Wormholes, Warp Drives and Energy Conditions

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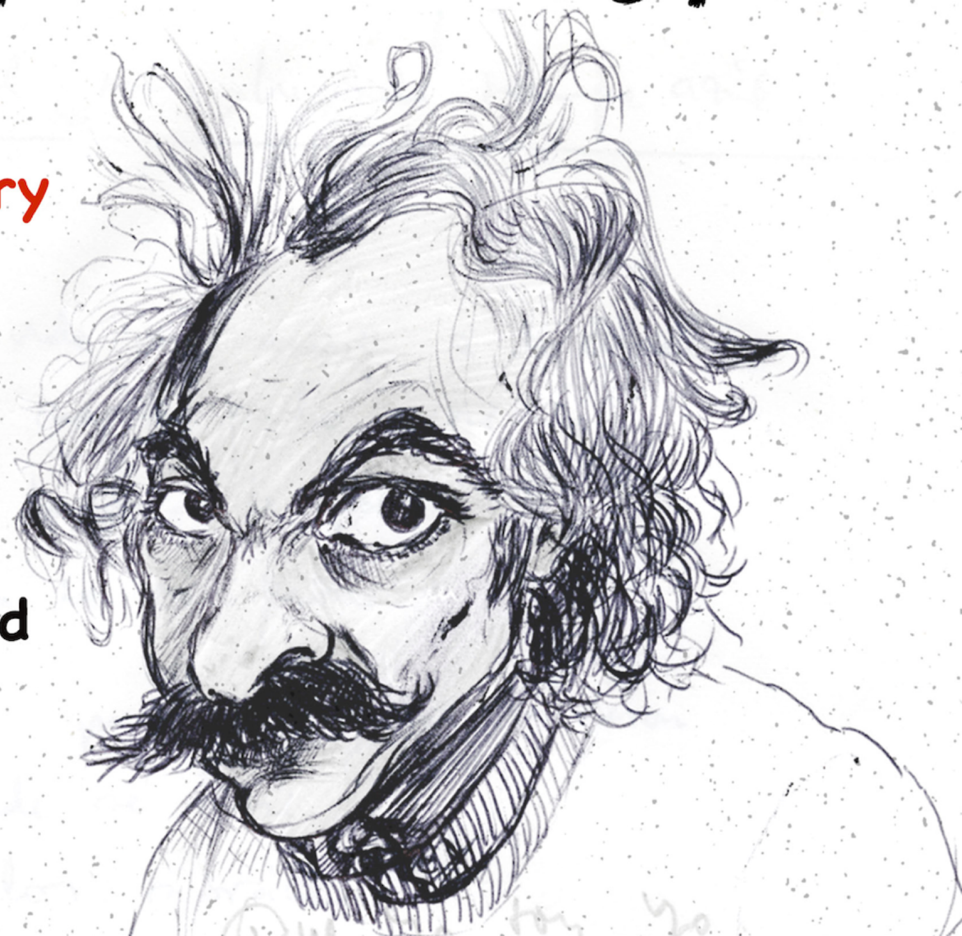
Travelling through Pedro's universes

5th December 2018

PTDC/FIS-OUT/29048/2017, UID/FIS/04434/2013

Travelling through
Pedro's universes:
from spectroscopy to cosmology

International and multidisciplinary
workshop in honour of
Prof. Pedro Félix González Díaz



Faculty of Physics
Complutense University of Madrid
3-5 December 2018

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Pedro's Angels

Outline of the talk

- History:
  - The (Flamm)-Einstein-Rosen bridge;
  - Geons and spacetime foam
- Renaissance and Basics:
  - Exotic matter;
  - Violation of the Energy Conditions; etc, etc
  - Wormholes in modified gravity
- Warp drive
  - Basics and interesting features.
- Pedro’s contributions to wormholes and warp drives:
  - Wormholes and ringholes;
  - Escaping the Big Rip with the Big Trip, etc,
  - Superluminal warp drive;
- Closed timelike curves and associated paradoxes.
- Conclusions

Setting the stage

Approach in solving the EFE: $G_{\mu\nu} = 8\pi T_{\mu\nu}$

- General Relativity (GR) has been an extremely successful theory, with a well established experimental footing, at least for weak gravitational fields.
- It's predictions range from the existence of black holes, gravitational radiation (now confirmed) to the cosmological models.
- All these solutions have been obtained by first considering plausible distributions of matter, i.e., a plausible stress-energy tensor, and through the EFE, the spacetime metric of the geometry is determined.

Setting the stage

Approach in solving the EFE:

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- General Relativity (GR) has been an extremely successful theory, with a well established experimental footing, at least for weak gravitational fields.
- It’s predictions range from the existence of black holes, gravitational radiation (now confirmed) to the cosmological models.
- All these solutions have been obtained by first considering plausible distributions of matter, i.e., a plausible stress-energy tensor, and through the EFE, the spacetime metric of the geometry is determined.
- **Reverse philosophy**: solve the EFE in the reverse direction, namely, one first considers an interesting metric, then finds the matter source responsible for the respective geometry.
- In this manner, it was found that some of these solutions possess a peculiar property, namely 'exotic matter', involving a stress-energy tensor that violates the null energy condition.
- These solutions are primarily useful as 'gedanken-experiments' and as a theoretician's probe of the foundations of GR, and include traversable wormholes and superluminal 'warp drive' spacetimes.

1. The Flamm-Einstein-Rosen bridge

- Wormhole physics can originally be tentatively traced back to Flamm in 1916, where his aim was to render the conclusions of the Schwarzschild solution in a clearer manner.

- In 1935, Einstein and Rosen (ER) were attempting to build a geometrical model of a physical elementary "particle" that is finite and singularity-free.

- Based their discussion in terms of a "bridge" across a double-sheeted physical space:
  - "… these solutions involve the mathematical representation of physical space by a space of two identical sheets, a particle being represented by a "bridge" connecting these sheets." ("The particle problem in general relativity", Phys.Rev. 1935)

- ER discussed two specific types of bridges, neutral and quasicharged. However, these can easily be generalized.

- Note that at the time ER were writing, the notions of "coordinate singularity" and "physical singularity" had not been cleanly separated.
  - To many physicists the event horizon *was* the singularity.

The (neutral) Einstein-Rosen bridge

- The neutral Einstein-Rosen bridge is a mere observation that a suitable coordinate change seems to make the Schwarzschild (coordinate) singularity disappear.

- In modern language: ER discovered that some coordinate systems naturally cover only two asymptotically flat regions of the maximally extended Schwarzschild spacetime.

- Consider the ordinary Schwarzschild geometry:

$$ds^2 = -\left(1 - 2M/r\right)dt^2 + \frac{dr^2}{1 - 2M/r} + r^2\, d\Omega^2$$

- Consider a coordinate change $u^2 = r - 2M$, this can be represented into the ER form:

$$ds^2 = -\frac{u^2}{u^2 + 2M}dt^2 + 4\left(u^2 + 2M\right)du^2 + \left(u^2 + 2M\right)^2\, d\Omega^2$$

with $u \in (-\infty, +\infty)$. This coordinate change discards the region containing the curvature singularity $r \in [0, 2M)$

The region near $u = 0$ is interpreted as a "bridge" connecting the asymptotically flat region near $u = +\infty$ with the asymptotically flat region near $u = -\infty$.

- To justify the “bridge” appellation, consider a spherical surface, with constant u.
  - The area of the surface is: $A(u) = 4\pi(2M+u^2)^2$.
  - The area has a minimum at u=0, with $A(0)= 4\pi(2M)^2$.
- One defines the narrowest part of the geometry as the “throat”, while the nearby region is denoted the bridge (or in modern terminology a “wormhole”).

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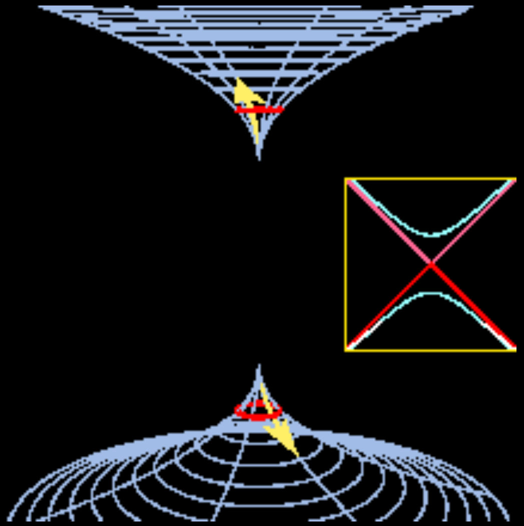
- The net result is that the neutral “Einstein-Rosen” bridge (aka as the “Schwarzschild wormhole”) is identical to a part of the maximally extended Schwarzschild geometry.
- Non-traversable: The throat will pinch off before an observer may traverse the throat.



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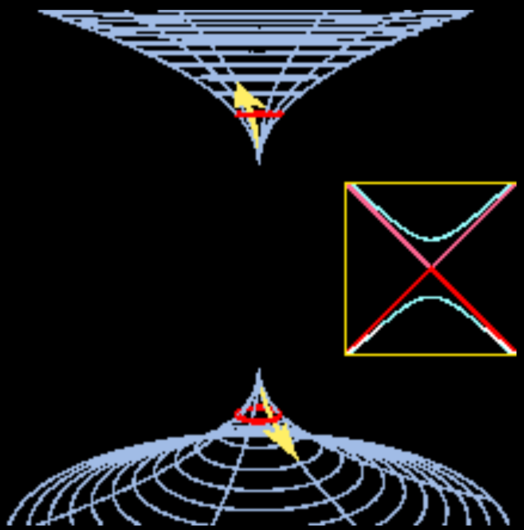
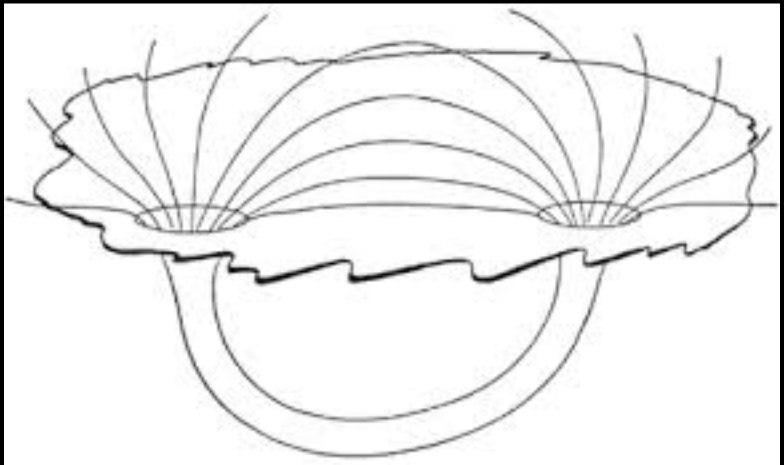


- Key ingredient of the bridge construction is the existence of an event horizon.

- IMPORTANT: The ER bridge is a coordinate artefact arising from choosing a coordinate patch, which is defined to double-cover the asymptotically flat region exterior to the BH event horizon.

2. *Geons* and *Spacetime foam*

- After the pioneering work by Einstein and Rosen, in 1935, the field lay dormant for a good 20 years.
- Interest in systems of this nature was rekindled in 1955 by John Wheeler, who was beginning to be interested in topological issues in General Relativity: (Resulted in the paper *"Geons", Phys.Rev. 1955*)
- The "geon" concept was coined by Wheeler to denote a "gravitational-electromagnetic entity".



2.1. “Classical Physics as Geometry” (The “*already* unified field theory”)

- Misner and Wheeler (Annals Phys. 1957) presented a *tour de force* wherein Riemannian geometry of manifolds of nontrivial topology was investigated with the view to explain *all* of physics.
  - This extremely ambitious idea was one of the very first uses of abstract topology, homology, cohomology, and differential geometry in physics.
  - Their point of view is best summarised by their phrase: “Physics is geometry”;
  - This is the first paper that introduces the term “wormhole” to the scientific community.

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  - Their point of view is best summarised by their phrase: “Physics is geometry”;
  - This is the first paper that introduces the term “wormhole” to the scientific community.

- **Ultimately, the aim was to use the source-free Maxwell equations, coupled to Einstein gravity, with the seasoning of nontrivial topology, to build models for classical electrical charges and all other particle-like entities in classical physics.**

- **More specifically, the existing well-established “already unified classical theory” allows one to describe in terms of empty curved space:**
  - **Gravitation without gravitation;**
  - **Electromagnetism without electromagnetism;**
  - **Charge without charge;**
  - **Mass without mass (around the mouth of the “wormhole” lies a concentration of eletromagnetic energy that gives mass to this region of space).**

2.2 *Spacetime foam*
(Quantum fluctuations in the spacetime metric)

- Wheeler: the overwhelming importance of the Planck scale in gravitational physics. At distances below the Planck length quantum fluctuations are so large, that linearized theory breaks down irretrievably and the quantum physics of full nonlinear Einstein gravity must be faced.
  - "…. On the atomic scale the metric appears flat, as does the ocean to an aviator far above. The closer the approach, the greater the degree of irregularity. Finally, at distances of the order of l_P, the fluctuations in the typical metric component, $g_{\mu\nu}$, become of the same order as the metric components themselves."

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- Once the metric fluctuations become nonlinear and strongly interacting, this might be expected to endow spacetime with a "foamlike" structure. Wheeler:
  - "… space resonates between one foamlike structure and another".

- More succinctly, one often encounters the phrase "spacetime foam" which refers to Wheeler's suggestion that the geometry, and topology, of space might be constantly fluctuating. (Topology change???)

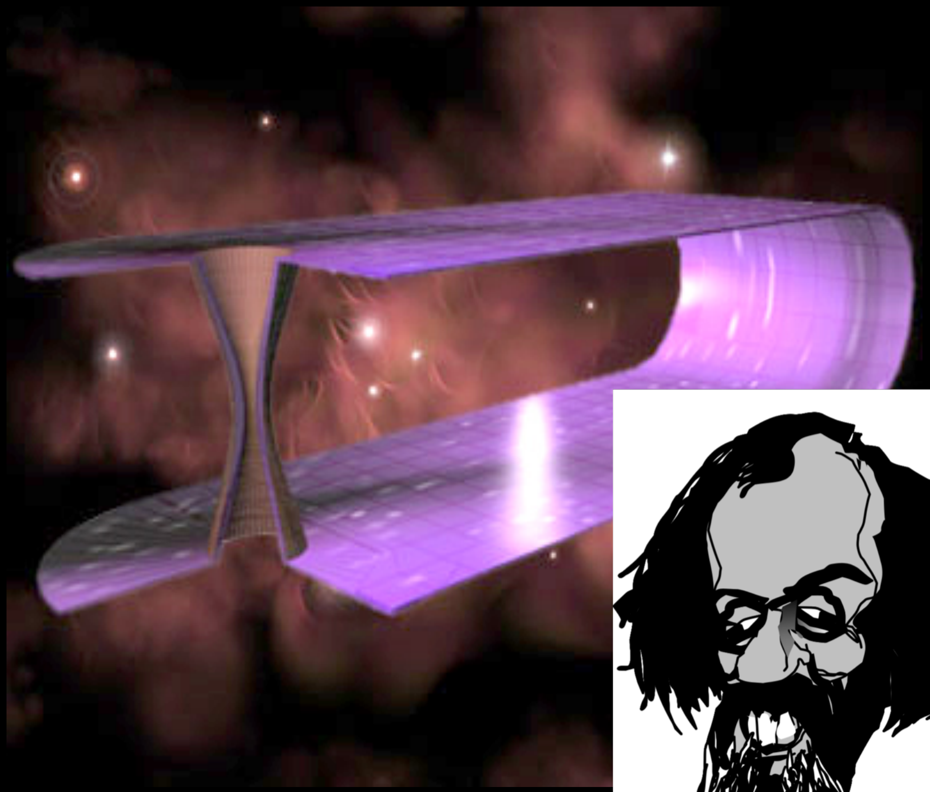
3. Interregnum and Renaissance

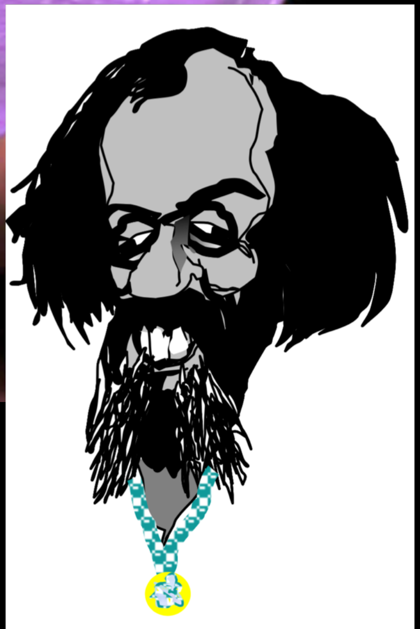
- 30 year gap between Wheeler's (and Misner's) 1955/1957 work and the 1988 Morris-Thorne renaissance of wormhole physics.
- Much was accomplished during this period, but little effort was devoted to Lorentzian wormholes.
- Considerable effort was invested in topics as attempting to understand the the "geon" concept and "spacetime foam" picture.
- **Isolated pieces of work do appear, such as the Homer Ellis' drainhole concept and Kirill Bronnikov's tunnel-like solutions, in the 1970s.**
- However, it is only in 1988 that a full-fledged renaissance of wormhole physics took place … and is still in full flight.

4. Traversable wormholes

- **Approach: What do the laws of physics permit?**
- **Wormholes: Useful primarily as a theoretician's probe of the foundations of general relativity.**
- **Very interesting physics involved!**
- **Pedagogical purposes: Useful for teaching General Relativity**

  (Morris &Thorne, AJP, 56, 395, 1988)





4.1.1 Spacetime metric:

$$ds^2 = -e^{2\Phi(r)}\, dt^2 + \frac{dr^2}{1 - b(r)/r} + r^2\,(d\theta^2 + \sin^2\theta\, d\phi^2)$$

- b(r): shape function – determines the shape of the wormhole
- Φ(r): redshift function – related to the gravitational redshift
- Wormhole throat, minimum radius: $r = b(r) = r_0$
- Impose traversability conditions (Total time in a traversal, acceleration and tidal accelerations felt by a traveller) !!

Field equations:

$$\begin{aligned}
\rho(r) &= \frac{1}{8\pi}\,\frac{b'}{r^2}\,, \\
\tau(r) &= \frac{1}{8\pi}\left[\frac{b}{r^3} - 2\left(1 - \frac{b}{r}\right)\frac{\Phi'}{r}\right], \\
p(r) &= \frac{1}{8\pi}\left(1 - \frac{b}{r}\right)\left[\Phi'' + (\Phi')^2 - \frac{b'r - b}{2r^2(1 - b/r)}\Phi' - \frac{b'r - b}{2r^3(1 - b/r)} + \frac{\Phi'}{r}\right].
\end{aligned}$$

4.1.2 Mathematics of embedding:

- Due to spherical symmetry, consider an equatorial plane $\theta=\pi/2$. Fixed time slice: t=const

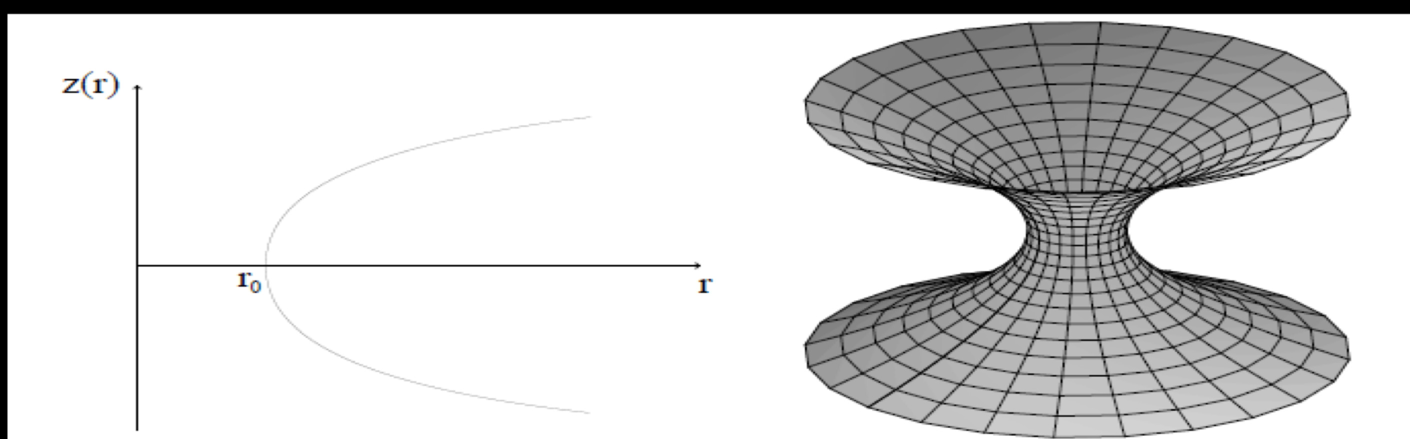
- **To visualise this slice:** Embed this metric into 3-d Euclidean space in cylindrical coordinates (r, ϕ, z), with surface equation z=z(r)

- Equation for the embedding surface:

$$\frac{dz}{dr} = \pm \left(\frac{r}{b(r)} - 1 \right)^{-1/2} .$$

- **Fundamental ingredient: Flaring-out condition!!**

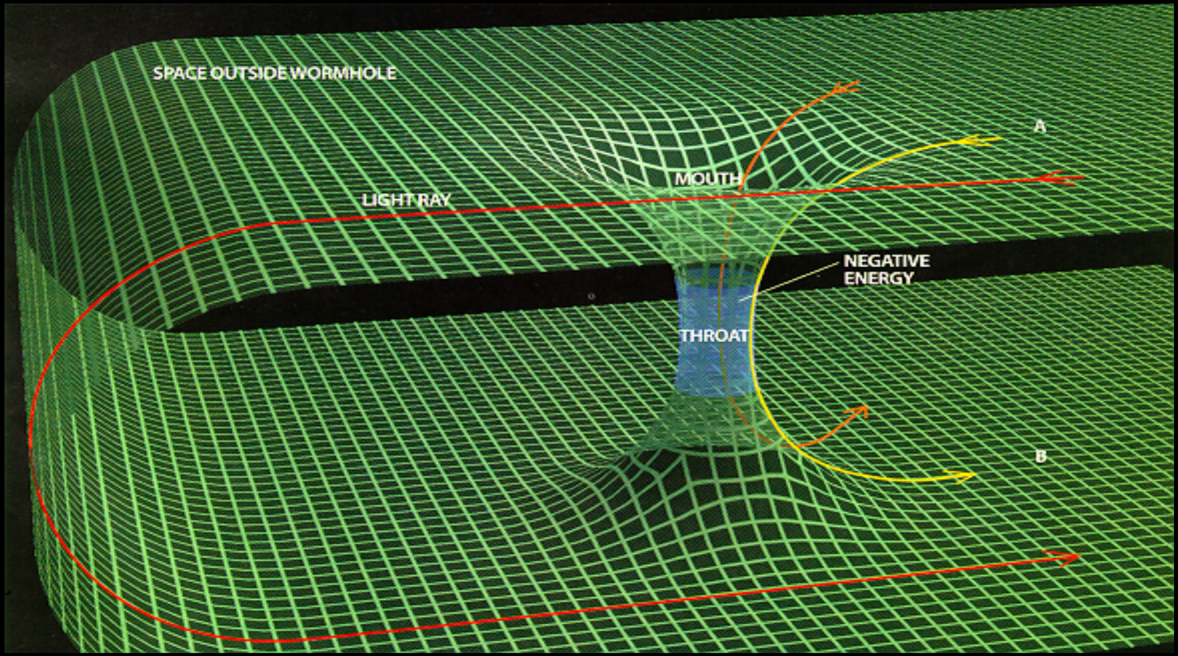
$$\frac{d^2 r}{dz^2} = \frac{b - b' r}{2b^2} > 0 \, .$$



4.1.3 Exotic matter

• **The flaring-out condition entails the violation of the NULL ENERGY CONDITION (NEC) at the throat:**
(negative energy densities not essential)

$$\rho + p_r < 0 \,!!$$



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- **Note that the null energy condition arises when one refers back to the Raychaudhuri equation: the positivity condition of the expansion term** $R_{\mu\nu}k^{\mu}k^{\nu} \geq 0$ **appears;**

- **In GR, through the EFE the positivity condition reflects the null energy condition:** $T_{\mu\nu}k^{\mu}k^{\nu} \geq 0$

4.1.3 Exotic matter

- **The flaring-out condition entails the violation of the NULL ENERGY CONDITION (NEC) at the throat:**
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- **In fact, it violates all of the pointwise energy conditions:**

$$1.\ \mathrm{NEC}: \rho + p_r \geq 0$$

$$2.\ \mathrm{WEC}: \rho \geq 0,\ \rho + p_r \geq 0$$

$$3.\ \mathrm{SEC}: \rho + p_r \geq 0,\ \rho + p_r + 2p_t \geq 0$$

$$4.\ \mathrm{DEC}: \rho \geq |p_r|$$

- **And the averaged energy conditions!**
- **(Quantum inequalities, semi-classical energy conditions, etc)**

4.1.3 Exotic matter

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As the violation of the energy conditions is a problematic issue, it is useful to minimize this violation.

Several approaches:

- ❑ **Rotating solutions;**
- ❑ **Evolving wormhole spacetimes;**
- ❑ **Thin-shell wormholes: cut-and-paste procedure; etc**

4.2.1. Rotating wormhole solutions (very messy!!)

Metric:

$$ds^2 = -N^2 dt^2 + e^{\mu} dr^2 + r^2 K^2 \left[d\theta^2 + \sin^2\theta (d\varphi - \omega dt)^2 \right]$$

EFE at the throat:

$$\begin{aligned}
8\pi T_{\hat{t}\hat{t}} &= -\frac{(K_\theta \sin\theta)_\theta}{r^2 K^3 \sin\theta} - \frac{\omega_\theta^2 \sin^2\theta}{4N^2} + e^{-\mu}\,\mu_r\,\frac{(rK)_r}{rK} + \frac{K^2 + K_\theta^2}{r^2 K^4}\,, \\
8\pi T_{\hat{r}\hat{r}} &= \frac{(K_\theta \sin\theta)_\theta}{r^2 K^3 \sin\theta} - \frac{\omega_\theta^2 \sin^2\theta}{4N^2} + \frac{(N_\theta \sin\theta)_\theta}{N r^2 K^2 \sin\theta} - \frac{K^2 + K_\theta^2}{r^2 K^4}\,, \\
8\pi T_{\hat{\theta}\hat{\theta}} &= \frac{N_\theta (K \sin\theta)_\theta}{N r^2 K^3 \sin\theta} + \frac{\omega_\theta^2 \sin^2\theta}{4N^2} - \frac{\mu_r\, e^{-\mu} (NrK)_r}{2NrK}\,, \\
8\pi T_{\hat{\phi}\hat{\phi}} &= -\frac{\mu_r\, e^{-\mu}\,(NKr)_r}{2NKr} - \frac{3\sin^2\theta\,\omega_\theta^2}{4N^2} + \frac{N_{\theta\theta}}{N r^2 K^2} - \frac{N_\theta K_\theta}{N r^2 K^3}\,, \\
8\pi T_{\hat{t}\hat{\phi}} &= \frac{1}{4N^2 K^2 r} \Big(6NK\,\omega_\theta\,\cos\theta + 2NK\,\sin\theta\,\omega_{\theta\theta} \\
&\quad - \mu_r e^{-\mu} r^2 N K^3 \sin\theta\,\omega_r + 4N\,\omega_\theta\,\sin\theta\,K_\theta - 2K\,\sin\theta\,N_\theta\,\omega_\theta \Big)\,.
\end{aligned}$$

NEC violation
(exotic matter confined to certain regions):

$$8\pi\, T_{\hat{\mu}\hat{\nu}} k^{\hat{\mu}} k^{\hat{\nu}} = e^{-\mu} \mu_r \frac{(rK)_r}{rK} - \frac{{\omega_\theta}^2 \sin^2\theta}{2N^2} + \frac{(N_\theta \sin\theta)_\theta}{(rK)^2 N \sin\theta}\,.$$

4.2.2 Evolving wormholes in a cosmological background

Metric:

$$ds^2 = \Omega^2(t)\left[-e^{2\Phi(r)}\,dt^2 + \frac{dr^2}{1-kr^2-\frac{b(r)}{r}} + r^2\,\left(d\theta^2+\sin^2\theta\,d\phi^2\right)\right]$$

EFE:

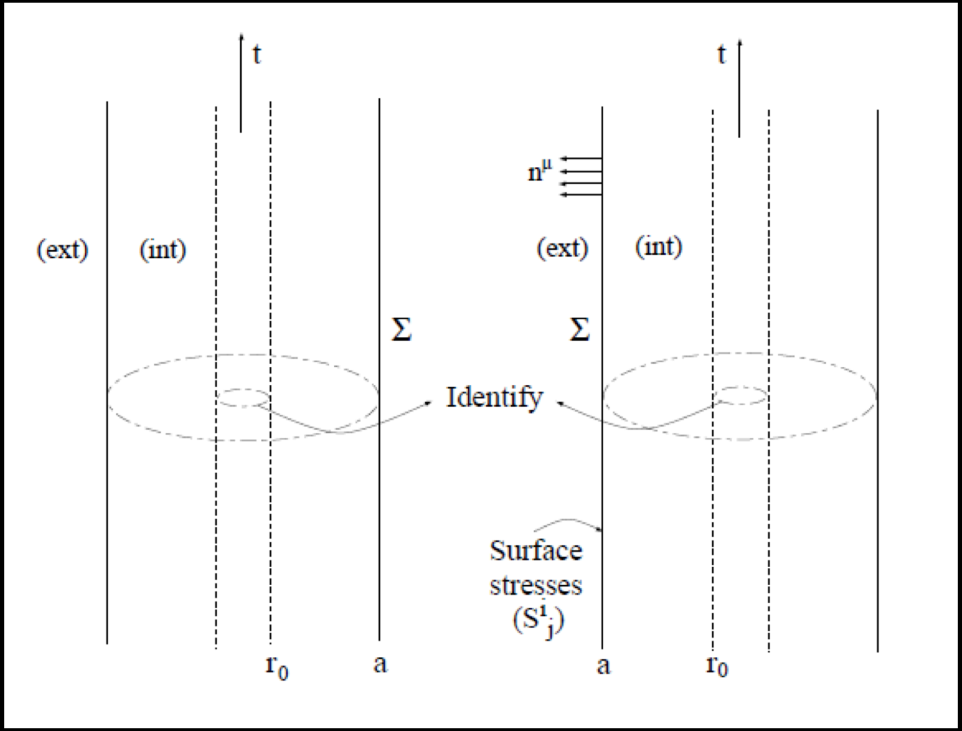
$$\begin{aligned}
\rho(r,t) &= \frac{1}{8\pi}\frac{1}{\Omega^2}\left[3e^{-2\Phi}\left(\frac{\dot\Omega}{\Omega}\right)^2 + \left(3k+\frac{b'}{r^2}\right)\right]\,,\\
\tau(r,t) &= -\frac{1}{8\pi}\frac{1}{\Omega^2}\left\{e^{-2\Phi(r)}\left[\left(\frac{\dot\Omega}{\Omega}\right)^2 - 2\,\frac{\ddot\Omega}{\Omega}\right] - \left[k+\frac{b}{r^3}-2\,\frac{\Phi'}{r}\left(1-kr^2-\frac{b}{r}\right)\right]\right\}\\
f(r,t) &= -\frac{1}{8\pi}\left[2\,\frac{\dot\Omega}{\Omega^3}\,e^{-\Phi}\,\Phi'\left(1-kr^2-\frac{b}{r}\right)^{1/2}\right]\,,\\
p(r,t) &= \frac{1}{8\pi}\frac{1}{\Omega^2}\left\{e^{-2\Phi(r)}\left[\left(\frac{\dot\Omega}{\Omega}\right)^2 - 2\,\frac{\ddot\Omega}{\Omega}\right] + \left(1-kr^2-\frac{b}{r}\right)\times\right.\\
&\qquad\left.\times\left[\Phi''+(\Phi')^2 - \frac{2kr^3+b'r-b}{2r(r-kr^3-b)}\Phi' - \frac{2kr^3+b'r-b}{2r^2(r-kr^3-b)} + \frac{\Phi'}{r}\right]\right\}.
\end{aligned}$$

Possible to show the existence of NEC violation flashes in time!

4.2.3 Thin-shell wormholes: Cut-and-paste procedure

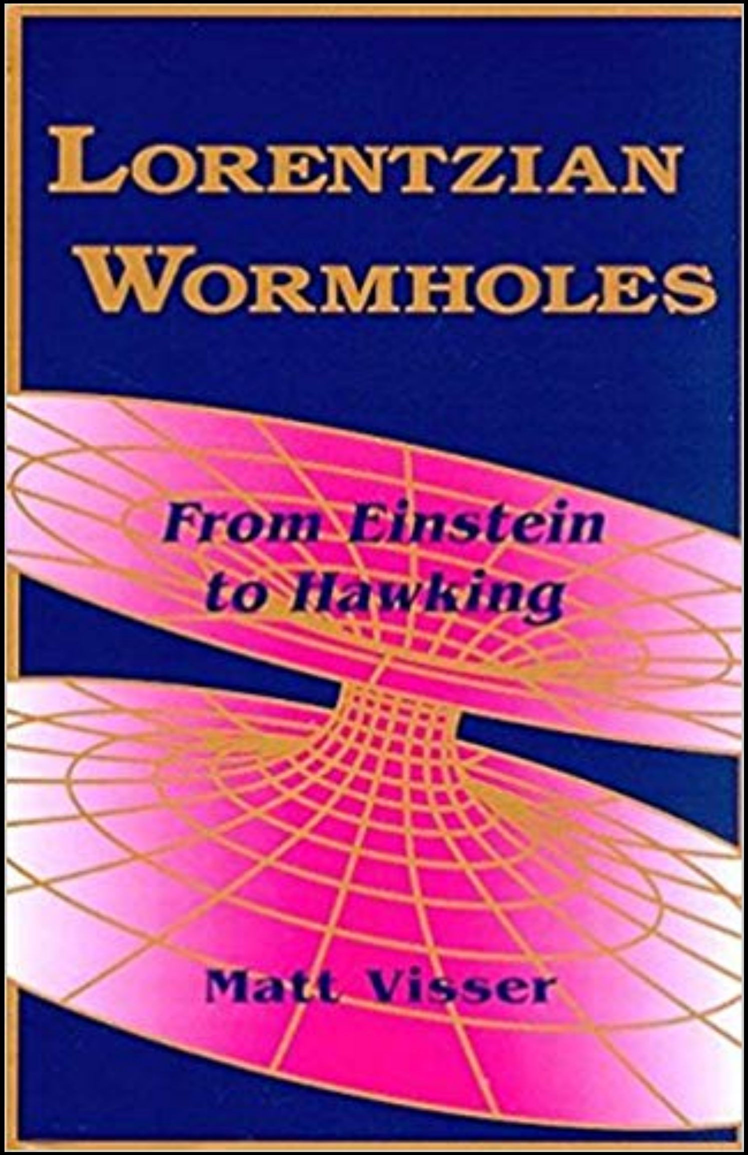
Thin-shells: matching conditions

Match an interior wormhole solution to a exterior vaccum spacetime:



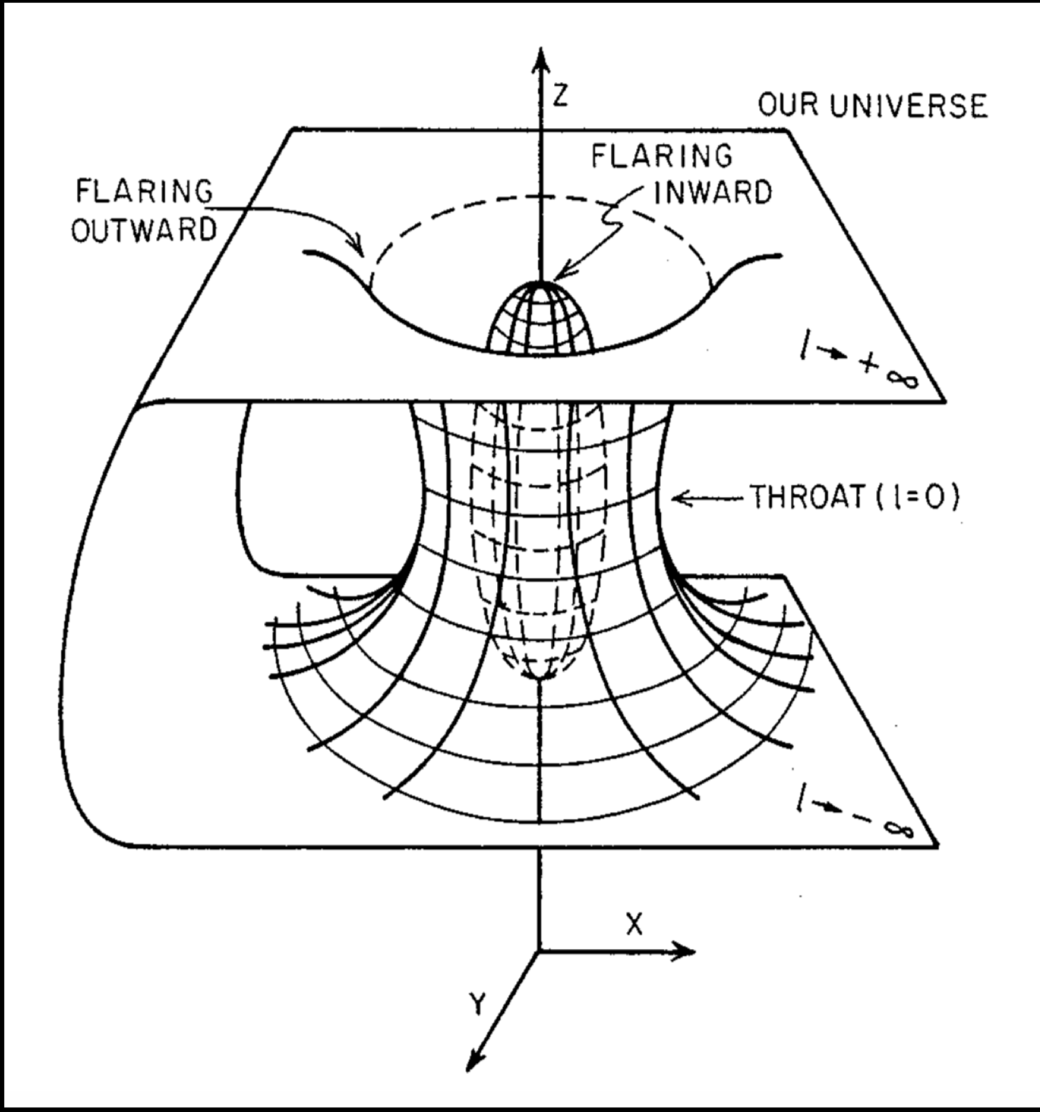
Recent work: Garcia, FL & Visser, PRD 2012; Martin-Moruno, Garcia, FL & Visser, JCAP 2012; Bouhmadi-Lopez, FL, Martin-Moruno, JCAP 2014

Review until 1996



Pedro's Ringholes (PRD, 1996)

- Toroidal topology (generalizes the Morris-Thorne wormhole);
- **Advantages**: an observer may encounter matter that satisfies the NEC (due to the peculiar topology, and consequently different flaring-out conditions);
- Lensing effects also different, depending on the angular coordinates, one may have converging/diverging properties;
- Generates CTCs.



Wormholes (and ringholes) and the Big Trip (PRD, 2003)

Consider the time-dependent spacetime metric:

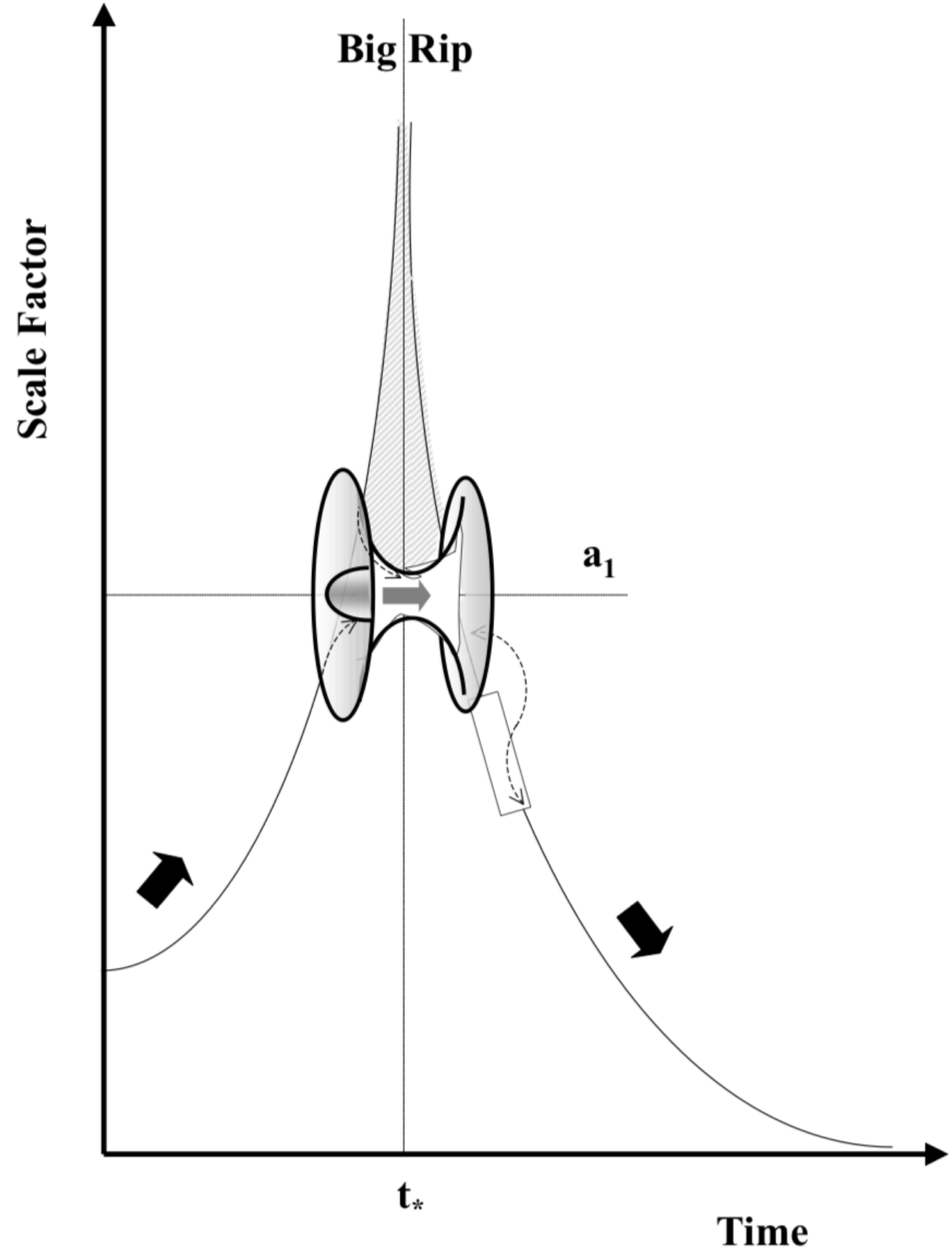
$$ds^2 = -e^{2\Phi(r)}dt^2 + g(t)^2 \left(\frac{dr^2}{1 - \frac{K(r)}{r}} + r^2 d\Omega_2^2 \right)$$

where

$$g(t)^2 = \left(1 + \frac{3(1+\omega)\sqrt{A}(t-t_0)}{2a_0^{3(1+\omega)/2}} \right)^{2/[3(1+\omega)]}$$

- Note that the shape of the wormhole, in an accelerating universe, is preserved with time;
- If the $\omega > -1$, then the size increase will be slower than in the inflating universe;
- If $\omega < -1$, then the wormhole will increase size more rapidly than in the inflating case. **Big trip: where the wormhole throat diverges will take place before the occurrence of the big rip singularity**

Escaping the Big Rip (with ringholes)



gr-qc arXiv version: 21st February 2005

PHYSICAL REVIEW D **71,** 043520 (2005)

Wormholes supported by a phantom energy

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(Received 24 January 2005; published 28 February 2005)

gr-qc arXiv version: 23rd February 2005

PHYSICAL REVIEW D **71,** 084011 (2005)

Phantom energy traversable wormholes

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(Received 1 March 2005; published 13 April 2005)

4.3. Modified theories of gravity:

Higher order actions may include various curvature invariants, such as:

$$\boxed{R^2, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}, etc.}$$

Consider f(R) gravity, for simplicity:

$$\boxed{S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa} f(R) + L_m(g^{\mu\nu}, \psi)\right]}$$

Appealing feature: combines mathematical simplicity and a fair amount of generality!

4.3.1. f(R) gravity

Action:

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa}f(R)+L_m(g^{\mu\nu},\psi)\right]$$

Gravitational field equations (vary action with $g^{\mu\nu}$):

$$FR_{\mu\nu}-\frac{1}{2}f\,g_{\mu\nu}-\nabla_\mu\nabla_\nu F+g_{\mu\nu}\nabla_\alpha\nabla^\alpha F=\kappa T^{(m)}_{\mu\nu}\,,\qquad F=\frac{df}{dR}$$

Effective Einstein equation: $G_{\mu\nu}=\kappa T^{eff}_{\mu\nu}$ **with** $T^{eff}_{\mu\nu}=\bar{T}^{(m)}_{\mu\nu}+T^{(c)}_{\mu\nu}$

$$\bar{T}^{(m)}_{\mu\nu}=T^{(m)}_{\mu\nu}/F\,,\quad \text{and}\quad T^{(c)}_{\mu\nu}=\frac{1}{\kappa F}\left[\nabla_\mu\nabla_\nu F-\frac{1}{4}g_{\mu\nu}\left(RF+\nabla_\mu\nabla^\mu F+\kappa T\right)\right]$$

Conservation law:

$$\nabla^\mu T^{(c)}_{\mu\nu}=\frac{1}{F^2}T^{(m)}_{\mu\nu}\nabla^\mu F$$

Energy condition violations

- In modified gravity, it is the effective energy-momentum tensor that comes into play and the NEC violation imposes:

$$T^{eff}_{\mu\nu} k^{\mu} k^{\nu} < 0$$

- Impose that normal matter satisfies the NEC: $T_{\mu\nu} k^{\mu} k^{\nu} \geq 0$

- Thus, it is the higher order curvature terms that are responsible for supporting the wormhole!

(Oliveira, FL, PRD 2009; PRD 2010)

4.3.2. Generalized modified gravity:

Consider the generalized gravitational field equations for a large class of modified theories of gravity (Harko, FL, Mak, Sushkov, PRD 2013):

$$g_1(\Psi^i)(G_{\mu\nu} + H_{\mu\nu}) - g_2(\Psi^j)\, T_{\mu\nu} = \kappa^2\, T_{\mu\nu}\,,$$

$H_{\mu\nu}$ is an additional geometric term that includes the geometrical modifications inherent in the modified gravitational theory under consideration;

$g_i\left(\psi^j\right)$ are multiplicative factors that modify the geometrical sector of the field equations;

ψ^j denote generically curvature invariants or gravitational fields such as scalar fields;

$g_2\left(\psi^j\right)$ covers the coupling of the curvature invariants or the scalar fields with the matter stress-energy tensor.

4.3.2. Generalized modified gravity:

Useful to rewrite the field equation as an effective Einstein field equation (Harko, FL, Mak, Sushkov, PRD 2013):

$$T^{\rm eff}_{\mu\nu} \equiv \frac{1+\bar{g}_2(\Psi^j)}{g_1(\Psi^i)}\, T_{\mu\nu} - \bar{H}_{\mu\nu}\,,$$

Where $\bar{g}_2\left(\psi^j\right) = g_2\left(\psi^j\right)/\kappa^2$ **and** $\bar{H}_{\mu\nu} = H_{\mu\nu}/\kappa^2$

Now, the violation of the generalized NEC $T^{eff}_{\mu\nu} k^\mu k^\nu < 0$ **implies:**

$$\frac{1+\bar{g}_2(\Psi^j)}{g_1(\Psi^i)}\, T_{\mu\nu} k^\mu k^\nu < \bar{H}_{\mu\nu} k^\mu k^\nu\,.$$

Specific examples:

- **Braneworlds** (FL, PRD 2007):
  - The stress energy tensor confined on the brane, threading the wormhole, is imposed to satisfy the NEC.
  - The local high-energy bulk effects and nonlocal corrections from the Weyl curvature in the bulk induce a NEC violating signature on the brane!

- **Conformal Weyl gravity** (FL, CQG 2008)

- **f(R) gravity** (Oliveira, FL, PRD 2009; PRD 2010)
  - the higher order curvature terms that are responsible for supporting the wormhole!

- **Curvature-matter couplings in f(R) gravity** (Garcia, FL, PRD 2010; CQG 2011)

$$S=\int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa}f_1(R)+\left[1+\lambda f_2(R)\right]L_m\right\}$$

- **Modified teleparralel gravity, f(T) gravity** (Boehmer, Harko & FL, PRD 2012)

- **Hybrid metric-Palatini gravity** (Capozziello, Harko, Koivisto, FL, Olmo, PRD 2012; JCAP 2013; Rosa, Lemos, FL, PRD 2018)

$$S=\frac{1}{2\kappa}\int d^4x\sqrt{-g}\left[\mathrm{R}+f(\mathcal{R})\right]+S_m$$

5. Warp drive basics

- Within the framework of GR, Alcubierre showed that it is in principle possible to warp spacetime in a small bubble-like region, in such a way that the bubble may attain arbitrarily large velocities.
- Inspired by the inflationary phase of the early universe, the enormous speed of separation arises from the expansion of spacetime itself.

Alcubierre spacetime metric:

$$ds^2 = -dt^2 + dx^2 + dy^2 + \left[dz - v(t)\, f\left(x, y, z - z_0(t)\right)\, dt\right]^2 .$$

Alcubierre form function is given by:

- ***R*** is the "radius" of the warp-bubble;
- ***σ*** can be interpreted as being inversely proportional to the bubble wall thickness.

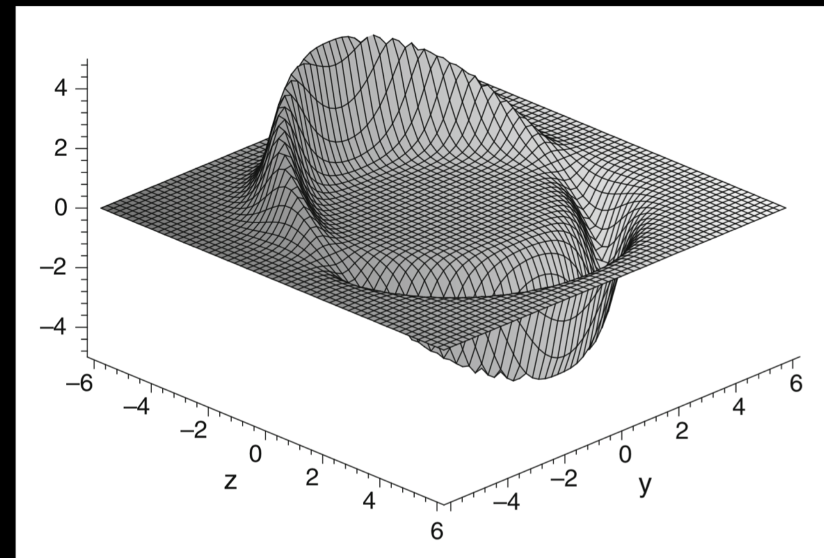
$$f(r) = \frac{\tanh\left[\sigma(r+R)\right] - \tanh\left[\sigma(r-R)\right]}{2\tanh(\sigma R)}$$

$$\lim_{\sigma \to \infty} f(r) = \begin{cases} 1, & \text{if } r \in [0, R], \\ 0, & \text{if } r \in (R, \infty). \end{cases}$$

5. Warp drive basics

Expansion of the volume elements:

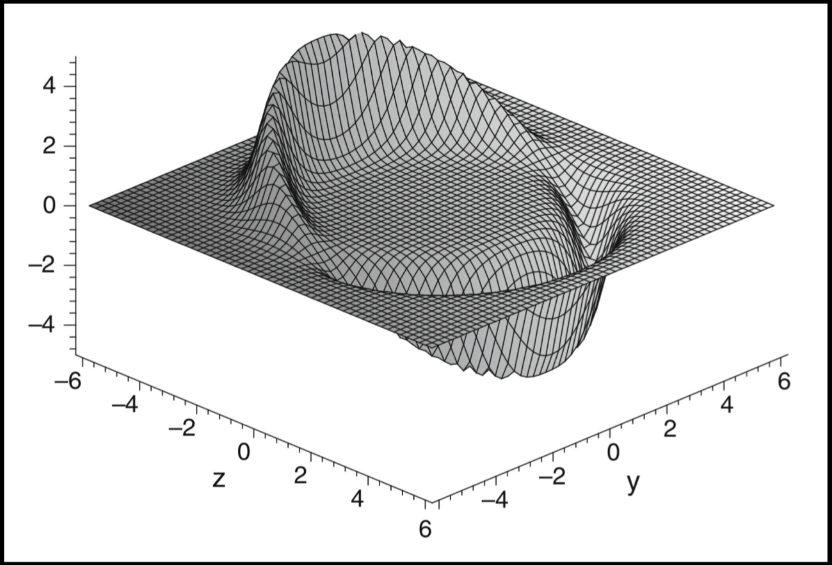
$$\theta = v \, \frac{z - z_0}{r} \, \frac{df(r)}{dr}.$$



5. Warp drive basics

Expansion of the volume elements:

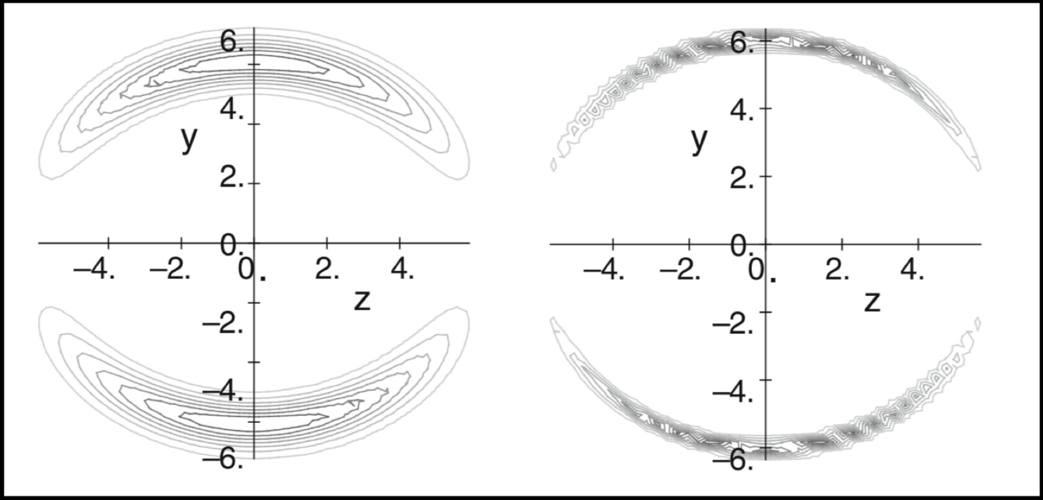
$$\theta = v \; \frac{z - z_0}{r} \; \frac{df(r)}{dr} .$$



Energy density:

Violation of the WEC!

$$T_{\mu\nu} \; U^\mu U^\nu = -\frac{1}{32\pi} \frac{v^2(x^2+y^2)}{r^2} \left(\frac{df}{dr}\right)^2 < 0 .$$



Interesting aspects of the Alcubierre spacetime:

- Superluminal travel in the warp drive:
  - The spaceship may travel faster than the speed of light. However, it moves along a spacetime temporal trajectory, contained within it's light cone, as light suffers the same distortion of spacetime.

- Quantify the "total amount" of energy condition violating matter in the warp bubble (FL, Visser, CQG 2004): $M_{\rm warp} \approx -v^2\ R^2\ \sigma,$

  Linearized warp drive: $v^2\ R^2\ \sigma \leq M_{\rm ship}$

  (provides extremely low bounds on the warp bubble velocity)

- The Krasnikov analysis (Krasnikov, PRD 1998):
  - Photons emitted in the forward direction by the spaceship never reach the outside edge of the bubble wall, which lies outside the forward light cone of the spaceship.
  - The bubble thus cannot be created (or controlled) by any action of the crew.
  - This behaviour is reminiscent of an *event horizon*.
  - Superluminal subway: The Krasnikov "tube".

- Warp drive and closed timelike curves (Everett, PRD 1996)

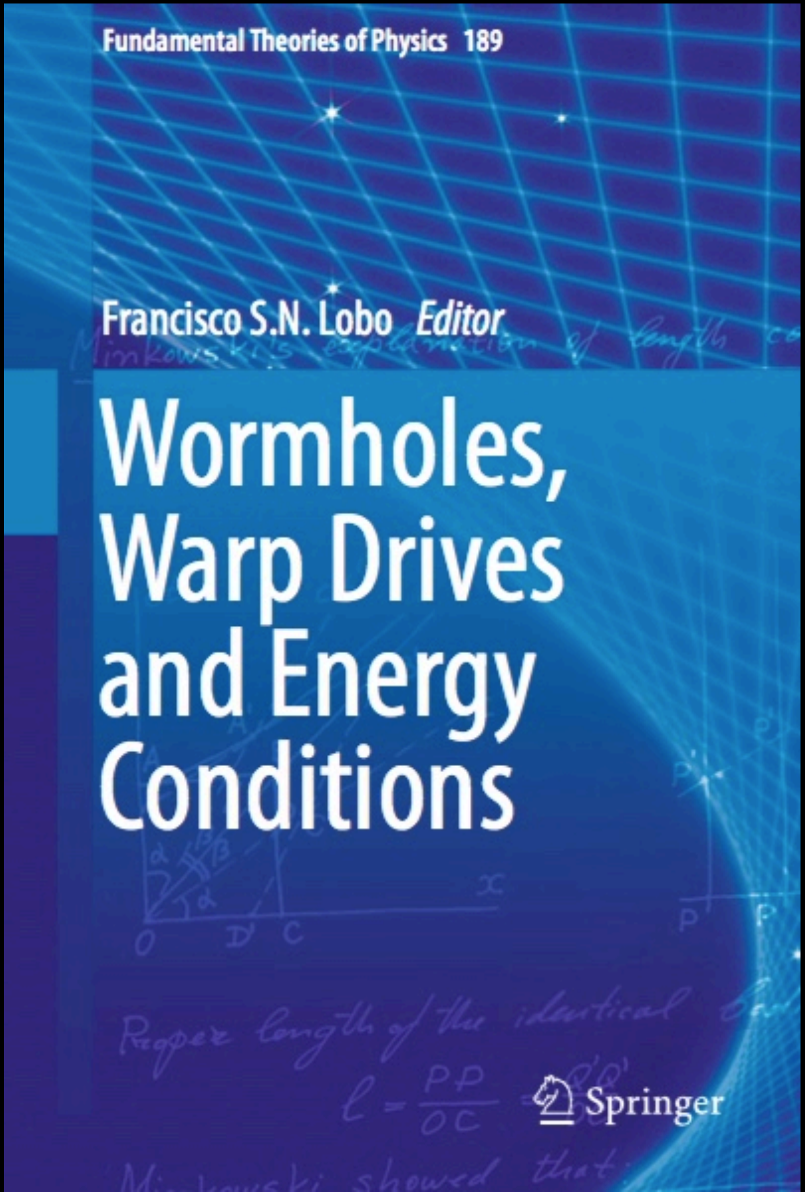
Interesting aspects of the Alcubierre spacetime:

Pedro's contribuitions:

- 2-dimensional spacetime metric:
  - Causal structure and quantum stability of the warp drive (PRD 2000)
  - Thermodynamic properties (PLB 2007);
  - Superluminal warp drive and dark energy (PLB 2007).

> The author thanks Carmen L. Sigüenza for useful discussions and the members of the theater group of Medellín, Spain, who allowed me to do this work in nice scientific–artistic working atmosphere. This work was supported by MCYT under Research Project No. FIS2005-01181.

Review until 2018



5. Closed timelike curves!!

(translation: “time travel”)

- In fact, a great variety of solutions to the Einstein Field Equations (EFEs) containing CTC exist, but, two particularly notorious features seem to stand out:

1. Solutions with a tipping over of the light cones due to a rotation about a cylindrically symmetric axis;

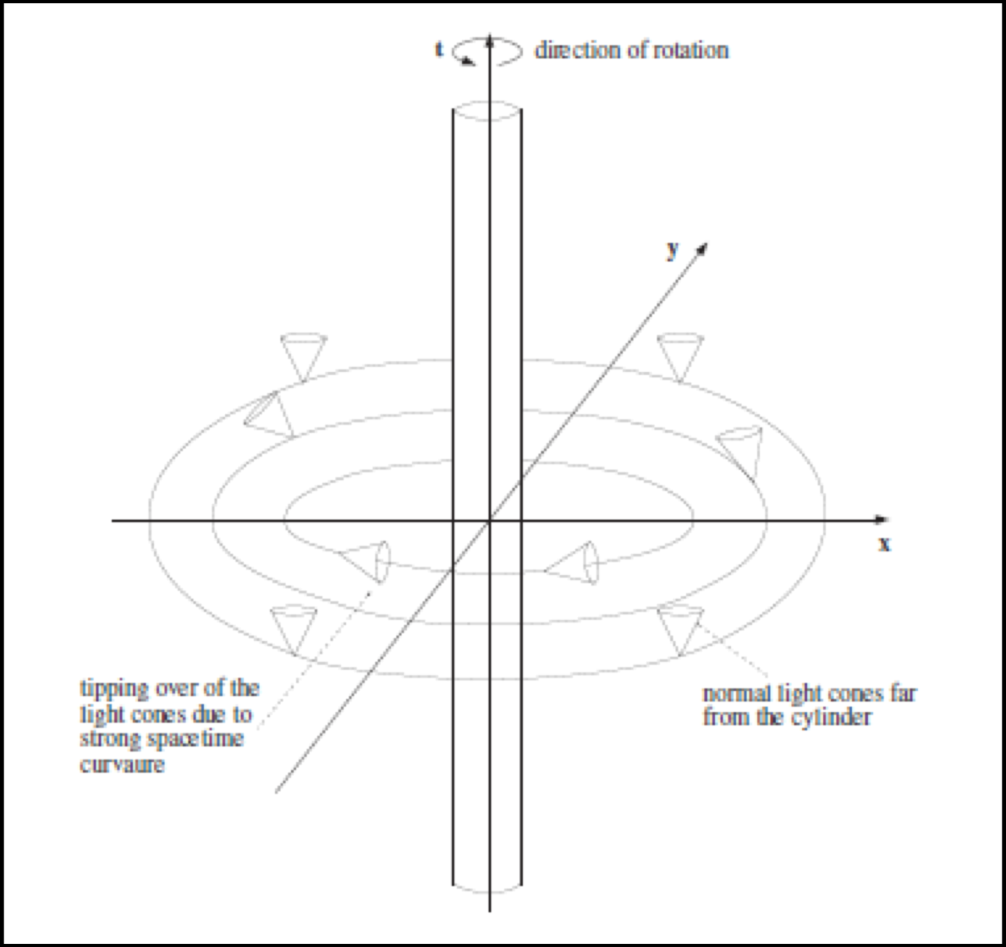
2. Solutions that violate the Energy Conditions of GR.

5.1. Rotating solutions

Van Stockum solution:

Rotating infinite dust cylinder

(tipping over of the light curves)



5.2. Wormholes induce closed timelike curves!!

Wormholes: Induce a timeshift between both mouths

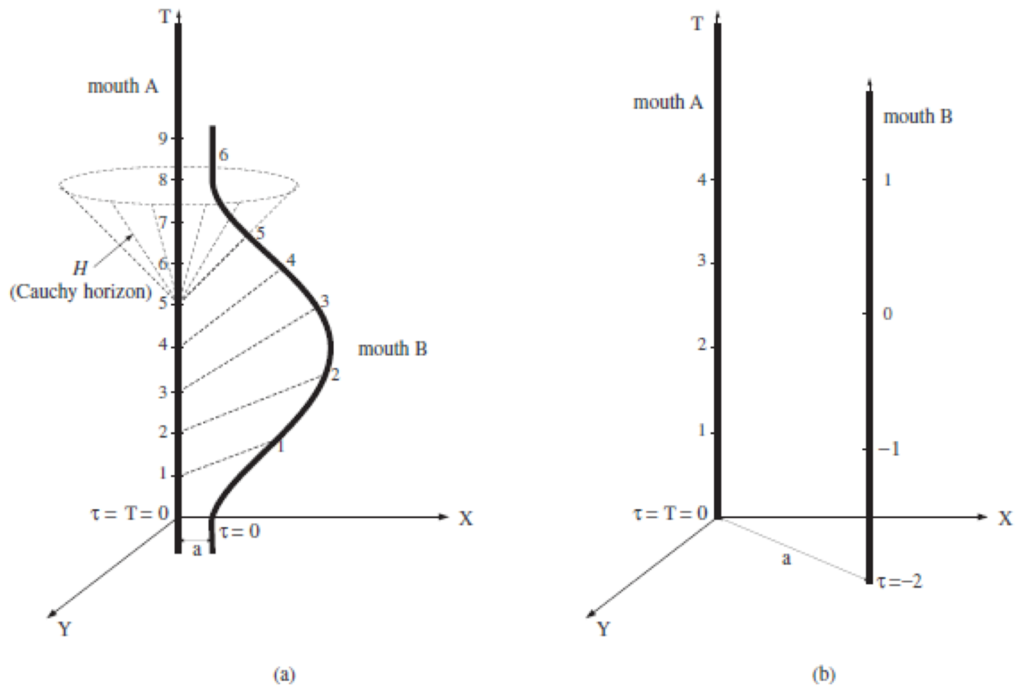


FIG. 12: Depicted are two examples of wormhole spacetimes with closed timelike curves. The wormholes tunnels are arbitrarily short, and its two mouths move along two world tubes depicted as thick lines in the figure. Proper time τ at the wormhole throat is marked off, and note that identical values are the same event as seen through the wormhole handle. In Figure (a), mouth A remains at rest, while mouth B accelerates from A at a high velocity, then returns to its starting point at rest. A time shift is induced between both mouths, due to the time dilation effects of special relativity. The light cone-like hypersurface H shown is a Cauchy horizon. Through every event to the future of H there exist CTCs, and on the other hand there are no CTCs to the past of H. In Figure (b), a time shift between both mouths is induced by placing mouth B in strong gravitational field. See text for details.

5.3. Paradoxes: Closed timelike curves!!

Opens Pandora's box and produces time travel paradoxes:

1. Causality violation (Grandfather paradox).

2. Causal loops.

Some conclusions

- **Lorentzian wormholes and *warp drives* are certainly speculative physics – there is zero(!) direct experimental evidence to support their existence**

- **Considerations on the energy conditions (much posterior work on energy conditions, averaged energy conditions, quantum inequalities, “volume integral quantifier”, semi-classical energy conditions, etc)**

- **Gell-Mann: “Everything not forbidden is compulsory”**
- **However, wormholes and *warp drive* spacetimes are primarily useful as a 'gedanken experiment' to explore the limitations of GR!**

- **Forces one to ponder seriously on the nature of time (manipulations of wormholes and warp drives induce closed timelike curves)**

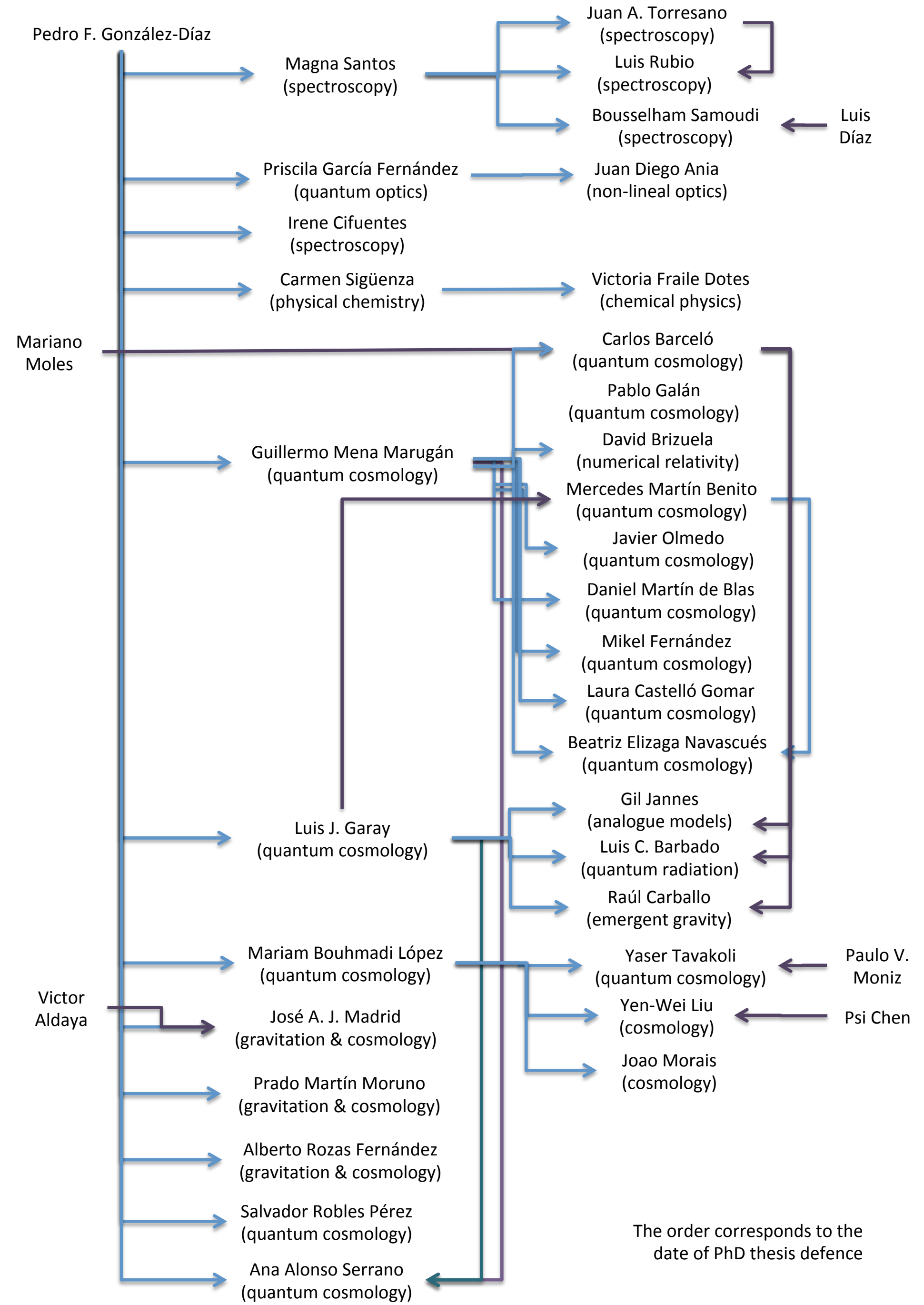
- **Fun physics!**

- **Grateful to Pedro for his unconventional style that provided so many insights into this exciting field of research!!**

Prospero:

“...   We are such stuff
As dreams are made ***on****; and our little life*
Is rounded with a sleep.”

Gracias por todo, Pedro



Thank you
for your time
and attention!

Prospero:

"Our revels now are ended. These our actors,
As I foretold you, were all spirits, and
Are melted into air, into thin air:
And like the baseless fabric of this vision,
The cloud-capp'd tow'rs, the gorgeous palaces,
The solemn temples, the great globe itself,
Yea, all which it inherit, shall dissolve,
And, like this insubstantial pageant faded,
*Leave not a rack behind. **We are such stuff***
As dreams are made on; and our little life
***Is rounded with a sleep**."*

The Tempest Act 4, scene 1, 148–158
Shakespeare

Prospero:

“Our revels now are ended. These our actors,
As I foretold you, were all spirits, and
Are melted into air, into thin air:...”

Prospero:

"...

And like the baseless fabric of this vision,...

The solemn temples, the great globe itself,

Yea, all which it inherit, shall dissolve..."

Prospero:

“…

And, like this insubstantial pageant faded,

Leave not a rack behind …”

Pedro's contribution: exploring the "event horizon"

Consider the 2-dimensional case:

$$ds^2 = -(1 - v^2 f^2)dt^2 - 2vf\,dz\,dt + dz^2$$

using the transformation:

$$dz = dr + v_b\,dt,$$

$$ds^2 = -A(r)\left[dt - \frac{v_b(1 - f(r))}{A(r)}\,dr\right]^2 + \frac{dr^2}{A(r)}$$

with $A(r) = 1 - v_b^2\,[1 - f(r)]^2$ and $d\tau = dt - \dfrac{v_b\,[1 - f(r)]}{A(r)}\,dr$

and

$$ds^2 = -A(r)\,d\tau^2 + \frac{dr^2}{A(r)}\,.$$