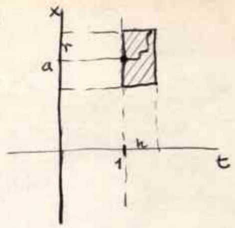
$\boxed{x' = \frac{2tx - x^2}{t^2}}$ $f(t,x)$ y $\frac{\partial f}{\partial x}(t,x)$ son continuas en todo $\mathbb{R}^2$ salvo en la recta $t=0$. Por tanto para cualquier punto $(1,a)$ existe un rectángulo $[1,1+h]\times[a-r,a+r]$ en el que f y $\frac{\partial f}{\partial x}$ son continuas (podemos tomar como h y como r cualquier par de reales positivos). El teorema de existencia y unicidad garantiza automáticamente que hay una única solución definida en un entorno a la derecha de $t=1$.



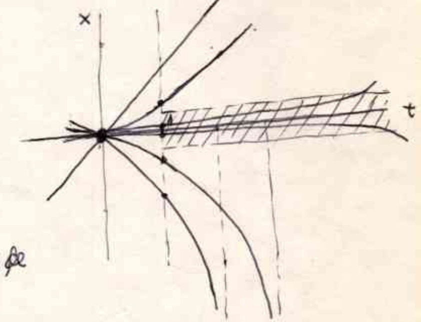
Para resolver se puede considerar como ecuación homogénea o de Bernoulli:

$x' = 2\left(\frac{x}{t}\right) - \left(\frac{x}{t}\right)^2$; $z = x/t$; $tz' + z = 2z - z^2$; $\frac{dt}{t} = \frac{dz}{z(1-z)} = \frac{1}{z} + \frac{1}{1-z}$; $c + \ln t = \ln \frac{z}{1-z}$;

$ct = \frac{x/t}{1 - x/t}$; $x = \frac{ct^2}{1+ct} = \frac{t^2}{t+c^*}$

$-\frac{x'}{x^2} = -\frac{2}{tx} + \frac{1}{t^2}$; $z = \frac{1}{x}$; $z' = -\frac{2z}{t} + \frac{1}{t^2}$; $z = ct^{-2} + t^{-2}\int \frac{t^2}{t^2}dt = \frac{C+t}{t^2}$; $x = \frac{t^2}{t+C}$

S: $x(1) = -1$; $\boxed{x = \frac{t^2}{t-2}}$



La solución con $x(1)=0$, que es $x(t)\equiv 0$ no es estable. La forma más sencilla de probarlo: cualquier solución (salvo $x\equiv 0$)

$x = \frac{t^2}{t+C} \xrightarrow[t\to\infty]{} \infty$ luego es imposible, sean [redacted] los que sean los valores iniciales de $x^*(t)$, que $|x^*(t) - 0| < \varepsilon$ para $t \geq 1$.

(además, se comprueba fácilmente que las soluciones que parten de $(1,a)$ con $a<0$ ni siquiera están definidas en $[1,\infty)$ ya que tienen una asíntota vertical).

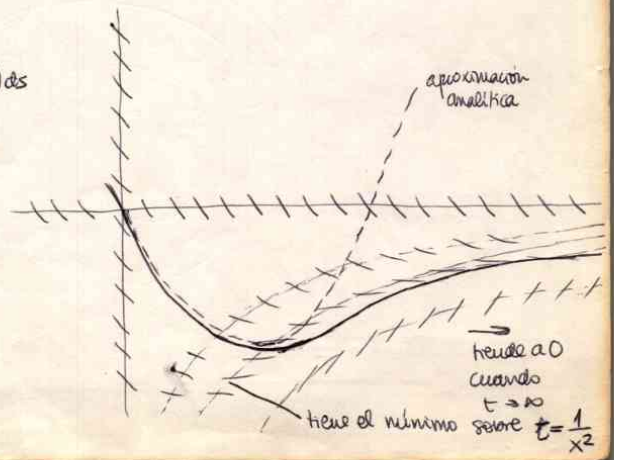
$\boxed{x' = tx^2 - 1}$

$x_{n+1}(t) = 0 + \int_0^t (s(x_n(s))^2 - 1)\,ds$

$x_1(t) = -t$

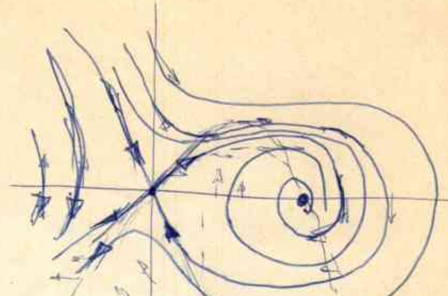
$x_2(t) = \frac{t^4}{4} - t$

Dibujo aproximado



$\boxed{x'' + x' - 2x + ax^2 = 0}$

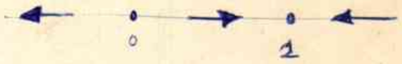
$\begin{cases} x' = v \\ v' = 2x - ax^2 - v \end{cases}$ $A = \begin{pmatrix} 0 & 1 \\ 2-2ax & -1 \end{pmatrix}$



P. sing: $\begin{cases} v = 0 \\ x = \frac{2}{a},\ x = 0 \end{cases}$ $(0,0)$ y $(\frac{2}{a},0)$ si $a\neq 0$

$A_{(0,0)} = \begin{pmatrix} 0 & 1 \\ 2 & -1 \end{pmatrix}$ $\lambda^2+\lambda-2=0$, $\lambda = \frac{-1\pm 3}{2} = 1, -2$ punto silla

$A_{(\frac{2}{a},0)} = \begin{pmatrix} 0 & 1 \\ -2 & -1 \end{pmatrix}$ $\lambda^2+\lambda+2=0$, $\lambda = \frac{-1\pm\sqrt{-7}}{2}$ foco estable



$\boxed{a=1}$ $(0,0)$, $(2,0)$ → Autovectores: $(1,1)$ est. $(1,-2)$ inest.

$\bar{V} = \begin{pmatrix} v \\ 2x - x^2 - v \end{pmatrix}$ $(x,0) \to \begin{pmatrix} 0 \\ 2x - x^2 \end{pmatrix}$

$(0,v) \to \begin{pmatrix} v \\ -v \end{pmatrix}$

$(x, 2x - x^2) \to \begin{pmatrix} 2x - x^2 \\ 0 \end{pmatrix}$

$H(x,v) = cte$ $\bar{V} = (\ )$

$2e^t + e^{-2t}$

$\boxed{a=0}$ $x'' + x' - 2x = 0$; sol general: $c_1e^t + c_2e^{-2t}$; $X_0 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$x' = \begin{pmatrix} 0 & 1 \\ 2 & -1 \end{pmatrix} x$; $\lambda = 1, -2 \to \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Sol. general: $x = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^t + c_2\begin{pmatrix} 1 \\ -2 \end{pmatrix}e^{-2t} = \begin{pmatrix} c_1e^t + c_2e^{-2t} \\ c_1e^t - 2c_2e^{-2t} \end{pmatrix}$

Laplace: $P^{-1} = \begin{pmatrix} -2 & -1 \\ -1 & 1 \end{pmatrix}\left(-\frac{1}{3}\right)$ $X = \underbrace{\begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}}_{P}\begin{pmatrix} e^t & \\ & e^{-2t} \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} e^t - e^{-2t} \\ e^t + 2e^{-2t} \end{pmatrix}$

$s^2X - 2s + 1 + sX - 2 - 2X = 0$; $(s^2+s-2)X = 2s+1$ General: $= c_1s + c_2$;

$X = \frac{2s+1}{s^2+s-2} = \frac{2s+1}{(s+2)(s-1)} = \frac{1}{s-1} + \frac{1}{s+2}$ $e^t + e^{-2t}$

$\boxed{(1-t)y''+ty'-y=e^{t}(1-t)^2}$ $\quad s\bar{y}'' - s(1+s)\bar{y}' + s\bar{y} = 0 \quad \lambda^2-\lambda-\lambda=0$

$y=\sum_0 c_k t^k \;;\; y'=\sum_1 k c_k t^{k-1} \;,\; y''=\sum_2 k(k-1)c_k t^{k-2}$

$\sum_2 k(k-1)c_k t^{k-2} - k(k-1)c_k t^{k-1} + \sum_1 k c_k t^k + \sum_0 -c_k t^k = 0$

$[2c_2-c_0] + [6c_3 - 2c_2 + c_1 - c_1]t + \cdots + [(k+2)(k+1)c_{k+2} - (k+1)k\,c_{k+1} + k c_k - c_k]t^k$

$c_2 = \frac{c_0}{2}$

$6c_3 = 2c_2 \;;\; c_3 = \frac{c_2}{3} = \frac{c_0}{6} \;;\; c_4 = \frac{3.2\,c_3 - c_2}{4.3} = \frac{c_0-\frac{c_0}{2}}{4.3} = \frac{c_0}{4!}$

$c_{k+2} = \frac{(k+1)k\,c_{k+1} - (k-1)c_k}{(k+2)(k+1)} = \cancel{\frac{kc_{k+1}}{k+2}} \cancel{\frac{(k-1)c_k}{(k+2)(k+1)}}$

Supongamos $c_n = \frac{c_0}{n!}$ $\;n=0\ldots k+1$ entonces $c_{k+2} = \frac{\frac{c_0}{(k-1)!} - \frac{(k-1)c_0}{k!}}{(k+2)(k+1)} = \frac{\frac{c_0}{k!}}{(k+2)(k+1)}$

$y=tu \;;\; y'=u+tu' \;;\; y''=2u'+tu''$

$2u'+tu''-2tu'-t^2u''+t^2u' = 0 \qquad = e^t(1-t)^2$

$u''(t-t^2) = u'(-t^2+2t-2) \;;\; u''=u'\left(1-\frac{2}{t}-\frac{1}{1-t}\right) + \left(\frac{1-t}{t}\right)e^t$

$\frac{u''}{u'} = \frac{-t^2+2t-2}{-t^2+t} = \boxed{1+\frac{t-2}{-t^2+t}} = 1-\frac{2}{t}-\frac{1}{1-t}$ $\quad \frac{e^t(1-t)}{2}$

$u' = \frac{c\,e^t(1-t)}{t^2} + \frac{e^t(1-t)}{t^2}\int\left(\frac{t^2(1-t)}{(1-t)t}\right)dt$ $\quad \frac{t^2}{2}$

$\ln u' = t + \ln t^{-2} + \ln(1-t) + C$

$u' = \frac{c\,e^t(1-t)}{t^2} \;;\; y = \left[c_1\int\left(\frac{e^t}{t^2}-\frac{e^t}{t}\right)dt + c_2\right]t = \left[c_1\frac{e^t}{t}+c_2\right]t =$

$-\frac{e^t}{t} + \int \cancel{\frac{e^t}{t}} \cancel{\frac{e^t}{t}}$ $\qquad \boxed{c_1e^t+c_2t}$

$y = c_1e^t + c_2t\left(1-\frac{t}{2}\right)$

$\frac{1}{2}\{e^t - te^t\} = \frac{1}{2}(e^t - te^t + e^t) = e^t\left(1-\frac{t}{2}\right)$

$W(t) = \begin{pmatrix} e^t & t \\ e^t & 1 \end{pmatrix} \;,\; |W|(t) = e^t(1-t)$

$W^{-1}(t) = \frac{1}{e^t(1-t)}\begin{pmatrix} 1 & -t \\ -e^t & e^t \end{pmatrix} \;;$

$t\int \frac{e^t(1-t)}{e^t(1-t)}e^t - e^t\int\frac{e^t(1-t)}{e^t(1-t)}t$

$-\frac{t^2}{2}e^t + te^t$

$y_p = W(t)\int_0^t \frac{1}{e^s(1-s)}\begin{pmatrix} -s \\ e^s \end{pmatrix} = \begin{pmatrix} e^t & t \\ e^t & 1 \end{pmatrix}\begin{pmatrix} -\frac{t^2}{2} \\ e^t-1 \end{pmatrix} =$

$\boxed{y_p = -\frac{e^t t^2}{2} + te^t - t}$

$y_p' = \cancel{-te^t} - \frac{e^t t^2}{2} + e^t + \cancel{te^t} - 1 = e^t\left(-\frac{t^2}{2}+1\right) - 1$

$y_p'' = -te^t - \frac{e^t t^2}{2} + e^t = e^t\left(-\frac{t^2}{2}-t+1\right)$

$-\frac{t^2}{2} - t + 1 + \frac{t^3}{2} + t^2 - t$

$e^t\left(\frac{t^3}{2}+\frac{t^2}{2}-2t+1\right) + e^t\left(-\frac{t^3}{2}+t\right) - t + \frac{e^t t^2}{2} - te^t + t$

$\underline{e^t(1-2t+t^2)} = e^t(1-t)^2$

$2e^t(1-t)^2 \quad \int_0^t \begin{pmatrix} -2se^{-2s} \\ 2e^{-s} \end{pmatrix}$

$y'' + \frac{t}{1-t}$

$\begin{pmatrix} e^{-2s}\left(s+\frac{1}{2}\right) \\ -2e^{-s} \end{pmatrix}$

$\boxed{y = c_1e^t + c_2t + te^t\left(1-\frac{t}{2}\right)}$ $\quad e^{-t}\left(t+\frac{1}{2}\right) - 2te^{-t}$

$e^{-t}\left(\frac{1}{2}-t\right)$

$-2e^{-2s}\cdot d\alpha^{-2s}$

$se^{-2s} - \int e^{-2s}$

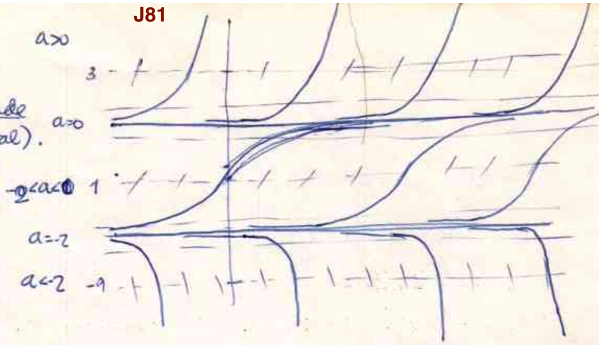
$+\frac{e^{-2s}}{2}$

J81

(A) $\boxed{x' = x^2(x+2)}$

Solución única ya que depende continuamente de los datos (trivial).

Problemas solo en prolongabilidad y estabilidad.



Si $a=0$, $-2<a<0$ ó $a=-2$ trivialmente prolongables. (teoremas)

Si $\underline{a>0}$: $x^3+2x^2 \geq x^3$; $\begin{cases} x' = x^3 \\ x(0)=a \end{cases}$ $\frac{x'}{x^3}=1$; $\frac{x^{-2}}{-2} = t+c$;

$x^2 = \frac{1}{c-2t}$; $x = \sqrt{\frac{1}{c-2t}}$

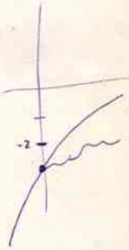
$x(0)=a$ $a = \sqrt{\frac{1}{c}}$; $c = \frac{1}{a^2}$; $x = \frac{1}{\sqrt{\frac{1}{a^2}-2t}}$ se va a ∞ en $t = \frac{1}{2a^2}$

y por tanto la nuestra también. No prolongable

Si $\underline{a<-2}$ $x^3+2x^2 \leq 2x^2$

$\begin{cases} x' = 2x^2 \\ x(0)=a \end{cases}$ $-\frac{1}{x} = 2t - c$; $x = \frac{1}{c-2t}$

$a = \frac{1}{c}$; $x = \frac{1}{\frac{1}{a}-2t}$



Se va a ∞ en $t = \frac{1}{2a}$ que está a la izquierda.

Esta es prolongable; de la nuestra no sabemos nada.

Comparamos con otra (el dibujo parece que nos da la no prolongabilidad).

Estabilidad : Si $a=0$ inestable (las soluciones con $x(0)=a>0$ se van a ∞).

Si $a=-2$ inestable (si $a>-2 \to 0$, si $a<-2 \to -\infty$).

Si $-2<a<0$ parece que es asintóticamente estable (la diferencia entre dos soluciones tiende a 0, pues las dos tienden a 0)

Si $a>0$ inestables pues ni siquiera están definidas hasta ∞.

Si $a<0$ por el dibujo parece lo mismo.

(B) $\boxed{ty''-(t+1)y'+y = t^2e^t}$

$y = e^t\int u$; $y' = e^t\int u + e^tu$; $y'' = e^t\int u + 2e^tu + e^tu'$

$te^t\int u + 2te^tu + te^tu' - \cancel{te^t\int u} - \cancel{e^t\int u} - te^tu - e^tu + \cancel{e^t\int u} = u'(te^t) - u(e^t - te^t) = t^2e^t$

$u' = (1/t - 1)u + t$; $u = te^{-t}c_1 + te^{-t}\int \frac{e^s}{s}\cdot s\,ds = c_1te^{-t} + t$

$y = e^t\left[c_1\int(te^{-t}+t) + c_2\right] = \boxed{\frac{t^2}{2}e^t + c_1(1+t) + c_2e^t}$

Comprobación: $t\left[\frac{t^2}{2}+2t+1\right]e^t - (t+1)\left[\frac{t^2}{2}+t\right]e^t + \frac{t^2}{2}e^t +$

$+ 0 - c_1(t+1)(1) + c_1(1+t) +$

$+ tc_2e^t - (t+1)c_2e^t + c_2e^t = \left[\cancel{\frac{t^3}{2}} + 2t^2 + \cancel{t} - \cancel{\frac{t^3}{2}} - \frac{3t^2}{2} - \cancel{t} + \frac{t^2}{2}\right]e^t =$

$= t^2e^t$

(C) $\boxed{\begin{matrix} x''' + 2x'' + 5x' = 5t \\ x(0)=0,\ x'(0)=0,\ x''(0)=1/5 \end{matrix}}$

$m^3+2m^2+5m=0$ $m=0$ $m=-1\pm 2i$

$x_p = t^2/2 - 2t/5$

$x = c_1 + (c_2\cos 2t + c_3 \operatorname{sen} 2t)e^{-t} + \frac{t^2}{2} - \frac{2t}{5}$

$x' = (-2c_2\operatorname{sen}2t + 2c_3\cos 2t - c_2\cos 2t - c_3\operatorname{sen}2t)e^{-t} + t - 2/5$

$x'' = ((-4c_2 - 2c_3 - 2c_3 + c_2)\cos 2t + \dots)e^{-t} + 1$

$x(0) = c_1 + c_2 = 0$
$x'(0) = 2c_3 - c_2 - 2/5 = 0$
$x''(0) = -3c_2 - 4c_3 + 1 = 1/5$

$\boxed{x = \frac{\operatorname{sen}2t\, e^{-t}}{5} + \frac{t^2}{2} - \frac{2t}{5}}$

$s^3X - 1/5 + 2s^2X + 5sX = \frac{5}{s^2}$; $X = \frac{s^2+25}{s^3(s^2+2s+5)}\cdot\frac{1}{5} = \frac{1}{5}\left[\frac{As^2(s^2+2s+5) + Bs(s^2+2s+5) + C(s^2+2s+5) + (Ds+E)s^3}{\dots}\right]$

$5C = 25$ $C=5$
$5B+2C=0$ $B=2$
$5A+2B+C=1$ $5A-4+5=1$, $A=0$
$2A+B+E=0$ $E=2$
$A+D=0$ $D=0$

$x = \frac{1}{5}L^{-1}\left[-\frac{2}{s^2} + \frac{5}{s^3} + \frac{2}{s^2+2s+5}\right] = -\frac{1}{5}L^{-1}\left[\frac{1}{s^2}\right] +$

$+ \frac{1}{2}L^{-1}\left[\frac{2}{s^3}\right] + \frac{1}{5}L^{-1}\left[\frac{2}{(s+1)^2+2^2}\right]$.

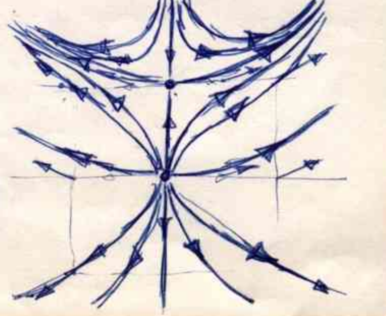
(D) $x' = 2x$
$y' = y(1-y) + x^2$

$x=0$ $y=0$ $y=1$ $(0,0)$ $(0,1)$ $\begin{pmatrix} 2 & 0 \\ 2x & 1-2y \end{pmatrix}$

$(0,0)$ $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ $\lambda = 2, 1$ punto inestable | $(0,1)$ $\begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$ $\lambda = 2, -1$ p. silla

$\lambda = 2$ $\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right\}$ $\lambda = 1$ $\left\{\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right\}$ L1 tangente | $\lambda=2$ $\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right\}$ inestable $\lambda=-1$ $\left\{\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right\}$ estable

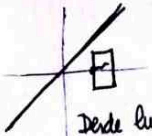
$V = \begin{pmatrix} 2x \\ y(1-y)+x^2 \end{pmatrix}$ $V(x,0) = \begin{pmatrix} 2x \\ x^2 \end{pmatrix}$ $V(x,1) = \begin{pmatrix} 2x \\ x^2 \end{pmatrix}$ $V(0,y) = \begin{pmatrix} 0 \\ y(1-y) \end{pmatrix}$ $V(1,-1) =$ $V(-1,-1) =$



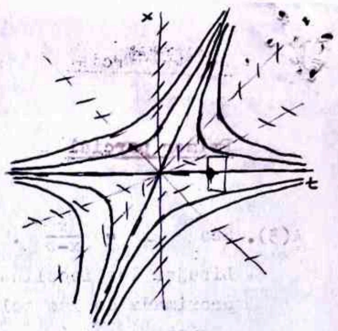
581

$\boxed{x' = \frac{x}{x-t}}$

$c=0$	$x=0$
$c=\infty$	$x=t$
$c=2$	$x=2t$
$c=1$	$t=0$
$c=-1$	$t=2x$
$c=\frac{1}{2}$	$t=-x$



Desde luego, f, $\frac{\partial f}{\partial x}$ continuas en entorno de $(1,0)$.



$(x-t)x' - x = 0$; $\frac{\partial}{\partial t}(x-t) = -1 = \frac{\partial}{\partial x}(-x)$.

$u(x,t) = \frac{x^2}{2} - xt + f(t)$

$u(x,t) = -xt + f(x)$

$\boxed{\frac{x^2}{2} - xt = C}$ solución general

(despejando $t = \frac{x}{2} - \frac{C}{x}$, o bien $x = t \pm \sqrt{t^2 + C}$)

(se podría resolver también $x - t = z$, $z' = \frac{t}{z}$, $z^2 - t^2 = C$; $x = t \pm \sqrt{t^2+C}$

o incluso considerando $\frac{dt}{dx} = \frac{x-t}{x} = -\frac{1}{x}t + 1$; $t = \frac{C}{x} + \frac{x}{2}$).

$\boxed{t^2x'' - 2tx' + 2x = \ln t}$ $x(1) = x'(1) = 1$ $\lambda^2 - \lambda - 2\lambda + 2 = 0$; $\lambda = \frac{3 \pm \sqrt{9-8}}{2} = {}^{2}_{1}$; $c_1 t + c_2 t^2$;

$x_p = \begin{pmatrix} t & t^2 \\ 1 & 2t \end{pmatrix} \int \begin{pmatrix} -\ln t/t^2 \\ \ln t/t^3 \end{pmatrix} = \begin{pmatrix} \ln t + 1 - \frac{\ln t}{2} - \frac{1}{4} \\ \end{pmatrix}$

$W = \begin{pmatrix} t & t^2 \\ 1 & 2t \end{pmatrix}$ $|W| = t^2$

$W^{-1} = \begin{pmatrix} 2/t & -1 \\ -1/t^2 & 1/t \end{pmatrix}$

$\int \ln t \, t^{-3} = -\frac{\ln t}{2} t^{-2} - \frac{1}{4} t^{-2}$

$\int \ln t \, t^{-2} = -\frac{\ln t}{t} - \frac{1}{t}$

$x = c_1 t + c_2 t^2 + \frac{\ln t}{2} + \frac{3}{4}$

$x(1) = c_1 + c_2 + \frac{3}{4} = 1$

$x'(1) = c_1 + 2c_2 + \frac{1}{2} = 1$

$c_2 = 1/4$; $c_1 = 0$

$\boxed{x = \frac{t^2}{4} + \frac{\ln t}{2} + \frac{3}{4}}$

(Siendo hábil, cambio $t = e^s$, queda (apuntes) $\frac{d^2x}{ds^2} - 3\frac{dx}{ds} + 2x = s$ de solución $c_1 e^{s} + c_2 e^{2s} + \frac{s}{2} + \frac{3}{4}$ tras probar solución particular $As + B$)

$\int_0^t 7 + 5\,\text{sen}\,t = 7t - 5\cos t - 5$, $e^{7t - 5\cos t - 5} \xrightarrow[t \to \infty]{} \infty$ es inestable.

Como $\int_0^{2\pi} 7 + 5\,\text{sen}\,t = 14\pi \neq 0 \Rightarrow$ la homogénea sólo la trivial $\Rightarrow$ la no homogénea sólo una solución 2π-p

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$\begin{cases} x' = y \\ y' = 1 - x + xy^2 + 5y \end{cases}$ único punto singular $(1,0)$; $A = \begin{pmatrix} 0 & 1 \\ -1+y^2 & 2xy+5 \end{pmatrix}_{(1,0)} = \begin{pmatrix} 0 & 1 \\ -1 & 5 \end{pmatrix}$, $\lambda^2 - 5\lambda + 1 = 0$

$\lambda = \frac{5 \pm \sqrt{25-4}}{2}$ nodo 2 tangentes inestable

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$\frac{1}{s^2+4s} + \frac{1}{s^2+4s+8} = \frac{1}{4}\frac{1}{s} - \frac{1}{4}\cdot\frac{1}{s+4} + \frac{1}{(s+2)^2+2^2}$; $L^{-1} = \boxed{\frac{1}{4} - \frac{e^{-4t}}{4} + \frac{e^{-2t}\,\text{sen}\,2t}{2}}$.

$\frac{1}{s^2+4s} = \frac{A}{s} + \frac{B}{s+4} = \frac{1}{4}\cdot\frac{1}{s} - \frac{1}{4}\frac{1}{s+4}$

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0 es p.r. de $t^2y'' + 2ty' + 4ty = 0$ pues 1, 2 y $4t$ son analíticas en 0. No lo es de $t^2y'' + t\cdot\frac{2}{t}y' + 4y = 0$.

$\lambda^2 - \lambda + 2\lambda + 0 = 0$, $\lambda = -1, 0$.