From Serrano street to the current Universe

Travelling through Pedro's universes
@ Madrid (Spain)

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ikerbasque Basque Foundation for Science

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You need not be afraid of phantom energy

Pedro F. González-Díaz
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(Dated: August 16, 2018)

Phantom energy which violates the dominant-energy condition and is not excluded by current constraints on the equation of state may be dominating the evolution of the universe now. It has been pointed out that in such a case the fate of the universe may be a big rid where the expansion is so violent that all galaxies, planet and even atomic nuclei will be successively ripped apart in finite time. Here we show however that there are certain unified models for dark energy which are stable to perturbations in matter density where the presence of phantom energy does not lead to such a cosmic doomsday.

PACS numbers: 98.80.-k, 98.80.Es

I. INTRODUCTION

WMAP [1] has confirmed it with the highest accuracy: Nearly seventy percent of the energy in the universe is in the form of dark energy -possibly one of most astonishing discoveries ever made in science. Moreover, recent observations do not exclude, but actually suggest a value even smaller than -1 for the parameter of the equation of state, ω, characterizing that dark energy [2]. That means that for at least a perfect-fluid equation of state, the absolute value for negative pressure exceeds that for positive energy density, i.e. $\rho + p < 0$, and hence it follows that the involved violation of the dominant-energy condition might allow the existence of astrophysical or cosmologi-

haviour of the universe after the big rip is in some respects even more bizarre than the big rip itself, as its size then steadily decreases from infinite down to zero at infinite time. In case that a generic perfect-fluid equation of state, $p = \omega\rho$, with $\omega < -1$ is considered, the above new behaviours show themselves immediately. For flat geometry, the scale factor is then given by [4]:

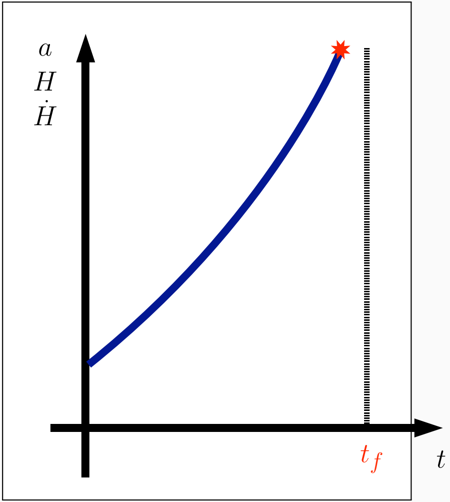
$$a(t) \propto \left[e^{C_1(1+\omega)t} - C_2 e^{-C_1(1+\omega)t}\right]^{2/[3(1+\omega)]}, \quad (1.1)$$

with $C_1 > 0$ and $0 < C_2 < 1$. We note that for $\omega < -1$, in fact $a \to \infty$ as $t \to t_* = \ln C_2/[C_1(1+\omega)]$. This marks the time at big rip and the onset of the contracting phase for $t > t_*$.

P. F. González-Díaz, PRD (RC) [arXiv:astro-ph/0305559]

Big rip singularity

- For this singularity the null energy condition is violated. The scale factor diverges in a finite time. It is accompanied with a divergence of the Hubble rate and the cosmic derivative of the Hubble rate
- For example this singularity might appear in a hologrphic Ricci dark energy model (C.Gao et al PRD, [arXiv:0712.1394])



Starobinsky, Grav. Cosmol. [astro-ph/9912054], Caldwell, PLB [astro-ph/9908168], Caldwell, Kamionkowski and Weinberg, PRL [astro-ph/0301273]

K-essential phantom energy: Doomsday around the corner? (his most cited work on phantom energy)

K-Essential Phantom Energy: Doomsday around the corner?

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December 1, 2003

Abstract

In spite of its rather weird properties which include violation of the dominant-energy condition, the requirement of superluminal sound speed and increasing vacuum-energy density, phantom energy has recently attracted a lot of scientific and popular interests. In this letter it is shown that in the framework of a general k-essence model, vacuum-phantom energy leads to a cosmological scenario having negative sound speed and a big-rip singularity, where the field potential also blows up, which might occur at an almost arbitrarily near time in the future that can still be comfortably accommodated within current observational constraints.

P. F. González-Díaz, PLB [arXiv:astro-ph/0312579]

Escaping the Big Rip?-1-

You need not be afraid of phantom energy

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haviour of the universe after the big rip is in some respects even more bizarre than the big rip itself, as its size then steadily decreases from infinite down to zero at infinite time. In case that a generic perfect-fluid equation of state, $p = \omega\rho$, with $\omega < -1$ is considered, the above new behaviours show themselves immediately. For flat geometry, the scale factor is then given by [4]:

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[3] R.R. Caldwell, M. Kamionkowski and N.N. Weinberg, astro-ph/0302506; G.W. Gibbons, hep-th/0302199; R.R. Caldwell, Phys. Lett. B545, 23 (2002).
[4] M. Bouhmadi, personal communication, paper in preparation.

P. F. González-Díaz, PRD (RC) [arXiv:astro-ph/0305559]

Escaping the Big Rip?-2-

Escaping the Big Rip?

Mariam Bouhmadi-López[1] and José A. Jiménez Madrid[2]

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(Dated: April 27, 2004)

We discuss dark energy models which might describe effectively the actual acceleration of the universe. More precisely, for a 4-dimensional Friedmann-Lemaître-Robertson-Walker (FLRW) universe we consider two situations: First of them, we model dark energy by phantom energy described by a perfect fluid satisfying the equation of state $P = (\beta - 1)\rho$ (with $\beta < 0$ and constant). In this case the universe reaches a "Big Rip" independently of the spatial geometry of the FLRW universe. In the second situation, the dark energy is described by a phantom (generalized) Chaplygin gas which violates the dominant energy condition. Contrary to the previous case, for this material content a FLRW universe would never reach a "big rip" singularity (indeed, the geometry is asymptotically de Sitter). We also show how this dark energy model can be described in terms of scalar fields, corresponding to a minimally coupled scalar field, a Born-Infeld scalar field and a generalized Born-Infeld scalar field. Finally, we introduce a phenomenologically viable model where dark energy is described by a phantom generalized Chaplygin gas.

PACS numbers: 98.80.-k,98.80.Es,11.10.-z

PU-ICG-04/09, astro-ph/0404540

M. B.-L. and J.A. Jiménez-Madrid, JCAP [arXiv:astro-ph/0404540]

Worse than a big rip?

2006

Worse than a big rip?

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(Dated: November 28, 2018)

We show that a generalised phantom Chaplygin gas can present a future singularity in a finite future cosmic time. Like the big rip singularity, this singularity would also take place at a finite future cosmic time, but unlike the big rip singularity, it happens for a finite scale factor. In addition, we define a dual of the generalised phantom Chaplygin gas which satisfies the null energy condition. Then, in a Randall-Sundrum 1 brane-world scenario, we show that the same kind of singularity at a finite scale factor arises for a brane filled with a dual of the generalised phantom Chaplygin gas.

M. B.-L., F. González-Díaz and P. Martín-Moruno, PLB [arXiv:gr-qc/0612135]

Summary of DE singularities-1-

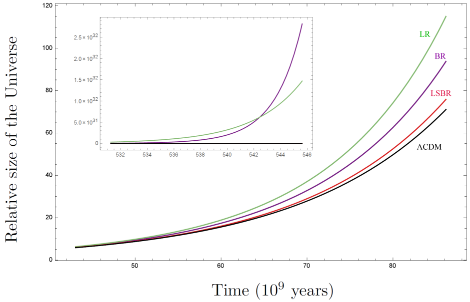
DE might induce a future cosmic singularity

Some of the cosmological parameters:

- t → Cosmic time
- a → Scale factor (relative size)
- H → Hubble parameter (growth rate)
- $\dot{H}$ → Time derivative of H

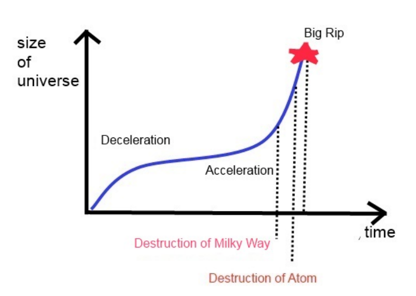
Singularity	t	a	H	$\dot{H}$	$\ddot{H}, \dddot{H}$ ...
Big Bang	0	0	∞	∞	∞
De Sitter (ΛCDM)	∞	∞	H_{ds}	0	0
Big Rip	t_s	∞	∞	∞	∞
LR	∞	∞	∞	∞	∞
LSBR	∞	∞	∞	$\dot{H}_s$	0
Big Freeze	t_s	a_s	∞	∞	∞
Sudden. S.	t_s	a_s	H_s	∞	∞
Type IV	t_s	a_s	H_s	$\dot{H}_s$	∞

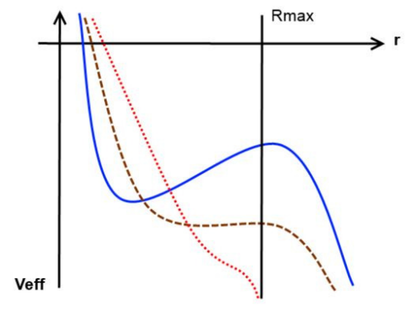
Asymptotic evolution of the scale factor



Nojiri, Odintsov and Tsujikawa, Phys. Rev. D 71 (2005) 06300, [hep-th/0501025].

Summary of DE singularities-2-





Pictures drawn by Che-Yu Chen (NTU)

Where are we standing now?

A brief sketch of the universe-1-

- The universe is homogeneous and isotropic on large scales (cosmological principle)
- The matter content of the universe:
  - Standard matter
  - Dark matter
  - Something that induce the late-time acceleration of the Universe
- The acceleration of the universe is backed by several measurments: $H(z)$, SneIa, BAO, CMB, LSS (matter power spectrum, growth function)...

A brief sketch of the universe-2-

- The effective equation of state of whatever is driving the current speed up of the universe is roughly -1. For example, for a wCDM model with w constant and $k = 0$, Planck (TT, TE, EE+lensing) + ext(BAO,H0,SNeIa) results implies w is very close to -1
- Such an acceleration could be due
  - A new component of the energy budget of the universe: dark energy; i.e. it could be Λ, quintessence or of a phantom(-like/effective) nature
  - A change on the behaviour of gravity on the largest scale. No new component on the budget of the universe but rather simply GR modifies its behaviour, within a metric, Palatini (affine metric) ....

A brief sketch of the universe-3-

2.16 base_plikHM_TT_lowl_lowE_lensing_post_BAO_Pantheon_zre6p5

Parameter	68% limits	Parameter	68% limits	Parameter	68% limits
$\mathbf{\Omega_b h^2}$	0.02222 ± 0.00019	$r_{drag}h$	99.78 ± 0.80	$H(0.51)$	89.67 ± 0.26
$\mathbf{\Omega_c h^2}$	0.1189 ± 0.0010	$\langle d^2 \rangle^{1/2}$	2.435 ± 0.021	$D_M(0.51)$	1981.5 ± 9.5
$\mathbf{100\theta_{MC}}$	1.04100 ± 0.00042	z_{re}	$7.88^{+0.62}_{-0.76}$	$H(0.61)$	95.27 ± 0.22
$\boldsymbol{\tau}$	$0.0563^{+0.0058}_{-0.0077}$	$10^9 A_s$	$2.102^{+0.025}_{-0.033}$	$D_M(0.61)$	2306 ± 10
$\mathbf{\ln(10^{10} A_s)}$	$3.046^{+0.012}_{-0.015}$	$10^9 A_s e^{-2\tau}$	1.878 ± 0.010	$H(2.33)$	235.72 ± 0.68
$\mathbf{n_s}$	0.9664 ± 0.0040	D_{40}	1227 ± 12	$D_M(2.33)$	5767 ± 11
$\mathbf{y_{cal}}$	1.0008 ± 0.0025	D_{220}	5725 ± 39	$f\sigma_8(0.15)$	0.4553 ± 0.0058
$\mathbf{A^{CIB}_{217}}$	48 ± 7	D_{810}	2537 ± 13	$\sigma_8(0.15)$	$0.7480^{+0.0049}_{-0.0058}$
$\boldsymbol{\xi}^{\mathbf{tSZ\times CIB}}$	—	D_{1420}	815.8 ± 4.9	$f\sigma_8(0.38)$	0.4740 ± 0.0049
$\mathbf{A^{tSZ}_{143}}$	$5.1^{+2.2}_{-2.0}$	D_{2000}	230.1 ± 1.7	$\sigma_8(0.38)$	$0.6632^{+0.0042}_{-0.0051}$
$\mathbf{A^{PS}_{100}}$	263 ± 28	$n_{s,0.002}$	0.9664 ± 0.0040	$f\sigma_8(0.51)$	0.4727 ± 0.0043
$\mathbf{A^{PS}_{143}}$	48 ± 8	Y_P	$0.245332^{+0.000084}_{-0.000075}$	$\sigma_8(0.51)$	$0.6207^{+0.0039}_{-0.0048}$
$\mathbf{A^{PS}_{143\times217}}$	43 ± 9	Y_P^{BBN}	$0.246658^{+0.000084}_{-0.000075}$	$f\sigma_8(0.61)$	0.4679 ± 0.0040
$\mathbf{A^{PS}_{217}}$	115 ± 10	$10^5 D/H$	2.614 ± 0.036	$\sigma_8(0.61)$	$0.5906^{+0.0037}_{-0.0046}$
$\mathbf{A^{kSZ}}$	< 4.79	Age/Gyr	13.806 ± 0.026	$f\sigma_8(2.33)$	$0.2979^{+0.0019}_{-0.0024}$
$\mathbf{A^{dust TT}_{100}}$	8.9 ± 1.8	z_*	1090.02 ± 0.27	$\sigma_8(2.33)$	$0.3071^{+0.0020}_{-0.0026}$
$\mathbf{A^{dust TT}_{143}}$	10.7 ± 1.8	r_*	144.82 ± 0.28	f^{143}_{2000}	30.9 ± 2.9
$\mathbf{A^{dust TT}_{143\times217}}$	18.3 ± 3.3	$100\theta_*$	1.04120 ± 0.00041	$f^{143\times217}_{2000}$	33.3 ± 2.0
$\mathbf{A^{dust TT}_{217}}$	93.3 ± 7.4	$D_M(z_*)/\mathrm{Gpc}$	13.909 ± 0.028	f^{217}_{2000}	107.9 ± 1.9
$\mathbf{c_{100}}$	0.99963 ± 0.00062	z_{drag}	1059.51 ± 0.43	$\chi^2_{lensing}$	9.23 ± 0.69
$\mathbf{c_{217}}$	0.99826 ± 0.00062	r_{drag}	147.54 ± 0.32	χ^2_{simall}	397.2 ± 1.9
H_0	67.63 ± 0.47	k_D	0.14028 ± 0.00043	χ^2_{lowl}	23.17 ± 0.86
Ω_Λ	0.6899 ± 0.0062	$100\theta_D$	0.16101 ± 0.00025	χ^2_{plik}	771.7 ± 5.2
Ω_m	0.3101 ± 0.0062	z_{eq}	3373 ± 25	χ^2_{JLA}	1036.34 ± 0.32
$\Omega_m h^2$	0.1418 ± 0.0010	k_{eq}	0.010296 ± 0.000075	χ^2_{6DF}	0.047 ± 0.059
$\Omega_m h^3$	0.09590 ± 0.00045	$100\theta_{eq}$	0.8182 ± 0.0045	χ^2_{MGS}	1.35 ± 0.45
σ_8	$0.8093^{+0.0055}_{-0.0063}$	$100\theta_{s,eq}$	0.4520 ± 0.0023	$\chi^2_{DR12BAO}$	4.6 ± 1.3
S_8	0.823 ± 0.011	$H(0.15)$	72.89 ± 0.40	χ^2_{prior}	7.3 ± 3.6
$\sigma_8 \Omega_m^{0.5}$	0.4506 ± 0.0062	$D_M(0.15)$	641.1 ± 4.0	χ^2_{CMB}	1201.3 ± 5.4
$\sigma_8 \Omega_m^{0.25}$	0.6039 ± 0.0060	$H(0.38)$	82.97 ± 0.31	χ^2_{BAO}	6.0 ± 1.0
$\sigma_8 / h^{0.5}$	0.9841 ± 0.0086	$D_M(0.38)$	1529.4 ± 8.1		

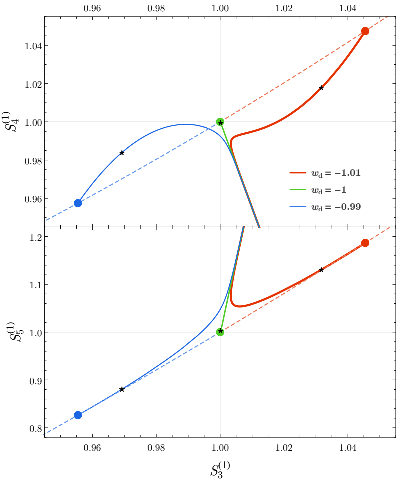
$\bar{\chi}^2_{eff} = 2250.91$; $R - 1 = 0.01942$

http://wiki.cosmos.esa.int/planck-legacy-archive

The late-universe: simplest approach within GR

Constant equation of state for DE: background

- State finders approach (Sahni, Saini and Starobinsky JETP Lett. [arXiv:astro-ph/0201498])
  - Scale factor: $\frac{a(t)}{a_0} = 1 + \sum_{n=1}^{\infty} \frac{A_n(t_0)}{n!} \left[H_0\,(t - t_0)\right]^n$, where $A_n := a^{(n)}/(a\,H^n)$, $n \in \mathbb{N}$.
  - State finders parameters: $S_3^{(1)} = A_3$, $S_4^{(1)} = A_4 + 3\,(1 - A_2)$, $S_5^{(1)} = A_5 - 2\,(4 - 3A_2)\,(1 - A_2)$
  - $\Omega_{\rm m} = 0.309$, $\Omega_{\rm d} = 0.691$ and $H_0 = 67.74$ km s^{-1} Mpc^{-1} (according to Planck).



Albarran, B.L. and Morais, EPJC, [arXiv:1706.01484]

Constant equation of state for DE: perturbations-1-

- The backgroud description of the late universe is very important but we need to go beyond. Why? Many reasons, let me just state a few:
    - Observational reasons: Despite the huge importance of measurments of SNeIa (luminosity distance), BAO (for example, the angular diamamter distance), H_0 and $H(z)$ to constrain the late-time cosmology of the universe, there are equally important measurments like the matter power spectrum, $f\sigma_8$ (...) that can help us as well to constrain or rule out some of the current models in the market.
    - Previous experience "The primordial inflationary epoch": While many models can describe the inflationary era at the background level only some can predict an observationally consistent spectral index and tilt for the curvature perturbations.
    - A perturbative approach has been quite efficient to disregard some extended theories of gravity or even DE models within GR.

Constant equation of state for DE: perturbations-2-

Consider the gauge invariant perturbed FLRW metric

$$ds^2 = a^2 \left[-(1+2\Phi) d\eta^2 + (1-2\Psi)\delta_{ij} dx^i dx^j \right] ,$$

where Φ and Ψ are the Bardeen potentials.

In the matter sector the perturbed energy momentum tensor is

$$\begin{aligned} {T^{(m)}}^0{}_0 &= -(\rho_m + \delta\rho_m) \,, \\ {T^{(m)}}^0{}_i &= -(\rho_m + P_m)\partial_i v_m \,, \\ {T^{(m)}}^i{}_j &= (P_m + \delta P_m)\delta^i_j \,, \end{aligned}$$

Constant equation of state for DE: perturbations-3-

- We worked on the Newtonian gauge and carried the first order perturbations considering DM, DE and radiation on GR. Radiation was included because our numerical integrations start from well inside the radiation dominated epoch
- We assumed initial adiabatic conditions for the different fractional energy density perturbations
- The total fractional energy density is fixed by Planck measurments; i.e. through A_s and n_s
- The speed of sound for DE:
  - The pressure perturbation of DE reads: $\delta p_d = c_{sd}^2 \delta\rho_d - 3\mathcal{H}\left(1+w_d\right)\left(c_{sd}^2 - c_{ad}^2\right)\rho_d v_d$, where $c_{sd}^2 = \left.\frac{\delta p_d}{\delta \rho_d}\right|_{r.f.}$ and $c_{aA}^2 = \frac{p_d'}{\rho_d'} = w$ (for these simple models)
  - Given that c_{sd}^2 is negative, we can end up with a problem (this is not intrisic to phantom matter as it can happen for example with fluids with a negative constant equation of state larger than -1)
  - We choose $c_{sd}^2 = 1$ as a phenomenological parameter

Constant equation of state for DE: perturbations-4-

- Evolution equations for EMT of the different components

$$
\begin{aligned}
\delta_{\rm r}' &= 4\left(\frac{k^2}{3}v_{\rm r}+\Phi'\right)\,, \\
v_{\rm r}' &= -\left(\frac{1}{4}\delta_{\rm r}+\Phi\right)\,, \\
\delta_{\rm m}' &= 3\left(\frac{k^2}{3}v_{\rm m}+\Phi'\right)\,, \\
v_{\rm m}' &= -\left(\mathcal{H}v_{\rm m}+\Phi\right)\,, \\
\delta_{\rm d}' &= 3\,(w-1)\,\delta_{\rm d}+3\,(1+w)\left\{\left[\frac{k^2}{3}+3\mathcal{H}^2\,(1-w)\right]v_{\rm d}+\Phi'\right\}\,, \\
v_{\rm d}' &= -\left(\frac{1}{1+w}\delta_{\rm d}+\Phi\right)+2\mathcal{H}v_{\rm d}\,,
\end{aligned}
$$

Constant equation of state for DE: perturbations-5-

- Evolution equations of the gravitational sector

$$\begin{aligned}\mathcal{H}\Phi' + \left(\mathcal{H}^2 + \frac{k^2}{3}\right)\Phi &= -\frac{1}{2}\mathcal{H}^2\delta_{\rm tot}\,,\\ \Phi' + \mathcal{H}\Phi &= -\frac{3}{2}\mathcal{H}^2\left(1 + w_{\rm tot}\right)v_{\rm tot}\,.\end{aligned}$$

where

$$\begin{aligned}w_{\rm tot} &= \frac{\sum_{\rm i=r,m,d}\rho_{\rm i}\,w_{\rm i}}{\sum_{\rm i=r,m,d}\rho_{\rm i}}\,,\\ \delta_{\rm tot} &= \frac{\sum_{\rm i=r,m,d}\rho_{\rm i}\,\delta_{\rm i}}{\sum_{\rm i=r,m,d}\rho_{\rm i}}\,,\\ v_{\rm tot} &= \frac{\sum_{\rm i=r,m,d}\rho_{\rm i}\left(1 + w_{\rm i}\right)\,v_{\rm i}}{\sum_{\rm i=r,m,d}\rho_{\rm i}\left(1 + w_{\rm i}\right)}\,.\end{aligned}$$

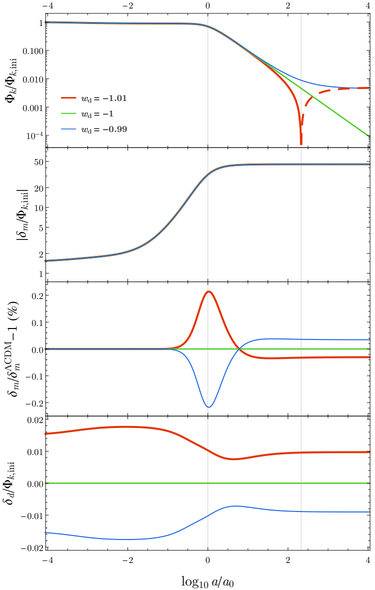
Constant equation of state for DE: perturbations-6-

- Initial conditions for δ are fixed through the amplitude and spectral index of the primordial inflationary power spectrum: $A_s = 2.143 \times 10^{-9}$, $n_s = 0.9681$ and $k_* = 0.05$ Mpc^{-1} (Planck values): $\Phi_{\rm ini} = \frac{2\pi}{3}\sqrt{2A_s}\left(\frac{k}{k_*}\right)^{n_s-1} k^{-3/2}$
- Well inside the radiation era: $\Phi_{\rm ini} \approx -\frac{1}{2}\delta_{\rm tot,ini}$ and $\Phi_{\rm ini} \approx -2\mathcal{H}_{\rm ini} v_{\rm tot,ini}$
- Initial adiabatic conditions (radiation dominated epoch):

$$\frac{3}{4}\delta_{\rm r,ini} = \delta_{\rm m,ini} = \frac{\delta_{\rm d,ini}}{1 + w_{\rm d,ini}} \approx \frac{3}{4}\delta_{\rm ini}$$

$$v_{\rm r,ini} = v_{\rm m,ini} = v_{\rm d,ini} \approx \frac{\delta_{\rm ini}}{4\mathcal{H}_{\rm ini}}$$

Results-1-: An example



- Example of the evolution of the perturbations: $k = 10^{-3}$ Mpc^{-1}
- ΛCDM model: Φ_k vanishes asymptotically
- Phantom model: Φ_k also evolves towards a constant in the far future but a change of sign occurs roughly at $\log_{10} a/a_0 \simeq 2.33$, corresponding to 8.84×10^{10} years in the future. A dashed line indicates negative values of Φ_k
- Quintessence model: Φ_k evolves towards a constant in the far future without changing sign

Results-2-:

- What about $f\sigma_8$ for the three different DE models?

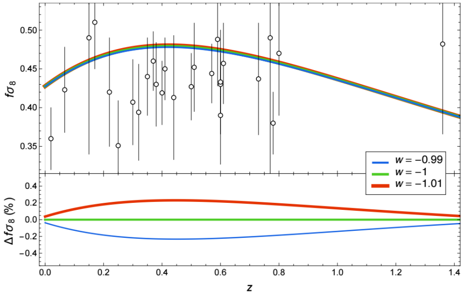


Figure 2: (Top panel) evolution of $f\sigma_8$ for low red-shift $z \in (0, 1.4)$ for three dark energy models: (blue) $w = -0.99$, (green) $w = -1$ and (red) $w = -1.01$. White circles and vertical bars indicate the available data points and corresponding error bars (cf. Table I of [13]). (Bottom panel) evolution of the relative differences of $f\sigma_8$ for each model with regard to ΛCDM ($w = -1$). $\Delta f\sigma_8$ is positive in the phantom case and negative in the quintessence case. For all the models, it was considered that σ_8 evolves linearly with δ_m and that $\sigma_8 = 0.816$ at the present time [7].

$$f \equiv \frac{d\left(\ln \delta_{\rm m}\right)}{d\left(\ln a\right)}, \qquad \sigma_8\left(z,\ k_{\sigma_8}\right) = \sigma_8\left(0,\ k_{\sigma_8}\right)\frac{\delta_{\rm m}\left(z,\ k_{\sigma_8}\right)}{\delta_{\rm m}\left(0,\ k_{\sigma_8}\right)}$$

$k_{\sigma_8} = 0.125$ hMpc^{-1}, $\sigma_8(0, k_{\sigma_8}) = 0.820$ (Planck)

Albarran, B.L. and Morais, EPJC [arXiv:1706.01484]

The late-universe through modified gravity: an example

$f(R)$-gravity: The action

The Einstein-Hilbert action is replaced by

$$S^{(f)} = \frac{1}{2\kappa^2} \int \mathrm{d}^4\mathbf{x} \sqrt{-g} f(R) \,, \qquad \kappa^2 = 8\pi G$$

where $f(R)$ is a generic function of the Ricci scalar R.

Among the general class of Modified Theories of Gravity, $f(R)$-gravity has been one of the most studied cases

- The simple equations of motion are easy to study;
- $f(R)$-gravity already captures interesting effects of MTG (e.g. the Starobinsky model for R^2 inflation);
- Possibility to avoid the solar system constraints (e.g. through the chameleon mechanism).

Nojiri and Odintsov, Int. J. Geom. Meth. Mod. Phys. 4, 115 (2007) [hep-th/0601213], Capozziello and De Laurentis, Phys. Rept. 509, 167 (2011) [arXiv:1108.6266 [gr-qc]].

$f(R)$-gravity: The different formalisms

Within the $f(R)$-theories of gravity we can identify different classes:

- metric formalism: the metric includes all the dynamical degrees of freedom.
- Palatini formalism: the connection Γ is independent of the metric g.
- metric-affine formalism: like the Palatini formalism but the matter Lagrangian density depends on both the metric and the connection.
- hybrid metric-Palatini formalism: a mixed action with terms that depend on the metric and the Levi-Civita connection and terms that depend on the metric and the independent connection.

$f(R)$-gravity: The modified Einstein equations

In the metric formalism, variation of the gravitational action with regards to the metric $g^{\mu\nu}$ leads to the modified Einstein equations:

$$R_{\mu\nu} f_R - \frac{1}{2} g_{\mu\nu} f - \left(\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box\right) f_R = \kappa^2 {T^{(m)}}_{\mu\nu} \,,$$

$${T^{(m)}}_{\mu\nu} := \frac{-2}{\sqrt{-g}} \frac{\delta \mathcal{L}^m}{\delta g^{\mu\nu}} \qquad \Box := g_{\mu\nu} \nabla_\mu \nabla_\nu \qquad f_R := \frac{df}{dR} \,.$$

The absence of extra coupling with the matter sector means that the usual matter conservation is verified

$$\nabla_\mu {T^{(m)}}^\mu_{\ \nu} = 0$$

$f(R)$-gravity: Cosmology

Consider the FLRW metric

$$ds^2 = -dt^2 + \frac{\delta_{ij} dx^i dx^j}{1 - \frac{\mathcal{K}}{4}\left(x^2 + y^2 + z^2\right)}\,, \qquad \mathcal{K} = -1, 0, +1\,.$$

Taking the (00) component of Einstein equation, we get the Friedmann equation

$$3\left(H^2 + \frac{\mathcal{K}}{a^2}\right) f_R + \frac{1}{2}\left(f - f_R R\right) + 3H\dot{R} f_{RR} = \kappa^2 \rho_m\,,$$

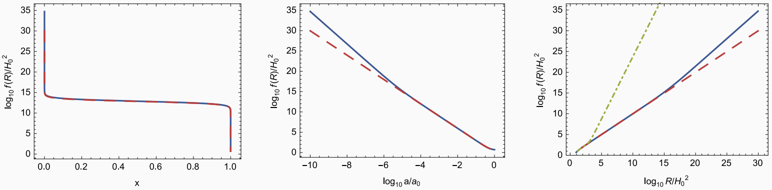
complemented by the conservation equation

$$\dot{\rho}_m + 3H\left(\rho_m + P_m\right) = 0\,,$$

where $\dot{x} := (\partial x)/(\partial t)$.

Reconstruction in $f(R)$: Chaplygin Gas as an example

Numerical solution where $f(R)$ plays the role of DE. The model contains as well DM and rad. We choose a mGCG that interpolates between radiation an a cosmological constant



Blue line indicates the numerical result while red line indicates the Einstein-Hilbert action.

Cosm. Perturbations: Perturbed Quantities

Consider the gauge invariant perturbed FLRW metric

$$ds^2 = a^2 \left[-(1 + 2\Phi)d\eta^2 + (1 - 2\Psi)\delta_{ij}dx^i dx^j\right] ,$$

where Φ and Ψ are the Bardeen potentials.

In the matter sector the perturbed energy momentum tensor is

$$\begin{aligned} {T^{(m)}}^0{}_0 &= -(\rho_m + \delta\rho_m) , \\ {T^{(m)}}^0{}_i &= -(\rho_m + P_m)\partial_i v_m , \\ {T^{(m)}}^i{}_j &= (P_m + \delta P_m)\delta^i_j , \end{aligned}$$

Cosm. Perturbations: GR vs $f(R)$

Taking into account the (ij), with $i \neq j$, component of the perturbed Einstein equations

$$\Phi - \Psi = \frac{\delta f_R}{f_R}$$

In GR with isotropic fluids the two Bardeen potentials are equal.

In $f(R)$ the existence of a new degree of freedom δf_R breaks the equality between the potentials.

The evolution equations of the metric perturbations can be obatined form the (00) and (0i) components of the Einstein perturbed equations

$$\left(\Psi^+\right)_N = -\Psi^+ - \frac{1}{4}\frac{(f_R)_N}{f_R}\left(2\Psi^+ - 3\Xi\right) - \frac{a^2\kappa^2(\rho_m + P_m)}{2f_R}\frac{v_m}{\mathcal{H}}\,,$$

$$\begin{aligned}(\Xi)_N &= \Xi + \frac{a^2\kappa^2(\rho_m + P_m)}{f_R}\frac{v_m}{\mathcal{H}} + \frac{1}{2}\frac{(f_R)_N}{f_R}\left(2\Psi^+ - 3\Xi\right) \\ &\quad -\frac{2}{3\mathcal{H}^2}\frac{f_R}{(f_R)_N}\left[2k^2\Psi^+ + \frac{a^2\kappa^2}{f_R}\left(\delta\rho_m - 3\mathcal{H}(\rho_m + P_m)v_m\right)\right] \\ &\quad +\frac{2}{\mathcal{H}}\frac{f_R}{(f_R)_N}\left(\mathcal{H} - \mathcal{H}_N\right)\Xi\,.\end{aligned}$$

where

$$\Psi^+ := \frac{\Phi + \Psi}{2}\,, \quad \Xi := \Phi - \Psi = \frac{\delta f_R}{f_R}\,. \quad \mathcal{H} := \frac{1}{a}\frac{\partial a}{\partial \eta}\,. \quad x_N := \frac{\partial x}{\partial \log(a/a_0)}\,.$$

In GR these two equations reduce to one constraint $\Psi^+(\delta\rho_m, v_m)$.

Cosm. Perturbations: Matter perturbations

For each individual fluid we can define peculiar velocity v_i and a relative energy density perturbation $\delta_i := \delta\rho_i/\rho_i$.

The total variables are related the individual fluid quantities by

$$\delta_m = \frac{\sum \rho_i \delta_i}{\sum \rho_i}\,, \qquad v_m = \frac{\sum(\rho_i + P_i)v_i}{\sum \rho_i + P_i}$$

The evolution equations for each fluid are the same as in GR:

$$(\delta_i)_N + 3\left(c_{s,i}^2 - w_i\right)\delta_i - (1+w_i)k^2\frac{v_i}{\mathcal{H}} = 3(1+w_i)\left(\Psi^+ + \frac{1}{2}\Xi\right)_N\,,$$

$$(v_i)_N + \left(1 - 3c_{s,i}^2\right)v_i + \frac{c_{s,i}^2}{1+w_i}\frac{\delta_i}{\mathcal{H}} = -\frac{1}{\mathcal{H}}\left(\Psi^+ - \frac{1}{2}\Xi\right).$$

where $w_i := P_i/\rho_i$ and $c_{s,i}^2 := \delta P_i/\delta\rho_i = \dot{P}_m/\dot{\rho}_m$

To obtain the evolution of the linear perturbations we evolve numerically the quantities

$$\Psi^{+}\,, \quad \Xi\,, \quad \delta_{dust}\,, \quad v_{dust}\,, \quad \delta_{rad}\,, \quad v_{rad}\,. \tag{1}$$

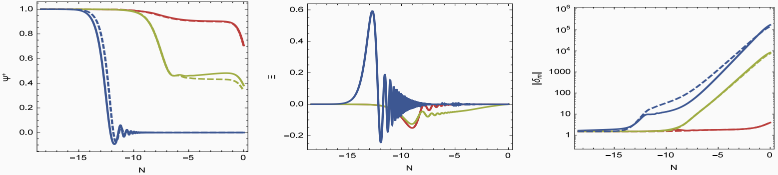
The initial conditions are set deep in the radiation dominated epoch, when all relevant modes are outside the horizon.

We consider adiabatic initial conditions.

No approximations are employed in the numerical integration of the equations until the present time.

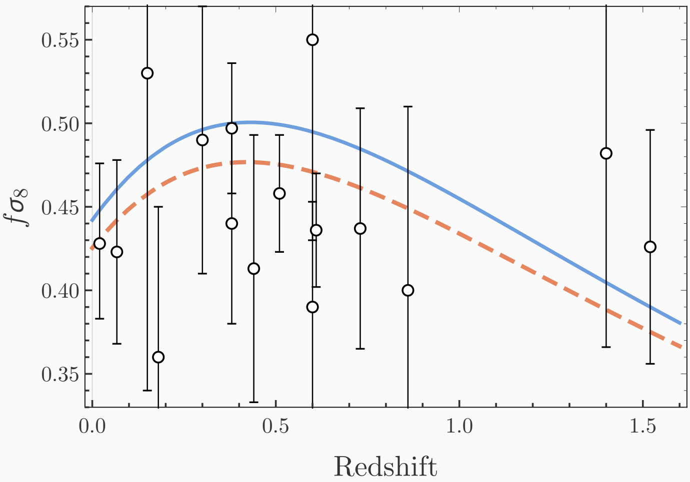
Modes corresponding to large scales are unaffected by the $f(R)$ corrections.

Cosm. Perturbations: Evolution II



Dashed lines represent the solutions in GR.

Growth of structure



Quantum cosmology of the late-universe: why not?

Summary of DE singularities

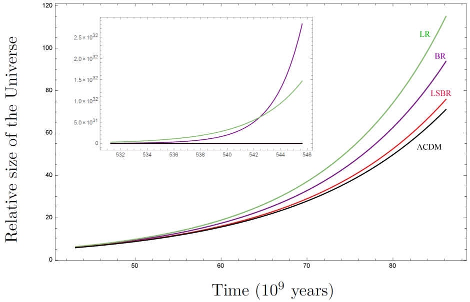
DE might induce a future cosmic singularity

Some of the cosmological parameters:

- t → Cosmic time
- a → Scale factor (relative size)
- H → Hubble parameter (growth rate)
- $\dot{H}$ → Time derivative of H

Singularity	t	a	H	$\dot{H}$	$\ddot{H}, \dddot{H}$ ...
Big Bang	0	0	∞	∞	∞
De Sitter (ΛCDM)	∞	∞	H_{ds}	0	0
Big Rip	t_s	∞	∞	∞	∞
LR	∞	∞	∞	∞	∞
LSBR	∞	∞	∞	$\dot{H}_s$	0
Big Freeze	t_s	a_s	∞	∞	∞
Sudden. S.	t_s	a_s	H_s	∞	∞
Type IV	t_s	a_s	H_s	$\dot{H}_s$	∞

Asymptotic evolution of the scale factor



Nojiri, Odintsov and Tsujikawa, Phys. Rev. D 71 (2005) 06300, [hep-th/0501025].

On the quantum fate of singularities in a dark-energy dominated universe

- There is no successful quantum gravity theory so far that would lead to THE theory of quantum cosmology
- There are, however, several approaches in this direction. Here we will follow the most conservative one which corresponds to the Wheeler deWitt approach.
- The Wheeler DeWitt equation is the equivalent to Schrödinger like equation

On the quantum fate of singularities in a dark-energy dominated universe within GR and beyond

Within the framework of quantum geometrodynamics and mainly within a Born Oppenheimer approximation

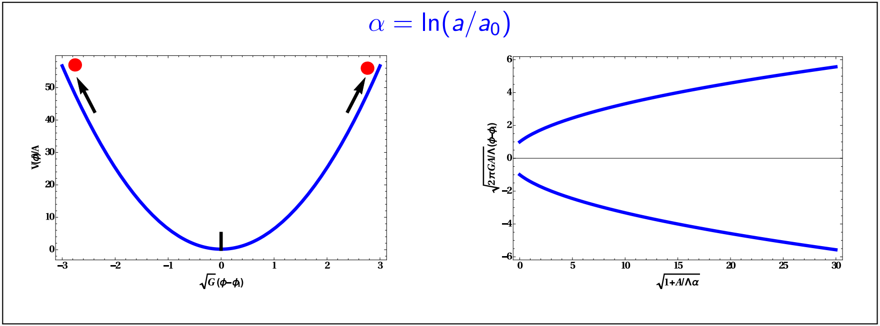
- It was shown that the big rip can be removed Dabrowski, Kiefer and Sandhöfer, PRD, [arXiv:hep-th/0605229], González-Díaz and Jiménez Madrid, IJMPD, [arXiv:0709.4038],Alonso, B.L. and Martín-Moruno, PRD, [arXiv:1802.03290 [gr-qc]].
- It was shown the avoidance of a big brake singularity Kamenshchik, Kiefer and Sandhöfer 07', PRD, [arXiv:0705.1688].
- It was shown also the avoidance of a big démarrage singularity and a big freeze BL, Kiefer, Sandhöfer and Moniz, PRD, [arXiv:0905.2421]
- Type IV singularity is removed BL, Krämer and Kiefer, PRD, [arXiv:1312.5976].
- It has been shown as well that LR can be removed Albarran, BL, Kiefer, Marto, Moniz, PRD, [arXiv:1604.08365].

Review to come soon on the topic by B.L., Kiefer and Martín-Moruno arXiv:18XX.XXXX

LSBR driven by a standard scalar field

- LSBR: H $\to \infty$ though $\dot{H}$ finite.
- Phantom scalar field ϕ: $\rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi)$, $\quad p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi)$

$$\ddot{\phi} + 3H\dot{\phi} - V'(\phi) = 0, \quad V(\phi) = \frac{A}{6} + 2\pi A G\,(\phi - \phi_1)^2$$



LSBR: Quantisation with a scalar field

- The Wheeler-DeWitt equation:

$$\frac{\hbar^2}{2}\left[\frac{\kappa^2}{6}\frac{\partial^2}{\partial\alpha^2}+\frac{\partial^2}{\partial\phi^2}\right]\psi(\alpha,\phi)+a_0^6 e^{6\alpha}V(\phi)\psi(\alpha,\phi)=0$$

- Can be solved within the BO:
  - The gravitational part are oscillatory or exponential functions.
  - The matter part can be written as parabolic cylinder functions that decay to zero at large value of the scale factor.
- It can be shown that there are solutions (wave functions) that vanishe close to the classically abrupt event. Therefore, the DeWitt condition is fullfilled. This result can be interpreted as an "abrupt event" avoidance.

Albarran, BL, Cabral, Martín-Moruno, JCAP, [arXiv:1509.07398] (minimally coupled scalar field)

B.L., Brizuela and Garay, JCAP, [arXiv:1802.05164 [gr-qc]] (3-forms)

Conclusions

Conclusions-1-

- We have summarised two of the most cited works of Pedro in phantom DE.
- Late-time acceleration of the Universe described by a DE component.
- Late-time acceleration of the Universe described by a modified theory of gravity.
- The late-time acceleration might end up in a singularity where a quantum regime will dominate.
- DE singularities seem to be unharmful in a quantum world.

Conclusions-2-

- Q: Does the laws governing the universe allow us to predict exactly what is going to happen to us in the future?
- R: The short answer is no, and yes, in principle. the laws allow us to predict the future. But in practice the calculations are often too difficult.

"Brief answers to the big questions", Stephen Hawking.

Thank you for your atention and Prado for having worked so hard to bring us all here. Of course, thank you also to Francisco for reminding us during the last Iberian Cosmology meeting in Lisbon that we had to organise a conference in honour of Pedro.

Chukran Pedro for all what you gave me!