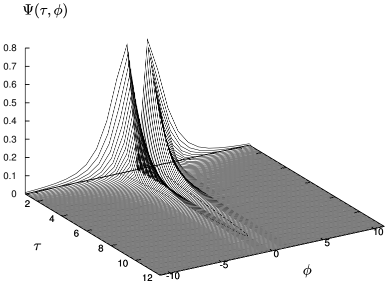
- Vanishing of the wave function at the point of the classical singularity (dating back to DeWitt 1967)
- Spreading of wave packets when approaching the region of the classical singularity
- (Semiclassical) time then comes to an end!

Example: big-brake cosmology

Normalizable solutions of the Wheeler–DeWitt equation vanish at the classical singularity



(c) Classical trajectory



(d) Wave packet

(Kamenshchik, C.K., Sandhöfer 2007)

The quantum cosmological path integral

Minisuperspace path integral:

$$\psi(a,\phi) = \int \mathrm{d}N \int \mathcal{D}a\mathcal{D}\phi \, \mathrm{e}^{-I[a(\tau),\phi(\tau),N]}$$

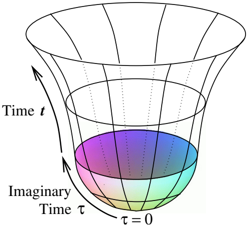
- Time (resp. shift N) is integrated over; this is an aspect of the timelessness of the theory.
- The quantum gravitational path integral is not a propagator and does not obey a composition law; it resembles an energy Green function.
- In general one has to integrate over complex contours.

No-boundary condition

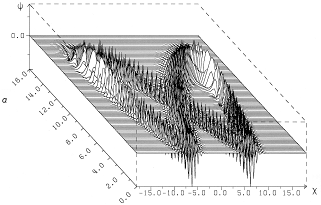
- “There ought to be something very special about the boundary conditions of the universe and what can be more special than the condition that there is no boundary.”
  (S. W. Hawking, Vatican conference, 1982)

  Saddle point approximation:

$$\psi_{\text{no-boundary}} \propto \left(a^2 V(\phi) - 1\right)^{-1/4} \exp\left(\frac{1}{3V(\phi)}\right) \cos\left(\frac{\left(a^2 V(\phi) - 1\right)^{3/2}}{3V(\phi)} - \frac{\pi}{4}\right)$$



Can one get wave-packet solutions from the path integral such as the following?



Answer: No! The no-boundary wave function either diverges at infinity or along the “lightcone” in minisuperspace. (C.K. 1991) The wave function therefore cannot reflect the classical behaviour.

Recent discussion

- Feldbrugge *et al*.: no-boundary wave function unstable under perturbations (using Picard-Lefschetz theory to define the Lorentzian path integral in a semiclassical expansion);
- Diaz Dorronsoro *et al*: the no-boundary proposal is stable under perturbations

- Tunneling condition: Only *outgoing* modes near singular boundaries of superspace (Vilenkin 1982 and others); e.g.

$$\psi_{\text{tunnel}} \propto (a^2V(\phi)-1)^{-1/4} \exp\left(-\frac{1}{3V(\phi)}\right) \exp\left(-\frac{\mathrm{i}}{3V(\phi)}(a^2V(\phi)-1)^{3/2}\right)$$

While the no-boundary state is *real*, the tunneling state is *complex* (distinguishes a direction in superspace).

However, without the reference phase $\exp(-\mathrm{i}Et/\hbar)$, the sign of the imaginary unit i has no intrinsic meaning (the word *tunneling* is thus only a metaphor)

Inflation from quantum cosmology?

Does one of these boundary conditions predict the occurrence of inflation?

- No-boundary condition: since $\psi_{\text{no-boundary}} \sim \exp\left(\frac{1}{3V(\phi)}\right)$, it favours *small* values of ϕ unsuitable for inflation
  (Hartle, Hawking, Hertog (2008): small amount of inflation possible after re-weighting the probability)
- Tunneling condition: since $\psi_{\text{tunnel}} \sim \exp\left(-\frac{1}{3V(\phi)}\right)$, it favours *large* values of ϕ potentially suitable for inflation

Beyond tree-level approximation?

- Barvinsky and Kamenshchik (1990):
  $\rho(\phi) \sim \mathrm{e}^{\pm I - \Gamma_{\text{1-loop}}} \sim \mathrm{e}^{\pm I} \phi^{-Z-2}$: normalizable state for $Z > -1$
- Barvinsky and Kamenshchik (1998): For non-mimimal coupling, the tunneling wave function is peaked around values suitable for inflation also at the one-loop order

Inflation from the tunnelling proposal

Higgs inflation Non-minimal coupling of the Standard-Model Higgs field to gravity; application of the above procedure leads to initial values of the Higgs field, which are high enough for inflation

(Barvinsky, Kamenshchik, C.K., Steinwachs 2010)

Natural inflation

$$V(\varphi) = \Lambda^4 \; [1 + \cos(\varphi/f)],$$

with $f = \mathcal{O}(M_{\mathrm{P}})$ and $\Lambda \approx M_{\mathrm{GUT}} \sim 10^{16}$ GeV

(Freese, Frieman, Olinto 1990)

Compatible with PLANCK data; sharp peak of the tunneling wave function already at the tree level

(Calcagni, C.K., Steinwachs 2014)

In quantum cosmology, arbitrary superpositions of the gravitational field and matter states can occur. How can we understand the emergence of an (approximate) classical Universe?

Decoherence in quantum cosmology

- ‘System’: global degrees of freedom (scale factor, inflaton field, …)
- ‘Environment’: small density fluctuations, gravitational waves, …

(Zeh 1986, C.K. 1987)

Example: scale factor a of a de Sitter universe ($a \propto \mathrm{e}^{H_{\mathrm{I}}t}$) (‘system’) experiences decoherence by gravitons (‘environment’) according to

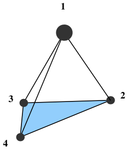
$$\rho_0(a,a') \to \rho_0(a,a')\exp\left(-CH_{\mathrm{I}}^3 a(a-a')^2\right)\ ,\ C>0$$

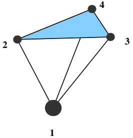
The Universe assumes classical properties at the beginning of inflation

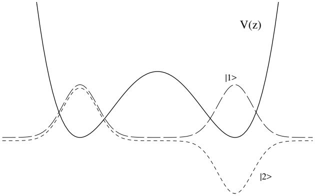
(Barvinsky, Kamenshchik, C.K. 1999)

Time from symmetry breaking

Analogy from molecular physics: emergence of chirality







dynamical origin: decoherence through scattering by light or air molecules

Quantum cosmology: decoherence between $\exp(\mathrm{i}S_0/G\hbar)$- and $\exp(-\mathrm{i}S_0/G\hbar)$-components of the wave function through interaction with e.g. weak gravitational waves

Example for decoherence factor:
$\exp\left(-\frac{\pi m H_0^2 a^3}{128\hbar}\right) \sim \exp\left(-10^{43}\right)$ (C.K. 1992)

Decoherence of primordial fluctuations

Because of decoherence, primordial quantum fluctuations from inflation behave like classical stochastic quantities and yield the seeds for the structures in the Universe.

See e.g. C.K. Polarski, Starobinsky (1998), . . .

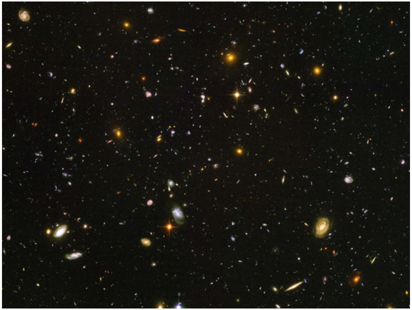


Figure credit: NASA, ESA

Interpretation of quantum cosmology

Both quantum general relativity and string theory preserve the linear structure for the quantum states;
strict validity of the superposition principle

only interpretations so far: Everett interpretation (with decoherence as an essential part) and Bohm interpretation

B. S. DeWitt 1967:
Everett's view of the world is a very natural one to adopt in the quantum theory of gravity, where one is accustomed to speak without embarassment of the 'wave function of the universe.' It is possible that Everett's view is not only natural but essential.

The direction of time

Paul Cézanne, *Nature morte au crâne* (Barnes Foundation, Pennsylvania)

Arrows of time

Almost all the fundamental laws of Nature are time-symmetric; but we observe classes of phenomena that distinguish a direction of time:

- Radiation (advanced versus retarded)
- Thermodynamics (increase of entropy)
- Quantum theory (measurement process)
- Gravity (expansion of the Universe;
  formation of structure; black holes)
- …

Master arrow of time?

Arrow of time from cosmology

Where does the Sun come from?

$\Downarrow$

Gravitational instability of dust clouds

$\Downarrow$

Cosmology

Ludwig Boltzmann (1898):

That in Nature the transition from a probable to an improbable state does not happen equally often as the opposite transition, should be sufficiently explained by the assumption of a very improbable initial state of the whole Universe surrounding us . . . .

How special is the Universe?

Penrose (1981):

Entropy of the observed part of the Universe is maximal if all its mass is in one black hole; the probability for our Universe would then be (updated version from C.K. arXiv:0910.5836)

$$\frac{\exp\left(\frac{S}{k_{\mathrm{B}}}\right)}{\exp\left(\frac{S_{\mathrm{max}}}{k_{\mathrm{B}}}\right)} \sim \frac{\exp\left(3.1\times 10^{104}\right)}{\exp\left(1.8\times 10^{121}\right)} \approx \exp\left(-1.8\times 10^{121}\right)$$

Arrow of time from quantum cosmology

Fundamental asymmetry with respect to "‘intrinsic time"’:

$$\hat{H}\Psi = \left(\frac{\partial^2}{\partial\alpha^2} + \sum_i \left[-\frac{\partial^2}{\partial x_i^2} + \underbrace{V_i(\alpha, x_i)}_{\to 0 \text{ for } \alpha \to -\infty} \right] \right) \Psi = 0$$

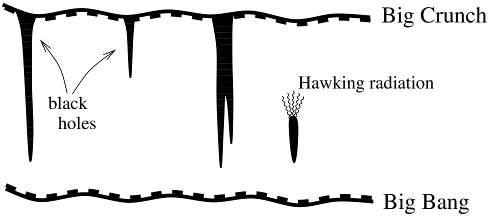
Is compatible with simple boundary condition:

$$\Psi \overset{\alpha \to -\infty}{\longrightarrow} \psi_0(\alpha) \prod_i \psi_i(x_i)$$

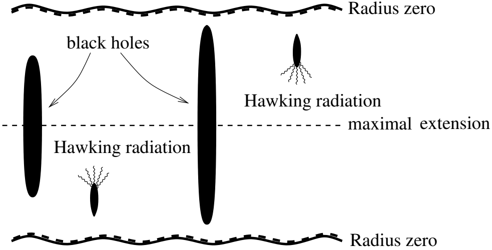
Entropy increases with increasing α, since entanglement with other degrees of freedom increases;
this defines the direction of time

Is the expansion of the Universe a tautology?

Arrow of time in a recollapsing quantum universe



(Penrose 1979)



(C.K. and Zeh 1995)

John Wheeler (1968):

These considerations reveal that the concepts of spacetime and time itself are not primary but secondary ideas in the structure of physical theory. These concepts are valid in the classical approximation. However, they have neither meaning nor application under circumstances when quantum-geometrodynamical effects become important. . . . There is no spacetime, there is no time, there is no before, there is no after. The question what happens “next” is without meaning.

Bryce DeWitt (1999):

. . . one learns that time and probability are both *phenomenological* concepts.