Can we see other universes?

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Travelling through Pedro's universes
Faculty of Physics – UCM, 3-5 Dec. 2018

(Yes, I can)

The model

The model

- A homogeneous and isotropic spacetime

$$ds^2 = -dt^2 + h_{ij}(t,\vec{x})\, dx^i dx^j \;,\;\; \varphi(t,\vec{x})$$

$h_{ij}(t,\vec{x}) \approx a^2(t)\delta_{ij}$ + small perturbations , $\varphi(t,\vec{x}) \approx \varphi_0(t)$ + small perturbations

$a(t)$, scale factor

$\varphi_0(t)$, homogeneous mode of the scalar field

- The field equations

$$\ddot{\varphi}_0 + 3\frac{\dot{a}}{a}\dot{\varphi}_0 + \frac{\partial V}{\partial \varphi_0} = 0$$

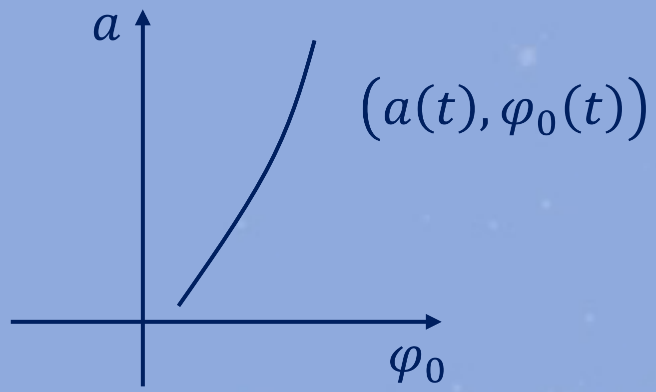
$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{k}{a^2} = \frac{1}{2}\dot{\varphi}_0^2 + V(\varphi_0)$$

$\Rightarrow$

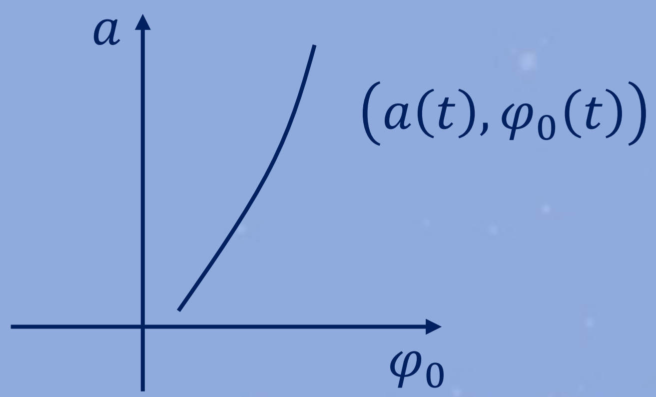
$a(t)$

$\varphi_0(t)$

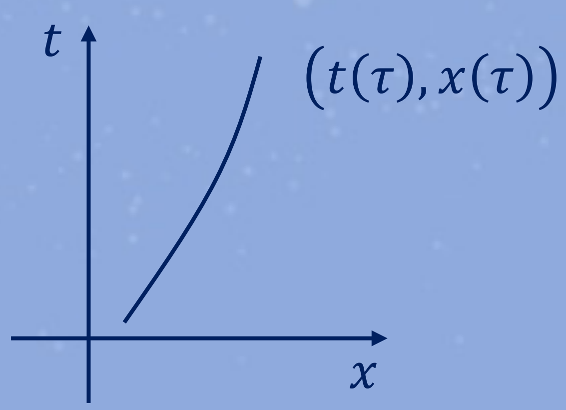
The minisuperspace

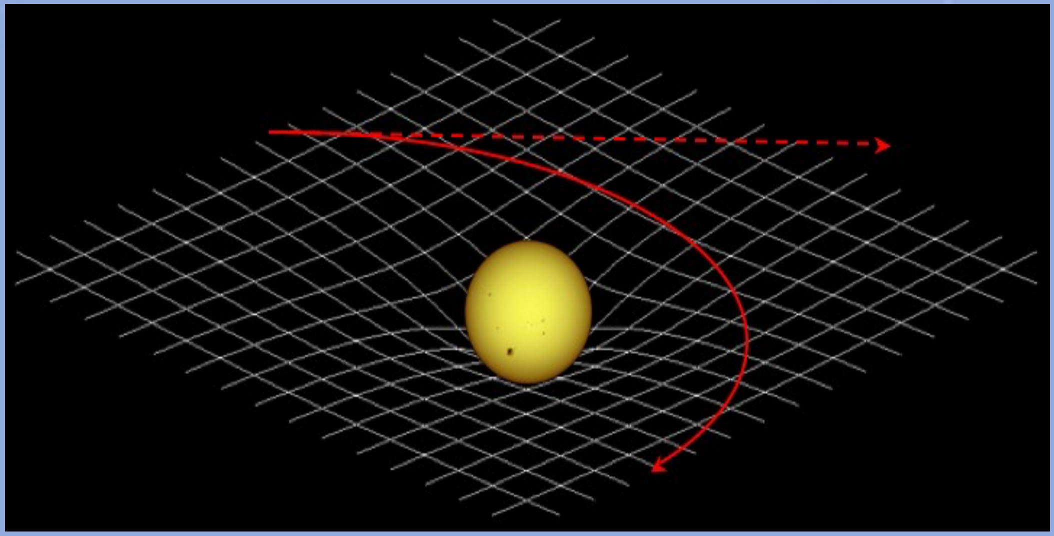


The minisuperspace

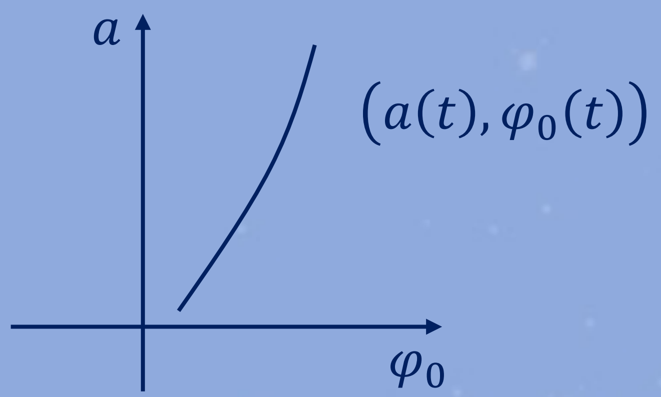


- spacetime

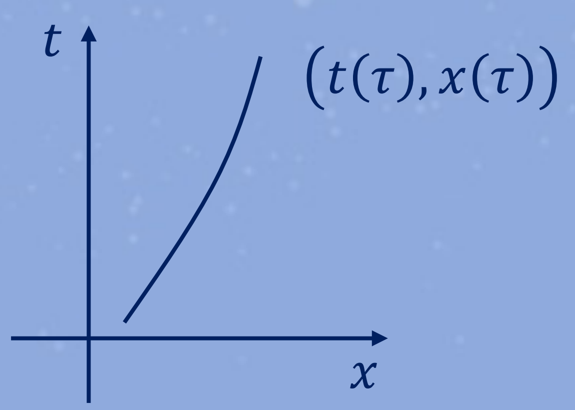




The minisuperspace



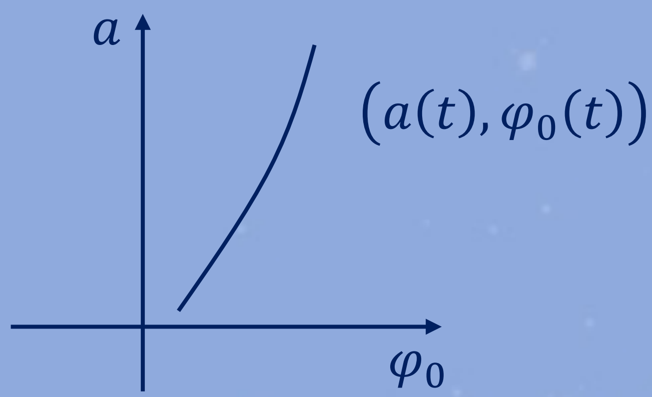
- spacetime



$$S = \frac{1}{2}\int d\tau \left(g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} - m^2 \right)$$

$$\delta S = 0 \;\Rightarrow\; \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0$$

The minisuperspace

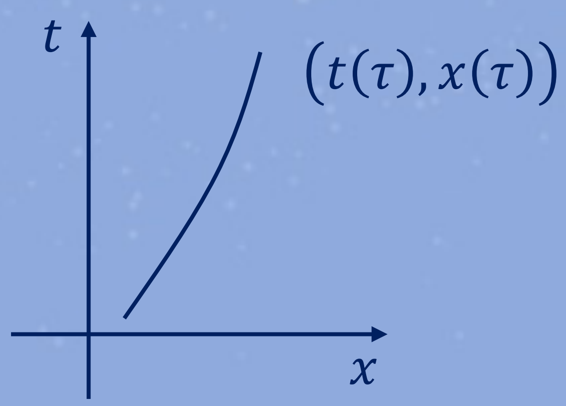


$$S = \int dt N \left(\frac{1}{2N^2} G_{AB} \frac{dq^A}{dt} \frac{dq^B}{dt} - V(q) \right)$$

$$q^A = \{a, \varphi_0\}\,,\ G_{AB} = \begin{pmatrix} -a & 0 \\ 0 & a^3 \end{pmatrix}$$

$$ds^2 = -a\, da^2 + a^3\, d\varphi_0^2$$

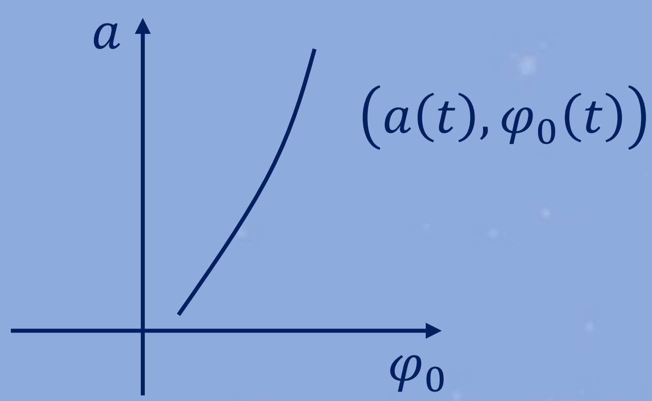
- spacetime



$$S = \frac{1}{2} \int d\tau \left(g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} - m^2 \right)$$

$$\delta S = 0 \Rightarrow \frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0$$

The minisuperspace



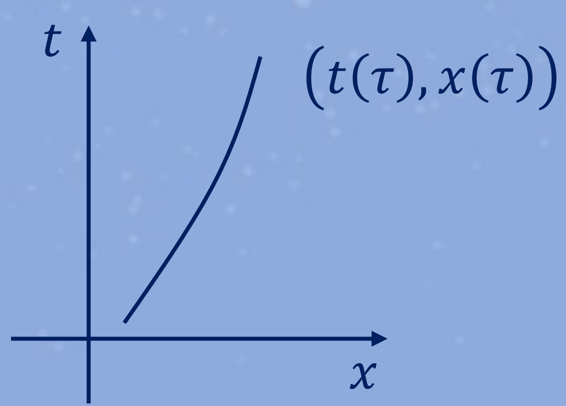
$$S = \int dtN\left(\frac{1}{2N^2}G_{AB}\frac{dq^A}{dt}\frac{dq^B}{dt} - V(q)\right)$$

$$\frac{d^2q^A}{dt^2} + \Gamma^A_{BC}\frac{dx^B}{dt}\frac{dx^C}{dt} = -G^{AB}\frac{\partial V}{\partial q^B}$$

$$\ddot{\varphi}_0 + 3\frac{\dot{a}}{a}\dot{\varphi}_0 + \frac{\partial V}{\partial \varphi_0} = 0\,,\ \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{2a^2} + \frac{1}{2a^2} = -3\left(\frac{1}{2}\dot{\varphi}_0 - V\right)$$

field equations

- spacetime



$$S = \frac{1}{2}\int d\tau\left(g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} - m^2\right)$$

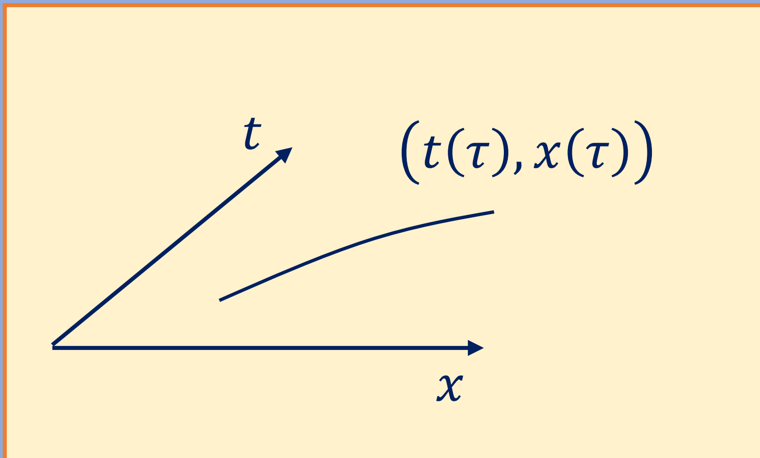
$$\delta S = 0 \quad \Rightarrow \quad \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0$$

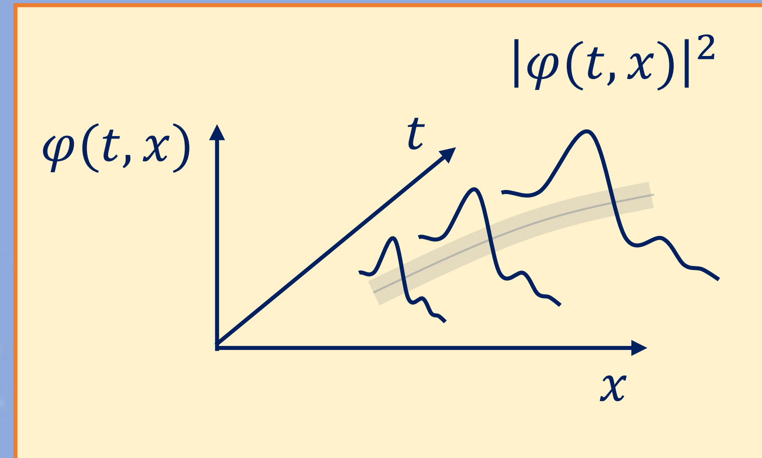
Minisuperspace vs. spacetime

spacetime	minisuperspace
$x^\mu = (t, \vec{x})$	$q^A = (a, \vec{\varphi})$
$S = \int d\tau \left(\frac{1}{2} g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} - m^2 \right)$	$S = \int dt \left(\frac{1}{2} G_{AB} \frac{dq^A}{dt} \frac{dq^B}{dt} - V(q) \right)$
$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & \ddots \end{pmatrix}$	$G_{AB} = \begin{pmatrix} -a & 0 & 0 \\ 0 & a^3 & 0 \\ 0 & 0 & \ddots \end{pmatrix}$
$ds^2 = -\,dt^2 + a^2(t)\, d\vec{x}^2$	$ds^2 = -a\, da^2 + a^3\, d\varphi_0^2$
Trajectory of a test particle $(t(\tau), \vec{x}(\tau))$	Evolution of the universe $(a(t), \vec{\varphi}(t))$

“Quantum cosmology in the light of quantum mechanics”

Quantum cosmology in the light of quantum mechanics





$$p_\mu \rightarrow -i\hbar \frac{\partial}{\partial x^\mu}$$

$$g^{\mu\nu} p_\mu p_\nu + m^2 = 0 \longrightarrow \ddot{\varphi} + \frac{3\dot{a}}{a}\dot{\varphi} - \frac{1}{a^2}\Delta\varphi + \frac{m^2}{\hbar^2}\varphi = 0$$

Momentum constraint

Under canonical quantisation

Klein-Gordon eq. (w. $\hbar^2$)

$$\varphi(t,x) = \int d\mu \left(e^{-ikx} \varphi_k(t)\, \hat{a}_k + e^{ikx} \varphi_k^*(t) \hat{a}_k^\dagger \right)$$

Quantum cosmology in the light of quantum mechanics

$$\varphi(t,x) = \int d\mu \left(e^{-ikx} \varphi_k(t)\, \hat{a}_k + e^{ikx} \varphi_k^*(t) \hat{b}_{-k}^\dagger \right)$$

$$[\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta_{kk'}$$

$$[\hat{b}_k, \hat{b}_{k'}^\dagger] = \delta_{kk'}$$

$$[\hat{a}_k^{(\dagger)}, \hat{b}_{k'}^{(\dagger)}] = 0$$

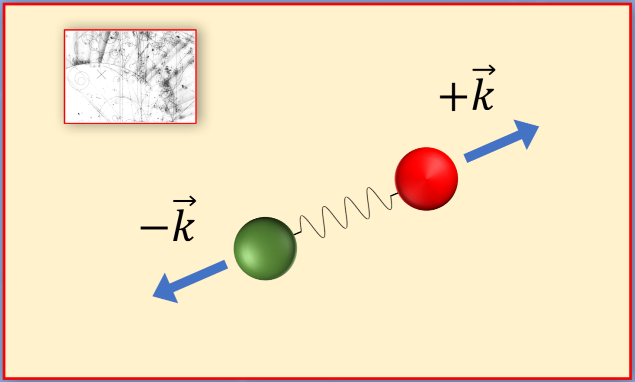
$\vdots \quad \vdots$

$|2_k\rangle$

$|1_k\rangle$

$|0_k\rangle$

Pair production



WKB solutions

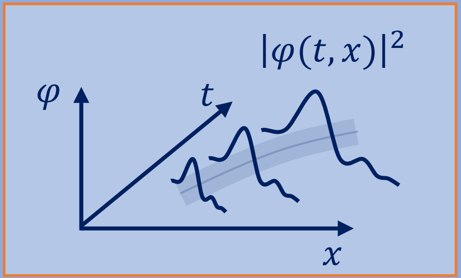
$$\varphi_k(t) = \frac{1}{\sqrt{a^3 \omega}} e^{\pm \frac{i}{\hbar} S(t)} \psi(t,x)$$

I. Garay & RP, 2018

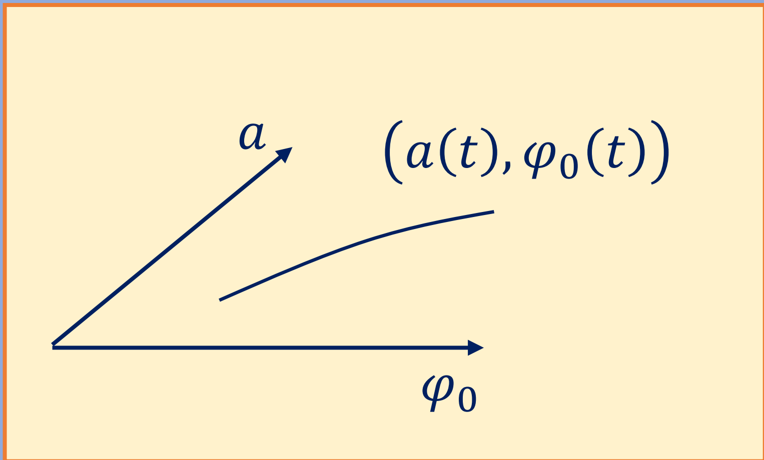
$\hbar^0$ — **geodesics**

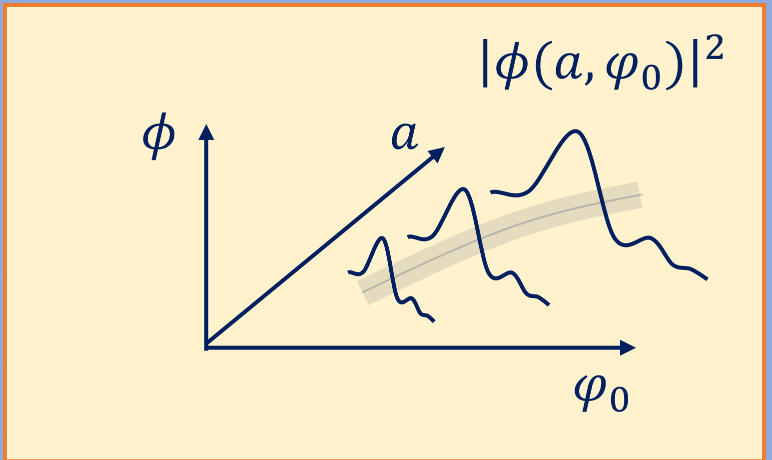
$\hbar^1$ — **Schrödinger eq.**

Particle following a classical geodesic with some uncertainty in the position



Quantum cosmology in the light of quantum mechanics





Momentum constraint

$$G^{AB} p_A p_B + V(q) = 0$$

$$p_A \to -i\hbar \frac{\partial}{\partial q^A}$$

Wheeler-De Witt eq.

$$\frac{-\hbar^2}{\sqrt{-G}} \partial_A\left(\sqrt{-G}\, G^{AB} \partial_B\right)\phi + V(q)\phi = 0$$

WKB

$$\phi(a, \varphi_0) = \frac{e^{-\frac{i}{\hbar} S(a,\varphi_0)}}{C(a, \varphi_0)} \psi(a, \varphi_0; x_k)$$

- Friedman eq.: a, φ_0
- Schrödinger eq.: x_k

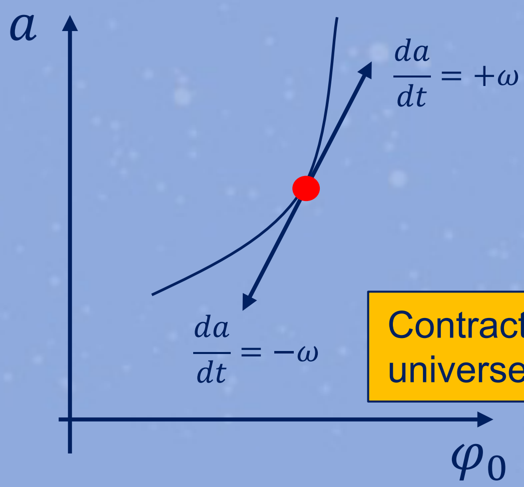
Quantum cosmology in the light of quantum mechanics

$$\varphi(a,\varphi_0) = \int d\mu \left(e^{-ik\varphi_0}\varphi_k(a)\,\hat{c}_k + e^{ik\varphi_0}\varphi_k^*(a)\hat{d}_{-k}^\dagger\right)$$

$$\left[\hat{c}_k, \hat{c}_{k'}^\dagger\right] = \delta_{kk'}, \left[\hat{d}_k, \hat{d}_{k'}^\dagger\right] = \delta_{kk'}, \left[\hat{c}_k^{(\dagger)}, \hat{d}_{k'}^{(\dagger)}\right] = 0$$

Expanding universe

Pair production



Contracting universe

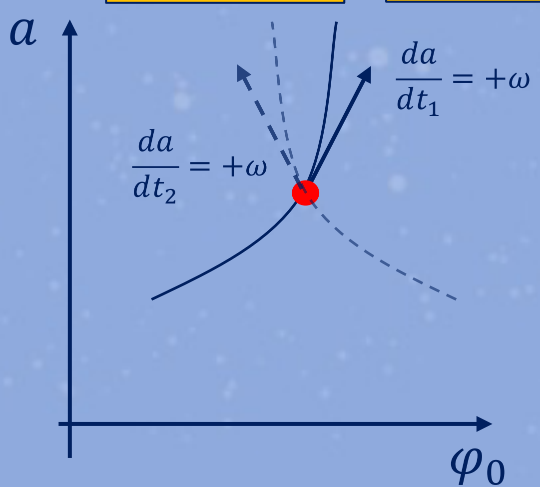
Quantum cosmology in the light of quantum mechanics

$$\varphi(a,\varphi_0) = \int d\mu \left(e^{-ik\varphi_0}\varphi_k(a)\,\hat{c}_k + e^{ik\varphi_0}\varphi_k^*(a)\hat{d}_{-k}^\dagger\right)$$

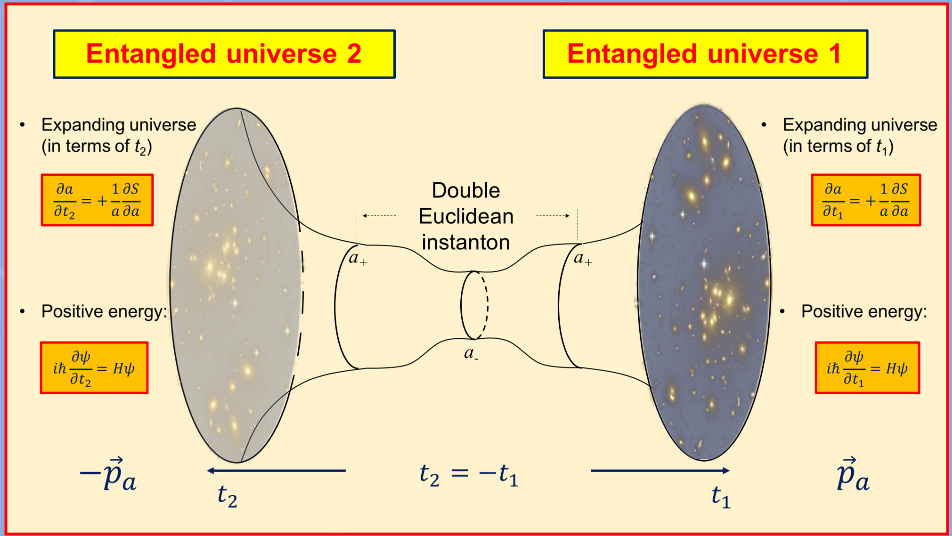
$$\left[\hat{c}_k, \hat{c}_{k'}^\dagger\right] = \delta_{kk'}, \left[\hat{d}_k, \hat{d}_{k'}^\dagger\right] = \delta_{kk'}, \left[\hat{c}_k^{(\dagger)}, \hat{d}_{k'}^{(\dagger)}\right] = 0$$

Expanding universe 2

Expanding universe 1



Pair production



Ok, but….
can we see other universes?

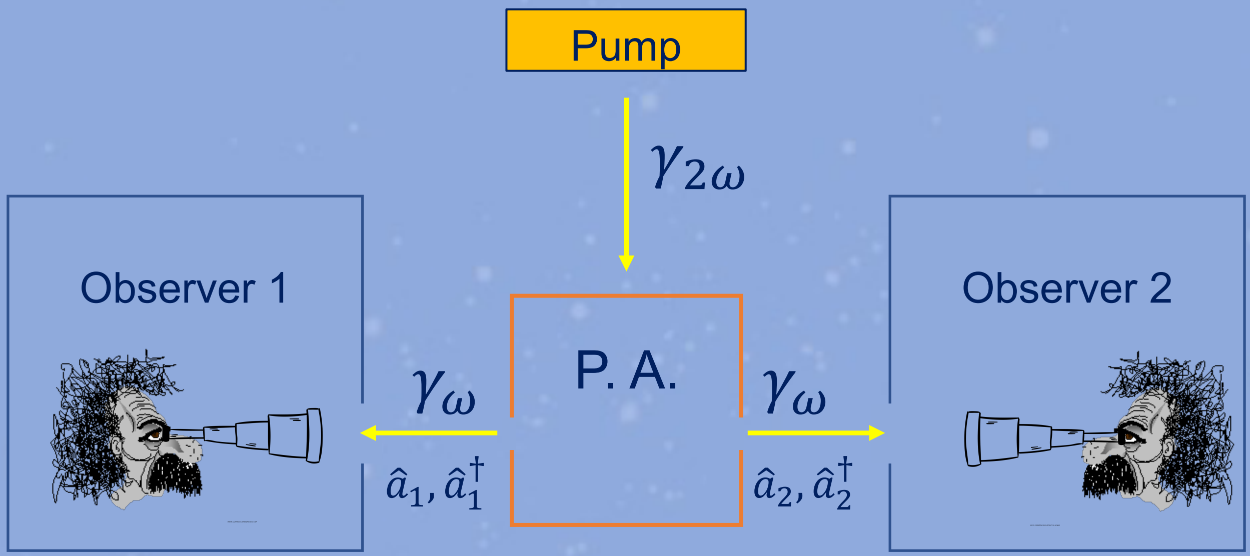
Quantum entanglement

$\{|\uparrow\rangle_1, |\downarrow\rangle_1\}$ $\{|\uparrow\rangle_2, |\downarrow\rangle_2\}$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 \pm |\downarrow\rangle_1|\uparrow\rangle_2)$$

Quantum entanglement

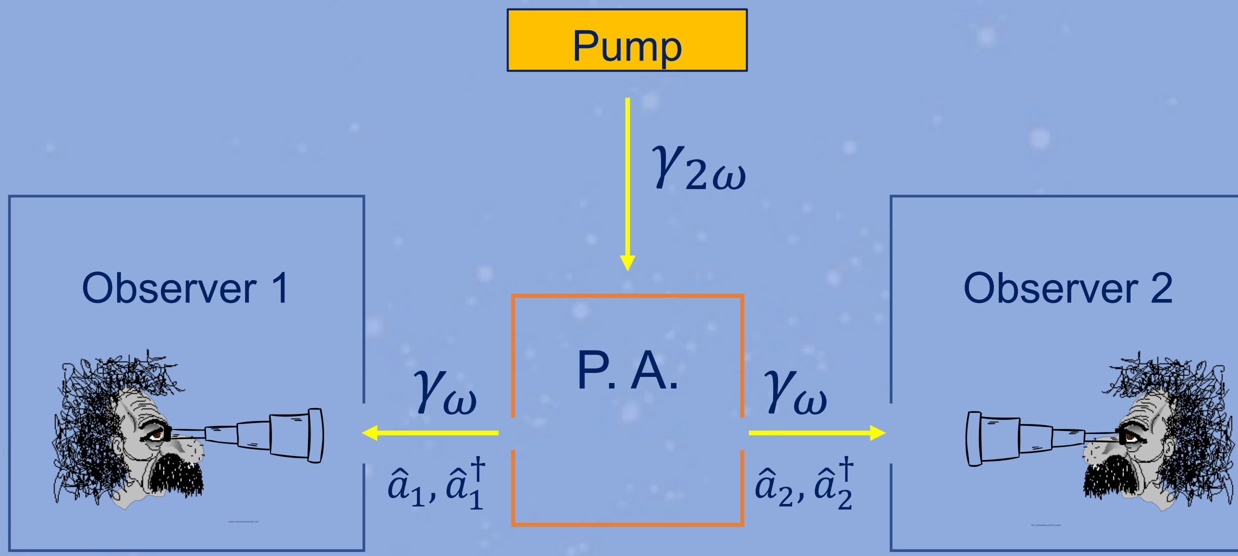
- The parametric amplifier of quantum optics



$$\hat{a}_1(t) = \hat{a}_1 \cosh \chi t + \hat{a}_2^\dagger \sinh \chi t, \; \hat{a}_2(t) = \hat{a}_2 \cosh \chi t + \hat{a}_1^\dagger \sinh \chi t$$

Quantum entanglement

- The parametric amplifier of quantum optics



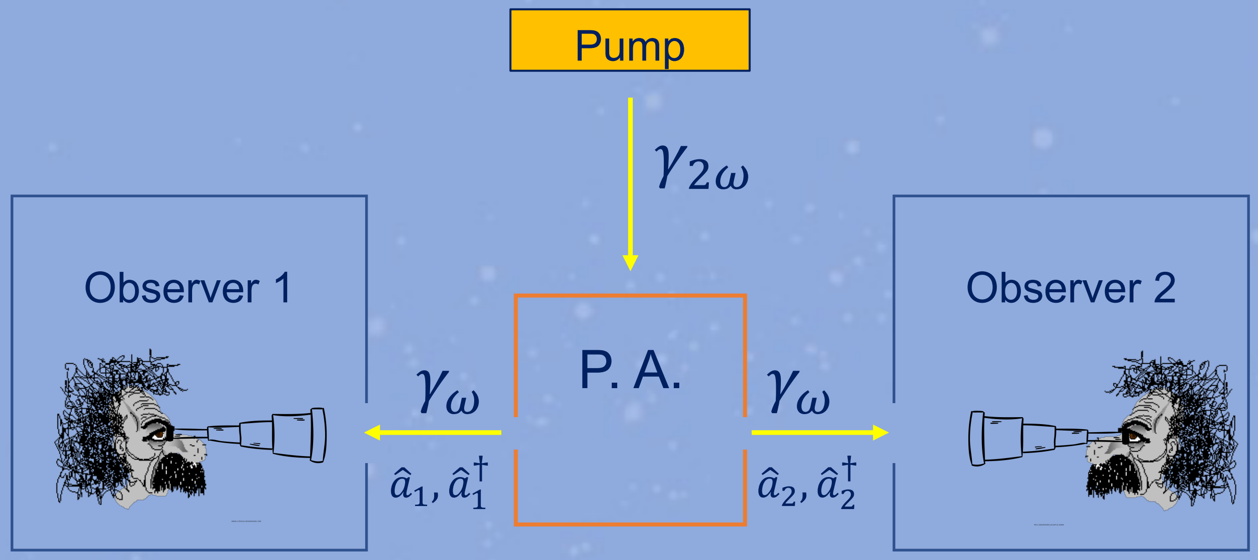
$$\hat{a}_1(t) = \hat{a}_1 \cosh \chi t + \hat{a}_2^\dagger \sinh \chi t, \ \hat{a}_2(t) = \hat{a}_2 \cosh \chi t + \hat{a}_1^\dagger \sinh \chi t$$

$$|\psi(t)\rangle = e^{\chi t\left(\hat{a}_1^\dagger \hat{a}_2^\dagger - \hat{a}_1 \hat{a}_2\right)} \underline{|0_1\rangle|0_2\rangle} = \frac{1}{\cosh \chi t} \sum_n (\tanh \chi t)^n \underline{|n_1\rangle|n_2\rangle}$$

Initial two mode vacuum state → Perfectly correlated number states

Quantum entanglement

❑ The parametric amplifier of quantum optics



$$T(t) = \frac{1}{\log \tanh^{-2} \chi t}$$

If entanglement is present

$$\rho_1 = \mathrm{Tr}_2\{|\psi(t)\rangle\langle\psi(t)|\} = \frac{1}{Z}\sum_n e^{-\frac{1}{T}\left(n+\frac{1}{2}\right)}|n_1\rangle\langle n_1|$$

If no entanglement is present

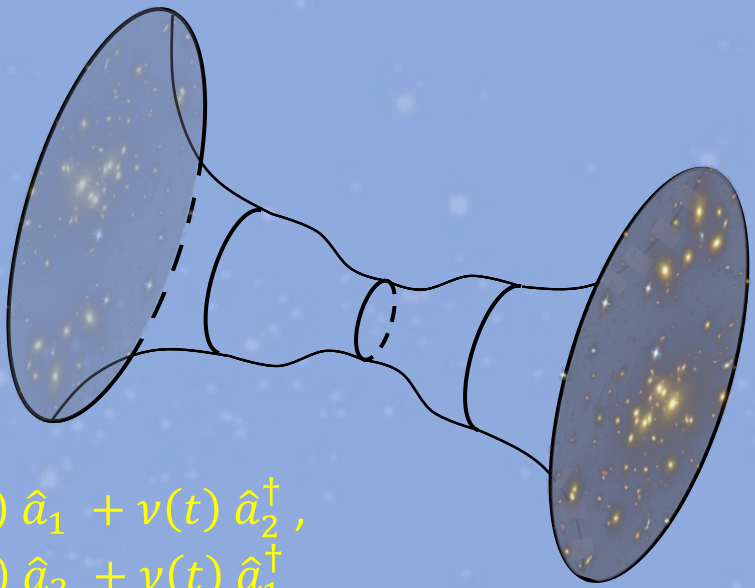
$$\rho_1 = |0_1\rangle\langle 0_1|$$

observers can infer the existence of the partner mode from the observed quasi-thermal state

The entangled universe

Cosmological perturbations in the entangled inflationary universe

Phys. Rev. D 97, 066018, 2018



The expansion of the universes mixes the modes of their matter fields

$$\hat{a}_1(t) = \mu(t)\,\hat{a}_1 + \nu(t)\,\hat{a}_2^\dagger\,,$$
$$\hat{a}_2(t) = \mu(t)\,\hat{a}_2 + \nu(t)\,\hat{a}_1^\dagger$$

If entanglement is present

$$\rho_1 = \mathrm{Tr}_2\{|\psi(t)\rangle\langle\psi(t)|\} = \frac{1}{Z}\sum_n e^{-\frac{1}{T}\left(n+\frac{1}{2}\right)}|n_1\rangle\langle n_1|$$

If no entanglement is present

$$\rho_1 = |0_1\rangle\langle 0_1|$$

$$T_{ent}^{-1}(t) = \log\frac{|\mu|^2}{|\nu|^2}$$

The choice of one or the other initial state would have consequences in the computed power spectrum of the CMB

Entangled and interacting universes

- P. F. González-Díaz et *al.*: Models of the multiverse
  - Class. Quant. Grav. 24, F41, (2007)
  - Phys. Lett. B 683, 1 (2010)
  - Phys. Rev. D 81, 083529 (2010)
- A. Alonso-Serrano et *al.*: Interactions between universes:
  - Phys. Lett. B 01, 013 (2013)
  - Phys. Lett. B 05, 091 (2016)
- S. Robles-Pérez et *al.*: Inter-universal entanglement models:
  - Phys. Rev. D 95, 083505 (2017)
  - Phys. Rev. D 96, 063511 (2017)
  - Phys. Rev. D 97, 066018 (2018)
- M. Bouhmadi-López et *al.*: Observational effects of interactions between universes:
  - Eur. Phys. J. C 77, 718 (2017)
  - Eur. Phys. J. C. 78, 240 (2018)
- Other works and works in progress…

Entangled and interacting universes

- The multiverse is a scientific proposal: testable and therefore falseable.

- It may be one of the major revolutionary ideas in cosmology since Copernicus

- It opens the door to a new wide variety of cosmic phenomena to be explored.